Hand-in assignement for chapter 3

March 3, 2015

1 Background

This problem deals with Cramer-Rao lower bound evaluations for frequency offset estimation of OFDM. In order to solve this problem, it is not needed to know what "'frequency offset"' or "'OFDM"' is, but I point out that OFDM is the modulation method of both WIFI and 4G systems (mobile phones), and that frequency offset (FO) is, e.g., an offset of the oscillator frequencies of the receiver and the transmitter. Further, for a reader with OFDM experience: The OFDM setup I use here is so overly simplified that you will hardly recognize it as OFDM.

In OFDM, estimation of the FO is critical[1]. In this problem we will derive the CRLB for the offset estimation. The main reason why I choose the offset estimation problem is that the weaknesses of the CRLB are well illuminated for this example.

2 Problem formulation

Consider a zero mean Gaussian $N \times 1$ vector $\mathbf{s}$ with covariance matrix $\mathbf{I}$. Let $D_\ell(\epsilon)$ denote an $N \times N$ diagonal matrix where the kth diagonal entry equals $\exp(-2\pi\jmath\epsilon(\ell\Delta + k/N))$, where $0 \leq k \leq N-1$. In the 4G "'mobile phone"' system, the value of Δ happens to be 3.21 for some reason. Now, the receiver is given two (vector-valued) signals and has the task to produce an estimate of ϵ. The two signals are

$$\mathbf{y}_0 = D_0(\epsilon)\mathbf{s} + \mathbf{n}_0$$

and

$$\mathbf{y}_1 = D_1(\epsilon)\mathbf{s} + \mathbf{n}_1$$

where $\mathbf{n}_0$ and $\mathbf{n}_1$ are two independent white Gaussian noise vectors with variance σ^2 per element.

3 Tasks

- Task 1. For N=64 and $\sigma^2 = 1$, find the CRLB. Hint: If you cannot solve the expectation in closed form, you can always do it by means of matlab simulations.

[1]For those of you who are interested, the main reason why it is so badly needed can be neatly seen in Figure 6 of "'Coded Orthogonal Frequency Division Multiplex"' by Le Floch, Alard, and Berrou.

- Task 2. Based on the differential of the log-likelihood, does it seem plausible that an MVU estimator exists?
- Task 3. Generate a pair of received signals $\mathbf{y}_0$ and $\mathbf{y}_1$. Now plot the log-likelihood function $\log p(\mathbf{y}_0, \mathbf{y}_1; \epsilon)$ for $-0.5 \leq \epsilon \leq 0.5$. From our discussions of the limitations of the CRLB, do you expect the CRLB to be a tight bound?
- Task 4. For $\epsilon = 0$, illustrate the CRLB as a function of the SNR.
- Task 5. Assume that the noise density σ^2 is also unknown. Find the CRLB for N=64. Illustrate the results using a 3D surface in Matlab. Hint: Use numerical evaluation of expectations and differentials.

4 Continuation

Once we have studied Maximum likelihood estimation, we will get back to this problem.