Exam in Digital Signal Processing, EITF75

Wednesday October 31

1. **Write clearly!** If I cannot read what you write, I **will consider it as not written at all**. My decision on this matter is final, you cannot argue that I should have been able to read it later.
2. It is important to **show the intermediate steps** in arriving at an answer, otherwise you may lose points.
3. When generating problems of the True/False form, I use Matlabs random number generator.
4. Providing two answers to a problem, where one of them is wrong, will result in points being deducted. Same holds for side-comments if you make side-comments that are not correct, points may be deducted. Same goes for writing too much about a problem. If you write down everything that you know, with the goal that at least something must be correct, points may be deducted for everything that is wrong.
5. Problems are not arranged in an order of ascending difficulty.
6. Allowed tools: Pocket calculator, Course book, Lecture slides, printed versions of Nedo's slides.
7. Answer the exam in English or Swedish

Problem 1 (1.0p)

We have "an LTE circuit" which we know implements a difference equation having at most 2 poles and at most 2 zeros. We give an input signal $x(n)$ to the circuit, turn on the circuit at time $n = 0$, and investigate the output signal.

a)

Suppose that $x(n) = 0$, for all n. The output signal is, however, $y(n) = (-1)^n$, $n \geq 0$. Provide as much information about the circuit as you can.

b)

Based on the information provided in a), is it possible to determine the exact input signal $x(n)$ for which the steady state output would become $y(n) = 0$? (With "exact", is meant that it should not be given in terms of some unknown variables).

c)

We apply the input signal $x(n) = u(n)$ (step function). The output signal is, for large values of n, $y(n) = 0$. What information about the circuit does this provide?

d)

We again apply the input signal $x(n) = u(n)$ (step function), but now observes the output signal $y(n)$ at all n. Surprisingly, we find that $y(n) = 0$ for all n. What information about the circuit does this provide?

Problem 2 (1.0p)

Assume an LTI system at rest (zero-state) with system equation

$$H(z) = \frac{2 + z^{-1} + 0.5z^{-2}}{1 + 0.5z^{-1} - 0.25z^{-2}}.$$

Assume that the output signal is

$$y(n) = [(0.5)^n + (-0.5)^n]\, u(n) + (0.5)^{n-1}\, u(n-1).$$

What is the input signal $x(n)$?

Problem 3 (1.0p)

Assume a two-user communication system in which two users are sending signals $x_1(t)$ and $x_2(t)$ to a receiving device. We further assume a noise-free universe. The receiving device is sampling its input signal $x(t) = x_1(t) + x_2(t)$ at a sampling frequency of F_s Hz.

a)

The two users are said to be "separable" if two time-discrete ideal filters $\mathcal{F}_1$ and $\mathcal{F}_2$ can be found such that the output of $\mathcal{F}_1$ is sufficient to reconstruct $x_1(t)$ perfectly - possibly with further processing necessary - without any interference from $x_2(t)$, and similarly for $\mathcal{F}_2$. Suppose that

$$\begin{cases} X_1(F) > 0, & 2000 \leq |F| \leq 4000 \\ X_1(F) = 0, & \text{otherwise} \end{cases}$$

$$\begin{cases} X_2(F) > 0, & 6000 \leq |F| \leq 8000 \\ X_2(F) = 0, & \text{otherwise} \\ X_2(7000+\delta) = X_2(7000-\delta) & 0 \leq \delta \leq 1000 \end{cases}$$

What is the smallest possible value for F_s such that the two users are separable?

b)

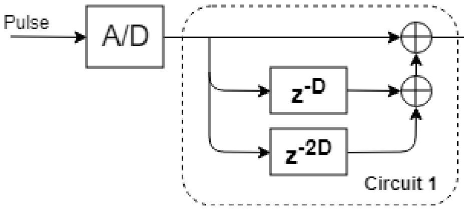
Assume now that $F_s = 20000$ Hz, and define "separability" as in a). Design, by selecting the zero-pole structure, a practical filter implementation of $\mathcal{F}_1$. (Note: Here, I am mostly interested in checking your understanding on the effect of zero-pole placement and filter characteristics - not to check whether you remember how to get a very good filter. However, your methodology for designing the filter should be clear).

c)

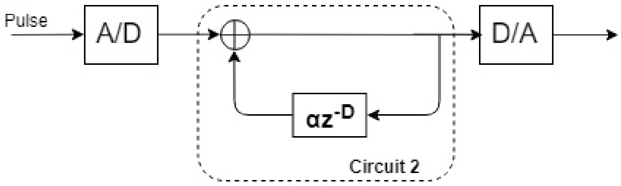
Continuation of b). Suppose now that the filter $\mathcal{F}_1$ has been ideally implemented, and that D/A reconstruction of $x_1(t)$ is desired. Unfortunately, there is no D/A converter available with such a high sampling rate as 20000 Hz. Show how succesful reconstruction can be obtained nonetheless using downsampling and a D/A converter with sampling rate < 20000 Hz.

Problem 4 (1.0)p

An acoustic company would like to implement an echo functionality to its devices. Two options are on the table, circuit I and circuit II. In circuit II, it is assumed that $\alpha > 0$. In both cases, $D = 1000$.



(a) Circuit 1: FIR filter



(b) Circuit 2: IIR filter

a)

Calculate the system functions for both cases, and compute zeros and poles.

b)

For circuit I, assume that a very short input signal $x(t)$ is used and that the first echo can be heard with a delay of 0.5 seconds. What is the sampling frequency F_s?

C)

For circuit II, provide a necessary and sufficient condition for the echo to fade out with time for a very short input signal $x(t)$.

D)

For circuit II, assume that the echo can be heard by a human as long as the power of echo is less than 20dB attenuated compared with the original signal. Provide a formula for the number of echos that one can hear for a short input signal $x(t)$.

Problem 5 (1.0)p

Assume that you have a signal $x(n)$ that satisfies $x(n) = 0$, $n < 0$ and $x(n) = 0$, $n \geq L$. Let X_k denote the N-DFT of the sequence $x(n)$, $N \geq L$.

a)

Assume that we construct $Y_k = H_k X_k$, $0 \leq k \leq N-1$. Let $y(n)$ be the IDFT of Y_k, and let $h(n)$ be the IDFT of H_k. Under what conditions is it true that

$$y(n) = x(n) * h(n).$$

b)

Assume that we construct $Y_k = X_k^2$, $0 \leq k \leq N-1$. Let $y(n)$ be the IDFT of Y_k. Under what conditions is it true that

$$y(n) = x(n) * x(n).$$

c)

Assume that $N = L = 5$, and that a device is provided with the sequence X_k, but that it does not have ability to compute an IDFT. Can the device compute the value $X(\omega_1)$ for $\omega_1 = 3\pi/5$?

d)

Explain, briefly, why the DFT is important when we can do essentially the same using a DTFT. Be concise, but precise - no essays please.

e)

Suppose that we construct $Y_k = H_k X_k$, $0 \leq k \leq N-1$ where

$$H_k = \frac{1 - \cos(\omega_0)\exp(-i2\pi k/N)}{1 - 2\alpha\cos(\omega_0)\exp(-i2\pi k/N) + \alpha^2\exp(-i4\pi k/N)}.$$

Describe the effect on $y(n)$.