Hand-in assignement for problem set 3

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1 Background

This problem continues the frequency offset estimation problem that we dealt with before. We will now do maximum likelihood estimation on a slightly more advanced model (but still not one that realistically models OFDM).

2 Problem formulation

Consider a zero mean Gaussian $N \times 1$ vector $\mathbf{s}$ with covariance matrix $\mathbf{I}$. Let $D_\ell(\epsilon)$ denote an $N \times N$ diagonal matrix where the kth diagonal entry equals $\exp(-2\pi\jmath\epsilon(\ell\Delta + k/N))$, where $0 \leq k \leq N-1$. In the 4G "'mobile phone"' system, the value of Δ happens to be 3.21 for some reason. Let $\mathbf{Q}$ denote the $N \times N$ IFFT matrix, and let $\mathbf{P}_0$ and $\vec{P}_1$ denote two diagonal matrices where all diagonal elements are independent random variables of the form $\exp(\jmath 2\pi u)$ where $u \sim U[0,1]$. The receiver knows both $\mathbf{P}_0$ and $\mathbf{P}_1$. Now, the receiver is given two (vector-valued) signals and has the task to produce an estimate of ϵ. The two signals are

$$\mathbf{y}_0 = D_0(\epsilon)\mathbf{Q}\mathbf{P}_0\mathbf{s} + \mathbf{n}_0$$

and

$$\mathbf{y}_1 = D_1(\epsilon)\mathbf{Q}\mathbf{P}_0\mathbf{s} + \mathbf{n}_1$$

where $\mathbf{n}_0$ and $\mathbf{n}_1$ are two independent white Gaussian noise vectors with variance σ^2 per element.

3 Tasks

- Task 1. For N=64, find the CRLB. Check if it is dependent on ϵ in this case?
- Task 2. Implement an MLE for the frequency offset. Compare its performance with that of the CRLB (think about what plots to present in order to "compare the performance with CRLB"). There are several things to consider:
    1. In the MLE, how should you generate the offset in order to compare with CRLB?
    2. Is the MLE unbiased? Can we get this information from a theorem in the book, or do we need to simulate and check?

3. How should we setup a simulation in order to verify unbiasedness?
4. If it is not unbiased, what does a comparison with the CRLB really give?

- Assume that the experiment is repeated M times, i.e., we keep ϵ fixed M times, but generate new $\mathbf{s}$, $\mathbf{P}_1$, $\mathbf{P}_2$, and noise vectors. Implement the MLE. Compare with CRLB and comment on the results. Make connections to results in the book.
- Now assume a uniform PDF for the offset. Implement the MAP estimator. Compare with CRLB and consider the same issues as for the MLE.
- Implement the MMSE estimator. Hint, you must (most likely) normalize the posterior numerically. Compare the mse-performance to MLE and MAP.
- Suppose that the OFDM system will work well if $|\epsilon - \hat{\epsilon}| > .025$ but breaks down otherwise. Find the estimator that maximizes the chance that the system will work.