# Final Matlab Problem

May 11, 2015

## 1 Background

This problem studies estimation with nuisance parameters, i.e., parameters that are unknown, but not of interest. The problem is taken from reciprocity calibration for Massive MIMO systems (but is heavily simplified). After lots of incomprehensible hardware-oriented arguments the following system model appears

$$\begin{aligned} y_{12} &= h_{12}x_1 + n_{12} \\ y_{21} &= h_{12}x_2 + n_{21} \\ y_{13} &= h_{13}x_1 + n_{13} \\ y_{31} &= h_{13}x_3 + n_{31} \\ y_{23} &= h_{23}x_2 + n_{13} \\ y_{32} &= h_{23}x_3 + n_{31} \end{aligned} \tag{1}$$

where $n_{ij}$ are independent complex Gaussian noise variables (variance $N_0$), the variables $h_{ij}$ are unknown but not of interest. Our task is to estimate $x_2$ and $x_3$ since it is known that $x_1 = 1$ (for the purpose of this assignmenet, $x_1$ can be though of as a training symbol).

We next put forth a generalized method-of-moments method to estimate $x_1$ and $x_2$. It is holds, e.g., that $y_{12}x_2 - y_{21}x_1 = n_{12}x_2 - n_{21}x_1$, and that this is zero-mean, we have (see lecture slides 6, slide 73-74) found a function $g(\cdot)$ that fulfills the requirements for the generalized methods-of-momemts. It follows that we can perform the estimation as

$$(\hat{x}_2, \hat{x}_3) = \arg\min_{x_1, x_2} \sum_{i=1}^{2} \sum_{j=i+1}^{3} |y_{ij}x_j - y_{ji}x_i|^2,$$

where $x_1 = 1$ by definition. This results in a quadratic form which can be easily solved (see rules for taking vector valued gradients at page 521 in the book).

## 2 Tasks

- Task 1. Assume that all variables $h_{ij}$ are complex Gaussians with unit variance and zero mean. Find the CRLB as a function of $x_2$ and $x_3$. Simplify as much as you can. Hint: use Section 3.9.

- Task 2. Select a few points of $x_1$ and $x_2$ and investigate the performance of the generalized method of moments compared with the CRLB. Show plots of the MSE against $1/N_0$.
- Task 3. Do a few tests and state whether you believe that the generalized method-of-moments is unbiased or not.
- Task 4. In this case we no longer assume that the variables $h_i j$ are complex Gaussians. Instead we let them be random phasors with length 1 and uniform phase, i.e., $h_{ij} \sim \exp(\jmath 2\pi\phi_{ij})$ where $\phi_{ij} \sim U(0, 1]$. Select some values of $x_2$ and $x_3$ and compute the CRLB. Hint: Use Monte Carlo simulations to integrate away the dependency on $\{h_{ij}\}$.
- Task 5. Compare CRLB with generalized method-of-moments for this case.