Lund University
Electrical and Information Technology

# Exam in
# Information Theory, EIT080

## August 25, 2010, kl 14-19

- Write your name on each paper.
- Start a new solution on a new sheet of paper. Use only one side of the paper.
- Solutions should clearly show the line of reasoning.
- Aid:
  - Course book.
  - Handed out material.
  - Lecture slides and articles from the home page.
  - Calculator.

## Good luck!

## Problem 1

The joint probability for the random variables $X$ and $Y$ is given by the following table

| $P(X,Y)$ | | $Y=0$ | $Y=1$ | $Y=2$ |
|---|---|---|---|---|
| $X$ | 0 | $1/8$ | $0$ | $1/8$ |
| | 1 | $1/4$ | $1/4$ | $1/4$ |

Calculate
(a) $H(X)$ and $H(Y)$
(b) $H(X|Y)$ and $H(Y|X)$
(c) $H(X,Y)$ and $I(X;Y)$

(3+4+3=10p)

## Problem 2

Consider a random variable $X$ taking the values according to the following table (also showing the corresponding probabilities)

| $x \in X$ | $P(x)$ |
|---|---|
| $a$ | 0.2 |
| $b$ | 0.2 |
| $c$ | 0.3 |
| $d$ | 0.3 |

(a) Construct an optimal source code for the variable.
(b) How many different optimal codes can be constructed?
(c) Are all optimal codes Huffman codes? If not, give an example of an optimum non-Huffman code.

(10p)

## Problem 3

Consider a summation channel, see Figure 3.1, where the output is

$$Y = X + Z \mod 64.$$

The input is a six bit unsigned integer, $X \in \{0, 1, \dots, 63\}$ and the disturbance $Z \in \{1, 5, 17, 26\}$ with equal probability. Calculate the capacity of the channel.

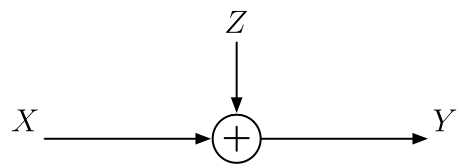


**Figure 3.1:** A summation channel.

(10p)

## Problem 4

Young Bob is going to the playground for the whole day. There are three different attractions; a swing, a rocking horse and a climbing net. At each time instant he will stay at the current attraction with probability $1/2$. If he will change attraction and he is currently on the swing or the rocking horse he chooses one of the other two with equal probability. But if he is changing from the climbing net he always chooses the rocking horse.

(a) What is the average time he will stay at an attraction?

(b) When Bob's mother comes to pick him up in the evening, what is the probability she will find him on the swing?

(c) At dinner Bob's father asks him what he has done during the day. What is the minimum number of bits required per time instant to describe his day?

(3+3+4=10p)

## Problem 5

Given $n$ independent probabilities, show that the binary entropy of the average probability is greater than the average of the binry entropy function of the individual probabilities. That is, show that

$$h\Big(\frac{p_1 + p_2 + \cdots + p_n}{n}\Big) \geqslant \frac{h(p_1) + h(p_2) + \cdots + h(p_n)}{n}$$

where $h(p)$ is the binary entropy function.

(10p)