Let M be the plaintext, K the key, and C the ciphertext ni some arbitrary cryptosystem.

Show that

$$0 \le H(\mathbf{M}|\mathbf{C}) \le H(K|\mathbf{C}).$$

• Give an example of a cryptosystem where

 $0 < H(\mathbf{M}|\mathbf{C}) < H(K|\mathbf{C}).$

• Give an example of a cryptosystem where

 $0 < H(\mathbf{M}|\mathbf{C}) = H(K|\mathbf{C}).$

Assume a source with output symbols A, B, C, D, where P(A) = 1/2, P(B) = 1/4, P(C) = P(D) = 1/8. Output symbols are chosen independently.

- Determine the unicity distance if the source is encrypted by a simple substitution cipher.
- To increase the unicity distance, explain how homophonic coding can be used.