Lund University
Electrical and Information Technology

Exam in
Information theory, EIT080

24 August, 2009, 14.00-19.00

- Write name and programme on all sheets.
- Start a new solution on a new sheet of paper. Use only one side of the paper
- Solutions should clearly show the line of reasoning.
- Aid:
  - Course book.
  - Handed out material.
  - Lecture slides and articles from the course web site.
  - Calculator.

Good luck!

Problem 1

A binary information source produces the symbols 0 and 1 with probabilities $P(X = 0) = \frac{3}{5}$ and $P(X = 1) = \frac{2}{5}$. The sequence from the source is split in blocks with three bits each. this sequence of three tuples should be compressed. In Table 1.1 a suggested encoding rule is shown.

U	W
000	0
001	100
010	110
011	11100

U	W
100	101
101	11101
110	11110
111	11111

Table 1.1: Table with a suggested encoding rule.

Is the suggested code optimally chosen for the source? If not, construct an optimal code. Also, derive the difference in average codeword lengths for the two encoders, and compare with the entropy of the source.

(10p)

Problem 2

A communication system is built with the (4,3) code in Table 2.1. The code sequence is to be transmitted over a BSC. The source messages sent over the syetm can be assumed to be equally likely.

w	$x(w)$
1	001
2	010
3	100
4	111

Table 2.1: Specification of the channel encoder..

(a) Specify an ML decoder for the code.
(b) How many different ML decoders are possible?
(c) Derive the block error probability P_{Be}.
(d) What is the block error probability without coding?

(3+2+3+2=10p)

Problem 3

Consider the following experiment. We have two coins, one true and one counterfeit. The true coin has equal probability for heads and tails, whereas the counterfeit has the probabilities $P(\text{heads}) = 3/4$. Choose randomly, with equal probability, one of the coins and toss it twice. How much information do we get about the identity of the coin from the total number of heads in the tosses?

(10p)

Problem 4

In Figure 4.1 a graph for a Markov source is shown. Derive the stationary distribution of the states and the entropy rate for the process, $H_\infty(X)$.

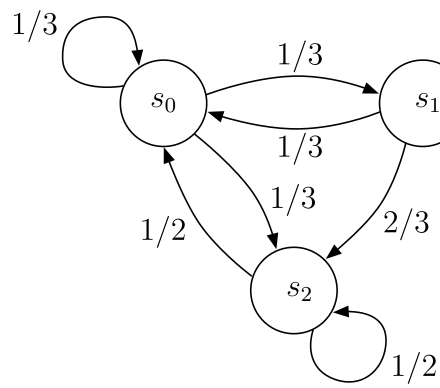

Figure 4.1: A graph for a Markov process.

(10p)

Problem 5

When constructing a receiver for a binary transmission system the weighting has become uneven. The resulting discrete channel therefore gets a skew distribution of the error probabilities, $P(y=1|x=0)=\alpha$ and $P(y=0|x=1)=\beta$, see Figure 5.1.

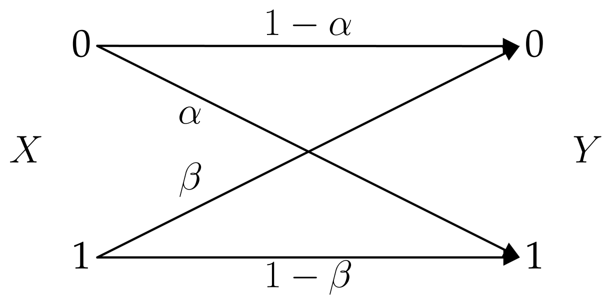

Figure 5.1: A binary asymmetric channel.

(a) Show that the mutual information for the channel is

$$I(X;Y) = h\Big(P_0(1-\alpha-\beta)+\beta\Big) - P_0 h(\alpha) - (1-P_0)h(\beta)$$

where $P_0 = P(X=0)$.

(b) Show that the result in (a) can be used to derive the channel capacity as

$$C = I(X;Y)\Big|_{P_0=\frac{1}{1-\alpha-\beta}\left(\frac{1}{1+2^A}-\beta\right)}$$

where

$$A = \frac{h(\alpha)-h(\beta)}{1-\alpha-\beta}$$

(Hint: $D(h(p)) = D(p)\log\left(\frac{1-p}{p}\right)$.)

(c) Verify the result in the previous sub-problems by deriving the information and capacity for the case when $\alpha=\beta=\delta$ and compare with the binary symmetric channel (BSC).

(4+4+2=10p)