Master's Thesis

IQ Imbalance Compensation: Blind versus Pilot-Based Algorithms, using Different IQ Imbalance Models

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By

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Abstract

Today, wireless communication systems try to cope with the high demands of the different applications that require more and more throughput. This trend result in developing different technologies in order to improve both the transmitter and the receiver side of the communication link. On the other hand, in order to reduce the price of the communication handsets, more cheap analog components with sub-optimal front-ends are used. To reduce the effects of these hardware impairments, more signal processing is required.

One of these hardware impairments is the IQ imbalance which result in distorting the constellation of the received signal in one or both of the I and the Q branches. The effect of the IQ imbalance on degrading the performance of the system becomes more pronounced when higher order modulation, for example 64 QAM, is used, where the constellation is more sensitive to symbol rotation and interference between the I and the Q branches.

In this thesis work, the performance of two IQ imbalance compensation algorithms are studied. One of them is a blind algorithm and the other one is a pilot-based algorithm. We also examine the performance of these algorithms under two IQ imbalance models: the Double Branch IQ imbalance Model (DBIQM), and the Single Branch IQ imbalance Model (SBIQM).

It was found that the two IQ imbalance models are related via a rotation and a scaling operations. It was also found that, since both algorithms implicitly use the SBIQM, their performance degrades significantly if the actual IQ imbalance follows the DBIQM. However, a simple modification is suggested to these algorithms that improves their performance when the IQ imbalance follow the DBIQM. In all cases, evaluating the performance is performed based on the bit-error-rate (BER), and the signal-to-distortion-ratio (SDR).

Acknowledgement

Thanks Allah, it is all your blessing…

I would like to express my deepest gratitude and appreciation to those who carried the holiest message in life, to those who paved the way of science and knowledge for us.

I would like to thank my home university, Soran University, to give me this great opportunity to study Wireless Communications at Faculty of Engineering (LTH), Lund University. I would also like to thank the Wireless Communications classmates of 2011.

I would also like to express the deepest gratitude and sincere respect to my examiner, Dr. Ove Edfors, for his supervision and professional guidance in this thesis. I am also extremely grateful to my supervisor, Dr. Ghassan Dahman, for his useful advises suggestions, comments and supervision in this thesis. I would also like to thank Michal Stala for his help.

Last and most important, I would like to express my love and deepest appreciation to my parents, my family, for their support and encouragement. I would like to thank all my dear friends and amazing people that I have met over the years especially in Lund who prayed for me.

With all my respect
Shireen Al-Majmaie
Lund, Sweden
July, 2014

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CHAPTER 1

1 Introduction

In the following sections, the purpose, aim and the outline of this thesis are explained

1.1 Purpose and aim

The purpose of this thesis work is twofold: First, to study, and find the relationship between, two models which are used to model the IQ imbalance: the Double-Branch IQ imbalance Model (DBIQM), and the Single-Branch IQ imbalance Model (SBIQM). Second, to evaluate, and compare the performance of two IQ imbalance compensation algorithms (a Blind and a Pilot Based algorithm). The comparison includes: illustrating the estimation and the compensation methods that are used by these two algorithms, and studying the effect of these two algorithms in compensating the imbalance of the received signal. The bit-error-rate (BER), and the signal-to-distortion-ratio (SDR) are used as measures to perform this comparison.

1.2 Thesis Outline

This thesis is structured as follows:

- Chapter 2 illustrates the background information about the receiver structure and the imbalance gain and phase of the local oscillator device in the receiver side.
- Chapter 3 describes two models of the IQ imbalance in the amplitude and phase for the received signal. This chapter also illustrates the relationship between these two models.

- Chapter 4 explains the two algorithms that are used to estimate and compensate the imbalance in the received signal, a Blind algorithm and a Pilot Based algorithm.
- Chapter 5 covers the comparison between the two algorithms that are described in chapter 4 and evaluates the performance of both of them. The comparison is based on the bit-error-rate (BER), and the signal-to-distortion-ration (SDR) of the compensated signal.
- Chapter 6 concludes this thesis work.

CHAPTER 2

2 Background Information

The significant development in the communication systems plays an important role in fulfilling the requirements of today's greedy wireless applications. One of the current challenges is to make the communication devices as simple and as cheap as possible, which means to develop transceivers with low cost, low power consumption, and with processing structures that can compensate for the impairments of cheap hardware [3].

In practice, one of the unavoidable problems is the misbalancing between the I component and Q component in the analog front-end [1]. This misbalancing is one result of the mixer operation when it mixes the Radio Frequency and the Local Oscillator Frequency to get Intermediate Frequency (IF) as in Figures 1.1-1.3. As a result of this misbalancing, the image signal appears on top of the required signal as an interference signal in the receiver. This image signal can be eliminated by using a Band Pass Filter before the mixer to eliminate the image frequency and another Band Pass Filter after the mixer to eliminate the unrequired frequency [2]. This operation is shown in the Figure 4, and will be explained in the next section.

However, the imbalance between the I and the Q components remains in the receiver [1], and in order to get a better performance for the system and a better image rejection, the IQ imbalance should be compensated for. Different algorithms have been proposed to compensate for this IQ imbalance. In this thesis we will focus on comparing the performance of two of them: a blind compensation algorithm, and a pilot-based compensation algorithm.

2.1 Basic Operation of the Mixer:

The Mixer is a nonlinear electrical device used to shift the signal frequency to another frequency in the spectrum and maintain the characteristics of the produced signal, like the amplitude and the phase, the same as the original signal as possible. It is used in radio frequency and microwave applications. The mixer has two input ports and one output port. The input signal with frequency f_{RF} and the other input signal with the frequency f_{LO} are multiplied to produce the output signal, which consist of the summation of these two frequencies as well as the differences between them as in Figure 1.1 [2].

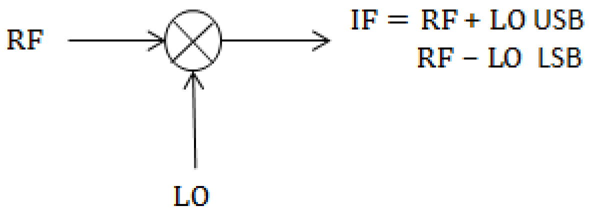

Figure 1.1: Ideal Mixer 'Multiplier'. It multiplies the RF signal with the LO signal and produces Upper Side Band and Lower Side Band Intermediate signals.

The mixer can do modulation"up conversion" when it is used in the transmitter side by mixing the two input signals: the baseband signal that want to be sent and the local oscillator signal producing the radio frequency signal. Or, it can do modulation "down conversion" when it is used in the receiver side by mixing the two input signals: the transmit band pass signal "radio frequency" with frequency f_{RF} , and the local oscillator signal with frequency f_{LO} producing output signal with the Intermediate Frequency f_{IF} . When $f_{RF} > f_{LO}$, the desired signal is $f_{IF} = f_{RF} - f_{LO}$; however, when the $f_{RF} < f_{LO}$, the desired signal is $f_{IF} = f_{LO} - f_{RF}$ as illustrates in Figures 1.2-1.3, respectively. The undesired signal is $f_{IF} = f_{RF} + f_{LO}$, which is with high frequency compared to the f_{IF} , and it can be removed after the mixer by using Band Pass Filter (BPF). Before the mixer, there is another BPF which is used to remove the image frequency at $f_{RF} - 2f_{IF}$, because the desired signal will be distorted if any other signal passes through the mixer that has same frequency as the image frequency as illustrates in Figure 1.4 [2].

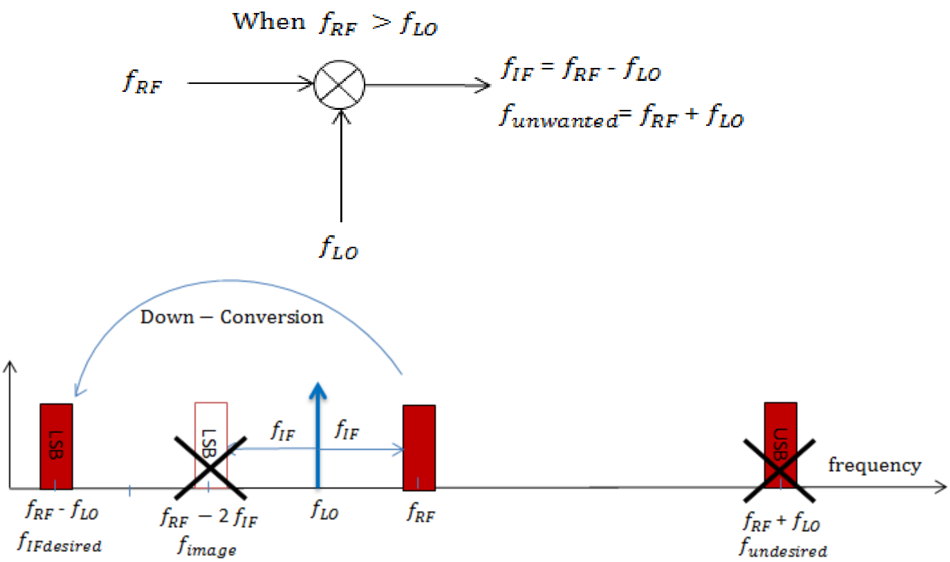

Figure 1. 1: Mixer operation: The inputs to the mixer are: radio frequency signal with radio frequency $\boldsymbol{f_{RF}}$ and the local oscillator signal with frequency $\boldsymbol{f_{LO}}$ and the output signal from the mixer is the multiplication of these two signals. The output signal from the mixer has Intermediate frequency $\boldsymbol{f_{IF}}$ is: $\boldsymbol{f_{RF} - f_{LO}}$ and undesired frequency $\boldsymbol{f_{RF} + f_{LO}}$, if $\boldsymbol{f_{RF} > f_{LO}}$

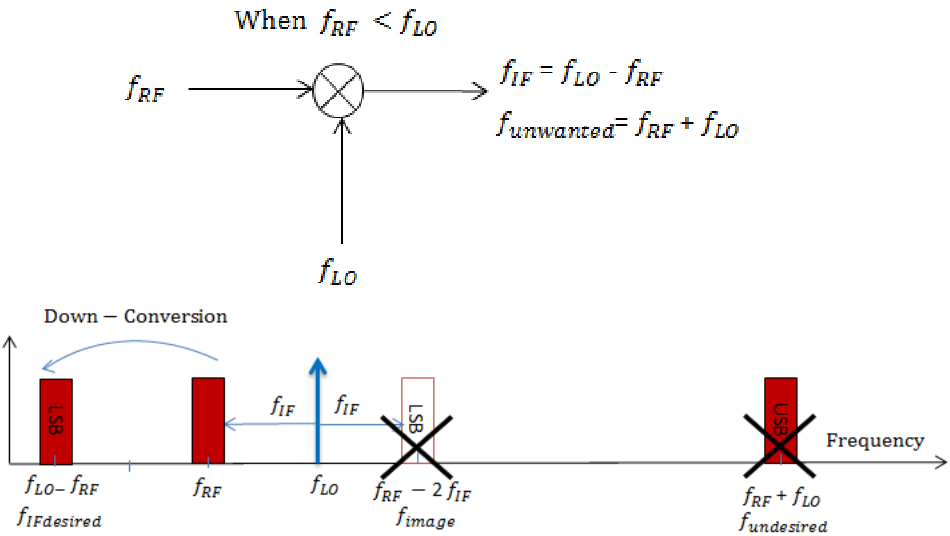

Figure 1.3: Mixer operations: The inputs to the mixer are: radio frequency signal with radio frequency $\boldsymbol{f_{RF}}$ and the local oscillator signal with frequency $\boldsymbol{f_{LO}}$ and the output signal from the mixer is the multiplication of these two signals. The output signal from the mixer has intermediate frequency $\boldsymbol{f_{IF}}$ is: $\boldsymbol{f_{LO} - f_{RF}}$ and undesired frequency $\boldsymbol{f_{RF} + f_{LO}}$, if $\boldsymbol{f_{RF} < f_{LO}}$

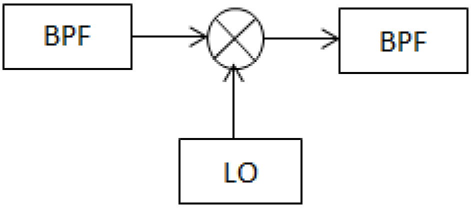

Figure 1.4: Band Pass Filter 'BPF' and Local Oscillator 'LO', The BPF before the LO is used to remove the image frequency and the BPF after the LO used to remove the unwanted output frequency

2.2 IQ imbalance

In the modern designs, wireless devices have three standard parts in the physical layer: antenna part, digital part of the transceiver and Front-End (i.e., analog domain) part of the transceiver.

The Front-End part of the transceiver suffers from errors that can take place in phase and amplitude in the local oscillator, which refers to as the IQ imbalance between the two branches. This imbalance is due to the local oscillator quadrature conversion that can happen in the transmitter side or it can happen in the receiver side as illustrate in Figure 1.5 and Figure 1.6, respectively. Or, it can happen in both sides (i.e., the transmitter and the receiver). The local oscillator quadrature conversion implementation is affected by the offsets of the gain and the phase causing IQ imbalance in it [2]. Only the IQ imbalance in the receiver side will be illustrated here.

In the receiver side the receiver signal has two distinct components in-phase component (I), and quadrature-phase component (Q). Each of them has a mixer, low pass filter, and analog to digital converter. Noisy mixers, noisy oscillator, and unbalances low pass filter unbalanced are the important sources of the IQ imbalance. The IQ imbalance between the low pass filter and the local oscillator in the I and Q channels can limit the quality of the signal and degradation it, which results in reducing the performance of the system.

These imbalances can be characterized by: a phase mismatch that can happen between the I and the Q components of the local oscillator signals, which becomes not exactly 90 degrees, and an amplitude mismatch due to the gain difference of the mixers of the I and Q branches.

In the ideal transmitter (transmitter without local oscillator gain and phase errors), let's suppose that we have baseband signal $x_L\,(t)$. The real part of $x_L\,(t)$ is named $x_I\,(t)$ and the imaginary part of is named $x_Q\,(t)$. To modulate the complex envelope $x_L\,(t) = x_I\,(t) + j\,x_Q\,(t)$ for the baseband signal $x_L\,(t)$ to the bandpass $x_{RF}\,(t)$ signal, the carrier signal with carrier factor $e^{j\omega_c t}$ at carrier frequency (f_c) has been used.

$$x_{RF}\,(t) = real\left\{x_L\,(t)\,e^{j\omega_c t}\right\} \tag{1.1}$$

$$= real\,\{(x_I\,(t) + j\,x_Q\,(t))(\cos\,(\omega_c\,t) + jsin(\omega_c\,t))\} \tag{1.2}$$

$$= real\,\{x_I\,(t)\cos(\omega_c\,t) + j\,x_I\,(t)\,sin(\omega_c\,t)$$

$$+\; jx_Q\,(t)\cos(\omega_c\,t) - x_Q\,(t)\,sin(\omega_c\,t)\} \tag{1.3}$$

After taking the real part of (1.3), we get the real signal that will be transmitted from the transmit antenna as in:

$$x_{RF}\,(t) = x_I\,(t)\,cos(\omega_c\,t) - x_Q\,(t)\;sin(\omega_c\,t) \tag{1.4}$$

$$= ½\{(x_I\,(t) + j\,x_Q\,(t))\,e^{j\omega_c t} + (x_I\,(t) - j\,x_Q\,(t))e^{-j\omega_c t}\} \tag{1.5}$$

where,

$\omega_c = 2\pi f_c$, $x_I\,(t)$ is the in phase component of the $x_L\,(t)$, and $x_Q\,(t)$ is the quadrature component of the$x_L\,(t)$.

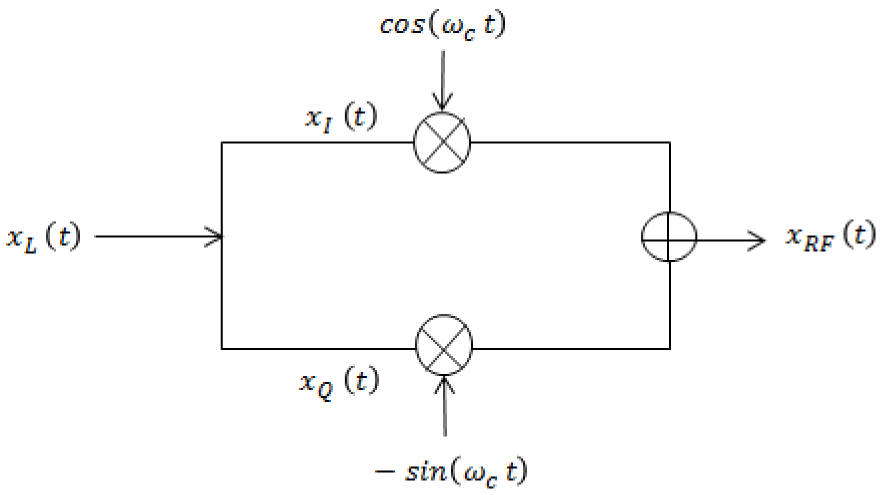

Figure 1.5: Ideal Transmitter

In the ideal receiver (without local oscillator gain and phase errors), the real signal $x_{RF}\,(t)$ is mixed with $x_{LO}\,(t) = e^{-j\omega_c t}$, the local oscillator signal in order to get converted to the low pass signal$x_{LP}\,(t)$.

$$x_{LP}\,(t) = x_{RF}\,(t)\, x_{LO}\,(t) \tag{1.6}$$

$$= x_{RF}\,(t)\, e^{-j\omega_c t} \tag{1.7}$$

After substituting the equation of the real signal $x_{RF}\,(t)$ as in (1.5) into (1.7), and the value of the local oscillator signal into (1.7), and then, simplifying the equations as in (1.8) to (1.10), the term corresponding to the -2f_c frequency, is cancelled:

$$x_{LP}\,(t) = ((½\,\{(x_I\,(t) + j\,x_Q\,(t))\,e^{j\omega_c t}$$

$$+ (½\,\{(x_I\,(t) - j\,x_Q\,(t))e^{-j\omega_c t}\})e^{-j\omega_c t} \tag{1.8}$$

$$= \left(x_I(t) + j\, x_Q(t)\right) e^{j\omega_c t} e^{-j\omega_c t}$$

$$+ \left(x_I(t) - j\, x_Q(t)\right) e^{-j\omega_c t}\, e^{-j\omega_c t} \qquad (1.9)$$

$$x_{LP}(t) = ½ \{\left(x_I(t) + j\, x_Q(t)\right) + \left(x_I(t) - j\, x_Q(t)\right) e^{-j2\omega_c t}\} \qquad (1.10)$$

Then by substituting the base band signal $x_L(t)$ instead of the In-phase and quadrature component $x_I(t) + j\, x_Q(t)$ we get:

$$x_{LP}(t) = ½ \{x_L(t) + x_L(t) e^{-j2\omega_c t}\} \qquad (1.11)$$

We get the the baseband signal:

$$x_{LP}(t) = x_L(t)\left(1 + e^{-j2\omega_c t}\right) \qquad (1.12)$$

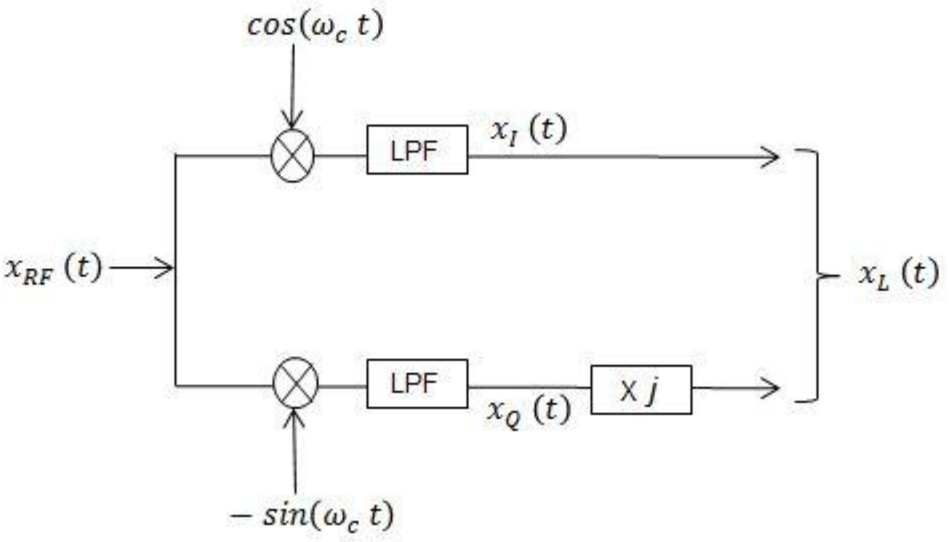

Figure 1.6 Ideal Receiver

In the non-ideal receiver (with local oscillator gain and phase errors), the real signal $x_{RF}\,(t)$ is mixed with the local oscillator signal $x_{LO}\,(t)$ to be converted into low pass signal $x_{LP}\,(t)$ from the real band pass signal $x_{RF}\,(t)$.

$$x_{LP}\,(t) = x_{RF}\,(t)\, x_{LO}\,(t) \tag{1.13}$$

Instead of having $x_{LO}\,(t) = \cos(\omega_c\, t) - j\sin(\omega_c\, t)$, and due to the gain of phase imbalance, it becomes:

$$x_{LO}\,(t) = (1 + \epsilon_R\,)\, cos(\omega_c\, t + \Delta\varphi_R\,) - j(1 - \epsilon_R\,)\; sin(\omega_c\, t - \Delta\varphi_R\,) \tag{1.14}$$

The imbalance takes place due to the local oscillator gain errors $(\mathbf{1} + \boldsymbol{\epsilon}_{\mathbf{R}}\,)$ and $(\mathbf{1} - \boldsymbol{\epsilon}_{\mathbf{R}}\,)$ in the I branch and in the Q branch, respectively, and due to the local oscillator phase errors as well ($\Delta\boldsymbol{\varphi}_{\mathbf{R}}$ and -$\Delta\boldsymbol{\varphi}_{\mathbf{R}}$) in the I branch and Q branch, respectively. As illustrate in the Figure 1.7, the IQ imbalance in the receiver makes the signal suffer from degradation [2]. By substituting (1.4) and (1.14) into (1.13), we get:

$$x_{LP}\,(t) = \left(x_I\,(t)\, cos(\omega_c\, t) - \; x_Q\,(t)\;\, sin(\omega_c\, t)\right)$$

$$\times\{\; (1 + \epsilon_R\,)\, cos(\omega_c\, t + \Delta\varphi_R\,) - \; j(1 - \epsilon_R\,)\, sin(\omega_c\, t - \Delta\varphi_R\,) \tag{1.15}$$

After simplified the equations, we get:

$$x_{LP}\,(t) = (cos(\Delta\varphi_R\,) - j\;\epsilon_R\;\; sin(\Delta\varphi_R\,))\;\left(x_I\,(t) + j\; x_Q\,(t)\right)$$

$$+(\epsilon_R\;\, cos(\Delta\varphi_R\,) + j\, sin(\Delta\varphi_R\,)\,)\;\left(x_I\,(t) - j\; x_Q\,(t)\right) \tag{1.16}$$

The term $(cos(\Delta\varphi_R\,) - j\;\epsilon_R\;\; sin(\Delta\varphi_R\,))$ will be denoted α_R , and the term $(\epsilon_R\;\, cos(\Delta\varphi_R\,) + j\, sin(\Delta\varphi_R\,))$ will be denoted β_R . After we substitute both α_R and β_R in the (1.16), we get the IQ imbalance equation as in (1.17).

In which: ϵ_R is the Amplitude offsets in the receiver side, and $\Delta\varphi_R$ is the phase offsets in the receiver side.

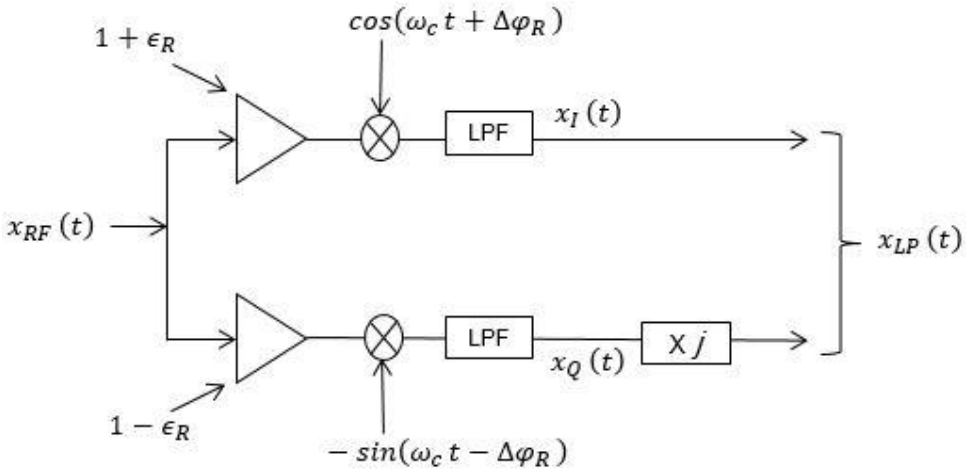

Figure 1.7: Non- ideal receiver illustrates the IQ imbalance, Gain and the Phase errors of the local oscillator in I and Q-branches.

The IQ imbalance in the receiver side will be modeled by substitute $\alpha_r + j\alpha_i$ instead of α_R that mean to represent α_R as real and imaginary , because the real components only can give linear model, not the complex conjugation. The same is performed for β_R , $\beta_r + j\beta_i$. By using β_R in the equation of the IQ imbalance after the low pass filter, we get:

$$x_{LP}(t) = \alpha_R \; x_L(t) + \beta_R \, x_L(t)^* \tag{1.17}$$

$$=(\alpha_r + j\alpha_i)\,(x_I(t) + j\,x_Q(t))$$

$$+\,(\beta_r + j\beta_i)\,(x_I(t) - j\,x_Q(t)) \tag{1.18}$$

$$= \alpha_r\, x_I\,(t) + j\,\alpha_r\,\ x_Q\,(t) + j\alpha_i\, x_I\,(t) - \alpha_i\ \ x_Q\,(t) + \beta_r\, x_I\,(t) - j\beta_r\ \ x_Q\,(t)$$

$$+\ j\beta_i\ \ x_I\,(t) + \beta_i\, x_Q\,(t) \qquad (1.19)$$

$$= (\ \alpha_r\ +\ \beta_r\,)\, x_I\,(t) - (\alpha_i\ -\ \beta_i\,)\, x_Q\,(t) + j\,(\alpha_i\ + \beta_i\ \,)\, x_I\,(t)$$

$$-j\,(-\,\alpha_r\ +\ \beta_r\,)\, x_Q\,(t) \qquad (1.20)$$

Saparating the real part and the imaginary part from the complex signal from (1.20), results in the following equation:

$$\begin{pmatrix} x_{LPI}\,(t) \\ x_{LPQ}\,(t) \end{pmatrix} = \begin{pmatrix} \alpha_r\ +\ \beta_r & -\,\alpha_i\ +\ \beta_i \\ \alpha_i\ +\ \beta_i & \alpha_r\ -\ \beta_r \end{pmatrix} \begin{pmatrix} x_I\,(t) \\ x_Q(t) \end{pmatrix} \qquad (1.21)$$

From (1.16) and (1.17) by substitute $(cos(\Delta\varphi_R\,) - j\ \epsilon_R\ sin(\Delta\varphi_R\,))$ that represents α_R and this term ($\epsilon_R\ cos(\Delta\varphi_R\,) + j\ sin(\Delta\varphi_R\,)$) that represents β_R in the (1.22) and simplified it we get the IQ imbalance equation in the receiver side:

$$\begin{pmatrix} x_{LPI}\,(t) \\ x_{LPQ}\,(t) \end{pmatrix} = \begin{pmatrix} (1+\epsilon_R\ )cos\big(\Delta\varphi_R\ \big) & (1+\epsilon_R\ )\ sin\big(\Delta\varphi_R\ \big) \\ (1-\epsilon_R\ )\ sin\big(\Delta\varphi_R\ \big) & (1-\epsilon_R\ )cos\big(\Delta\varphi_R\ \big) \end{pmatrix} \begin{pmatrix} x_I\,(t) \\ x_Q(t) \end{pmatrix} \qquad (1.22)$$

This equation in (1.22) is the general case when the imbalance exists in both I and Q branches, as will be describe in details in chapter 3.

CHAPTER 3

3 IQ Imbalance Models

In this chapter, two models of IQ imbalance are described. These two models vary according to the way of modeling the mismatch between the I component and the Q component. The first model will be called the Double-Branch IQ imbalance Model (DBIQM), in which the IQ imbalance is modeled as amplitude and phase errors that exist in both the I and the Q branches. The second model will be called the Single-Branch IQ imbalance Model (SBIQM), in which the IQ imbalance is modeled as amplitude and phase errors that exist only in one branch, the Q-branch. These two models will be described in this chapter and the relationship between them will be explained.

3.1 The Double-Branch IQ imbalance Model (DBIQM)

In this model the mismatch between the I and Q branches is represented by errors in both the I and Q branches (worst cases). In the I branch, the amplitude and phase mismatch are characterized by $1 + \epsilon_R$ and $\Delta\varphi_R$ respectively. In the Q branch, the amplitude and phase imbalance are characterized by $1 - \epsilon_R$ and $-\Delta\varphi_R$, respectively as in [2]. Where ϵ_R represents the amount of error in the amplitude, i.e., deviation from the perfectly balanced case due to the IQ imbalance. In case of not having any IQ imbalance, then the values of both ϵ_R , and $\Delta\varphi_R$ are zero.

Figure 3.1 dipicts a non- ideal receiver with amplitude and the phase errors in I and Q-branches using the DBIQM. The IQ-imbalanced signal using the DBIQM can be written as follows.

$$\begin{pmatrix} x_{LPI}(t) \\ x_{LPQ}(t) \end{pmatrix} = \begin{pmatrix} (1+\epsilon_R)cos(\Delta\varphi_R) & (1+\epsilon_R)\,sin(\Delta\varphi_R) \\ (1-\epsilon_R)\,sin(\Delta\varphi_R) & (1-\epsilon_R)cos(\Delta\varphi_R) \end{pmatrix} \begin{pmatrix} x_I(t) \\ x_Q(t) \end{pmatrix} \quad (3.1)$$

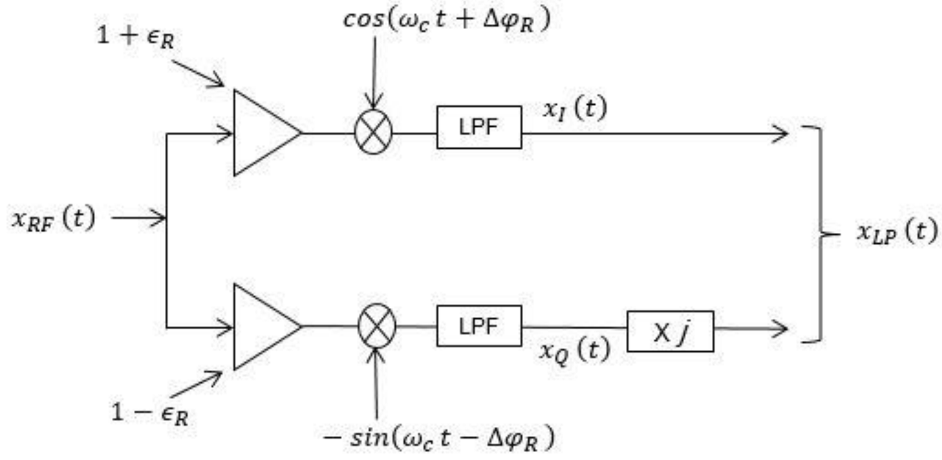

Figure 3. 1: **DBIQM**.

Figure 3.2 illustrate the effect of the DBIQM on the signal constellation. We examine several QAM sizes e.g., 4 QAM, 16 QAM, and 64 QAM. In all cases we consider three levels of IQ imbalance: low with $\epsilon_R = 0.03$ and $\Delta\varphi_R = 2$ degrees, medium with $\epsilon_R = 0.06$ and $\Delta\varphi_R = 4$ degrees, and high with $\epsilon_R = 0.1$ and $\Delta\varphi_R = 6$ degrees.

From Figure 3.2, we observe the following points:

1. As it is expected, the higher the IQ imbalanced, the higher the distortion in the QAM constellation.
2. The IQ imbalance using the DBIQM results in having each point in the constellation to be shifted from its original position in *both* the I and the Q axis.
3. The IQ imbalance has a more pronounced effect when higher order QAM constellation is considered.

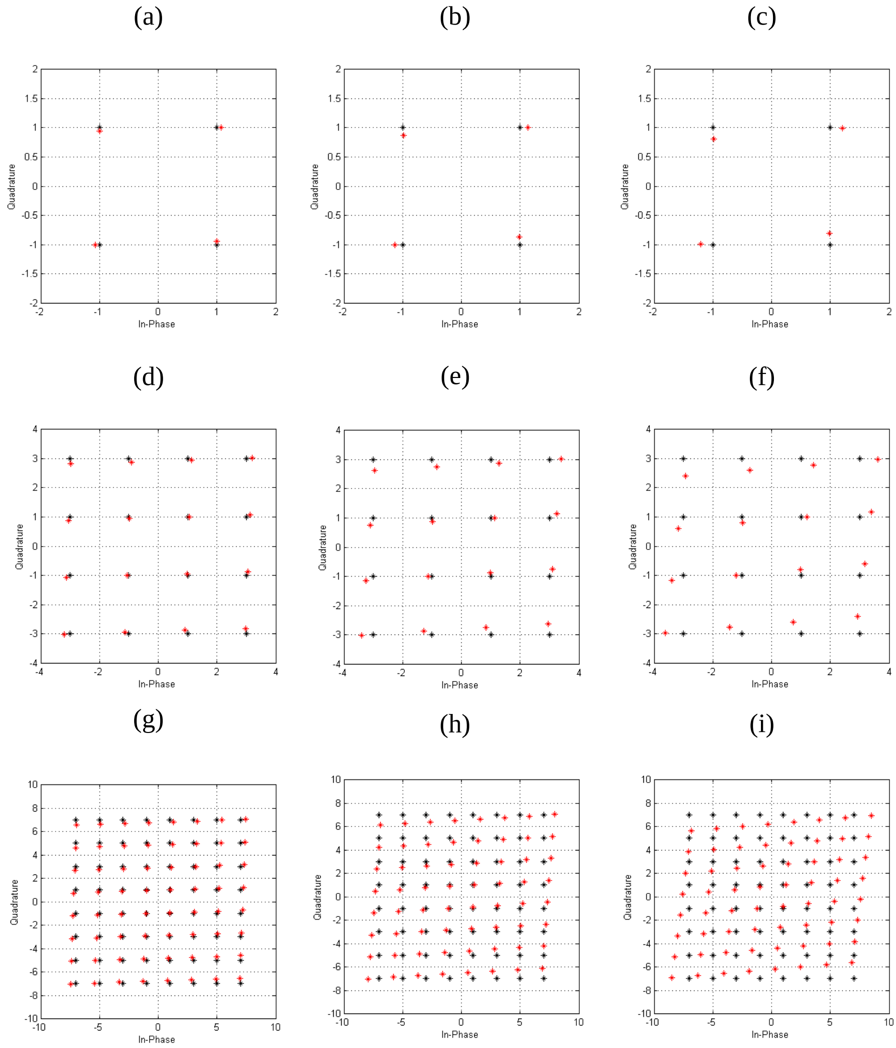

Figure 3.2: Constellation diagram for the transmitted symbols (Black Stars) and the corresponding IQ imbalanced symbols (Red Stars) using the DBIQM for: (a) 4-QAM, low IQ imb., (b) 4-QAM , medium IQ imb., (c) 4-QAM, high IQ imb., (d) 16-QAM, low IQ imb., (e) 16-QAM, medium IQ imb., (f) 16-QAM, high IQ imb., (g) 64-QAM , low IQ imb., (h) 64-QAM , medium IQ imb., and (i) 64-QAM, high IQ imb.

The Signal-to-Distortion-Ratio (SDR) amount (in dB) due to of the IQ imbalance using DBIQM can be found by the following formula [2].

$$SDR = 10\ log\left(\frac{1+{\epsilon_R}^2+{\epsilon_R}^2 tan^2(\Delta\varphi_R)}{{\epsilon_R}^2+tan^2(\Delta\varphi_R)}\right) \tag{3.2}$$

Figure 3.3 illustrates the SDR amount calculated as in (3.2), and is plotted in the amplitude error ϵ_R versus phase error $\Delta\varphi_R$ plane. In this plot we considered $\Delta\varphi_R$ to vary from 0 to 6 degrees, and ϵ_R to vary between 0 and 0.1. Within the considered range, the interference amount due to the IQ imbalance can roughly go up to 20 dB. The amplitude and the phase mismatch should be less than 3% and 2° respectively in order to get at least 30 dB for the SDR.

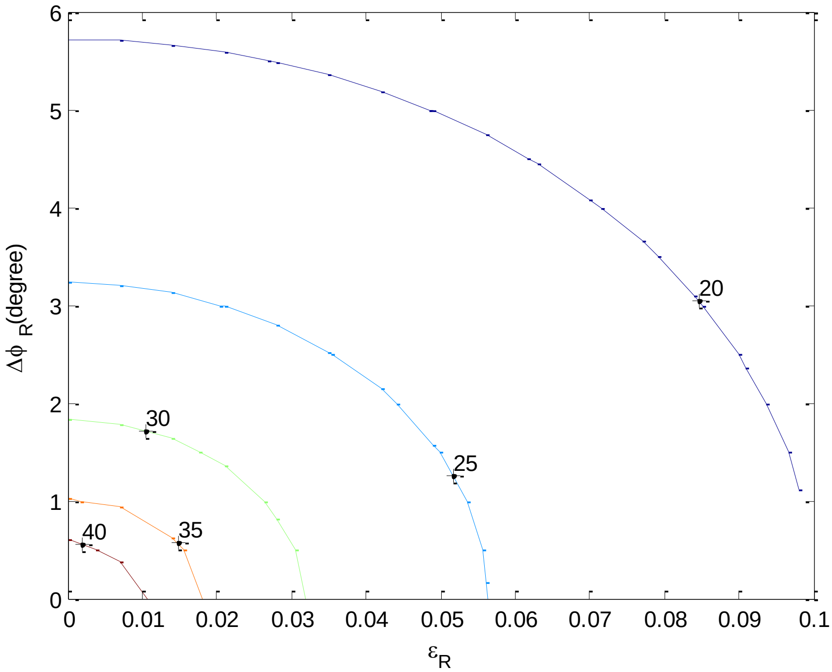

Figure3.3: Contour of **SDR** due to IQ imbalance in the amplitude error versus phase error plane (valid for both Tx and Rx IQ imbalance) [2].

3.2 The Single-Branch IQ imbalance Model (SBIQM)

In this model the mismatch between the I and the Q branches exists only in the Q branch, and there is no error in the I branch. The amplitude gain in the Q branch is characterized by g and the phase mismatch is characterized by φ [5]. Therefore, for this model, $1-g$ represents the amount of error in the amplitude, i.e., deviation of the amplitude of the Q branch from the perfectly balanced case due to the IQ imbalance.

$$\begin{pmatrix} x_{LPI}(t) \\ x_{LPQ}(t) \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -g\sin(\varphi) & g\cos(\varphi) \end{pmatrix} \begin{pmatrix} x_I(t) \\ x_Q(t) \end{pmatrix} \tag{3.3}$$

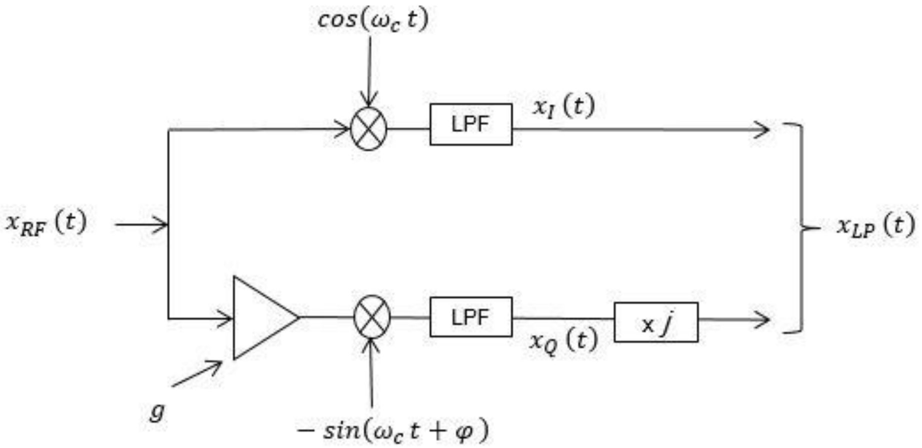

Figure 3.4: IQ imbalance using the **SBIQM**. The I branch has nor amplitude error nor phase error. The Q branch has amplitude error of $\mathbf{1-g}$, and phase error $\boldsymbol{\varphi}$.

Figure 3.5 illustrates the effect of the single-branch IQ imbalance on the signal constellation. We examine several QAM sizes e.g., 4 QAM, 16 QAM, and 64 QAM. In all cases we consider three levels of IQ imbalance: low with $1-g=0.0583$ and $\varphi=-4$ degrees, medium with $1-g=0.1132$ and $\varphi=-8$ degrees, and high with $1-g=0.1818$ and $\varphi=-12$ degrees. The selection of these specific values will be explained in the next section.

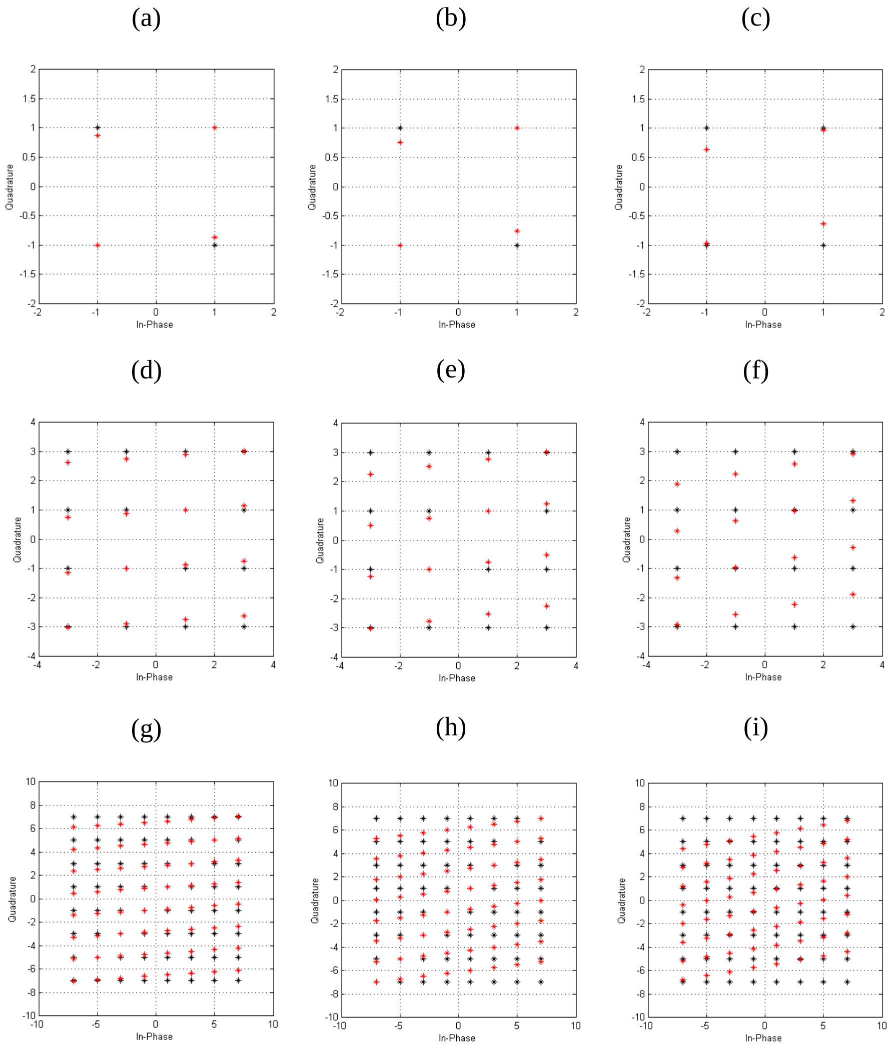

Figure 3.5: Constellation Diagram for the Transmitted symbols (Black Stars) and the IQ imbalanced symbols (Red Stars) using the SBIQM for: (a) 4-QAM, low IQ imb., (b) 4-QAM, medium IQ imb., (c) 4-QAM, high IQ imb., (d) 16-QAM, low IQ imb., (e) 16-QAM, medium IQ imb., (f) 16-QAM , high IQ imb., (g) 64-QAM, low IQ imb., (h) 64-QAM, medium IQ imb., and (i) 64- QAM, high IQ imb.

From Figure 3.5, we also observe that:

a) The higher the IQ imbalanced, the higher the distortion in the QAM constellation.
b) The IQ imbalance has a more pronounced effect when higher order QAM constellation is considered.

However, it should be noticed that, when the SBIQM is used to model the IQ imbalance, each point in the constellation is shifted from its original position *only* in the Q axis.

3.3 Relationship between the two models: DBIQM, and SBIQM

As explained earlier, the two models: the DBIQM, and the SBIQM represent the IQ imbalance in two different ways. The DBIQM allows for amplitude and phase errors to exist in both branches; however, the SBIQM assumes that the amplitude and phase error exists only in one branch. The block diagram in Figure 3.6 explains the relationship between the two models. The DBIQM is equivalent to the SBIQM proceeded by two blocks. The first block represents a Gain G, and the second block represents a Rotation matrix R. By using these two blocks in combination of the SBIQM, the resulting IQ imbalance will be equivalent to the DBIQM.

In matrix notations, the relationship between the DBIQM and the SBIQM can be expressed as follows.

$$\mathrm{DBIQM} = (\mathrm{G})(\mathrm{SBMIQ})(\mathrm{R}) \tag{3.4}$$

where,

$$\mathrm{SBIQM} = \begin{pmatrix} 1 & 0 \\ -g\sin(\varphi\ ) & g\cos(\varphi\ ) \end{pmatrix} \tag{3.5}$$

$$DBIQM = \begin{pmatrix} (1+\epsilon_R)cos(\Delta\varphi_R) & (1+\epsilon_R)\,sin(\Delta\varphi_R) \\ (1-\epsilon_R)\,sin(\Delta\varphi_R) & (1-\epsilon_R)cos(\Delta\varphi_R) \end{pmatrix} \tag{3.6}$$

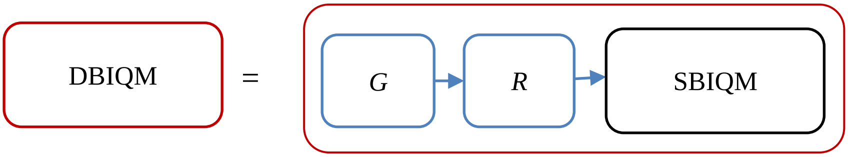

Figure 3.6: Relationship between the DBIQM and the SBIQM

Where the values of G and R are found from the following relationships:

$$G = 2/(1+g) \tag{3.7}$$

$$R = \begin{pmatrix} cos(-\,\varphi/2) & sin(-\,\varphi/2) \\ -\,sin(-\,\varphi/2) & cos(-\,\varphi/2) \end{pmatrix} \tag{3.8}$$

To switch between models:

- The parameters of the DBIQM are calculated from the parameters of the SBIQM as follows:

$$\epsilon_R = (1-g)/(1+g) \tag{3.9}$$

$$\Delta\varphi_R = -\,\varphi/2 \tag{3.10}$$

- The parameters of the SBIQM are calculated from the parameters of the DBIQM as follows:

$$g = (1-\epsilon_R)/(1+\epsilon_R) \tag{3.11}$$

$$\varphi = -2\Delta\varphi_R \tag{3.12}$$

Based on the above conversion equation, we should note that the low, medium, and high IQ imbalance levels that are assumed in Sections 1.1 and 1.2 for both the DBIQM and the SBIQM, respectively, are equivalent. These values are restated again in the following table. For more details on the relationship between the two models, please check Appendix 1.

Table 3.1. Values of the Different Parameters in the DBIQM, and the SBIQM for the Selected Low, Medium, and High IQ Imbalance Levels.

IQ imbalance level	DBIQM		SBIQM	
	Amplitude error ϵ_R	Phase error $\Delta\varphi_R$ (degrees)	Amplitude error $1 - g$	Phase error φ (degrees)
Low	0.03	2	0.0583	- 4
Medium	0.06	4	0.1132	- 8
High	0.10	6	0.1818	-12

CHAPTER 4

4 IQ Imbalance Estimation and Compensation Algorithms

This chapter describes two algorithms to estimate and compensate the IQ imbalance in the receiver side. These algorithms are based on the work of [5] and [4]. The first one is a blind algorithm [5], and the second one is a pilot-based algorithm [4]. The authors in [5] and [4] have derived both of the studied compensation algorithms based on the SBIQM, where the IQ imbalance is assumed to affect only one branch, the Q branch.

4.1 The Blind Algorithm

Today the blind algorithm became so popular because it is so simple, low complexity algorithm, and it is not required data training for the desired signal just needs the statistical properties. This algorithm is used for estimation and compensation of the IQ imbalance [6]. Figure 4.1 illustrates the block diagram of the blind algorithm which is introduced in [5]. First, three estimators are used to estimate the parameters θ_1,θ_2, and θ_3. Then, the values of c_1 and c_2 , which represent the coefficient parameters of the IQ compensator, are calculated.

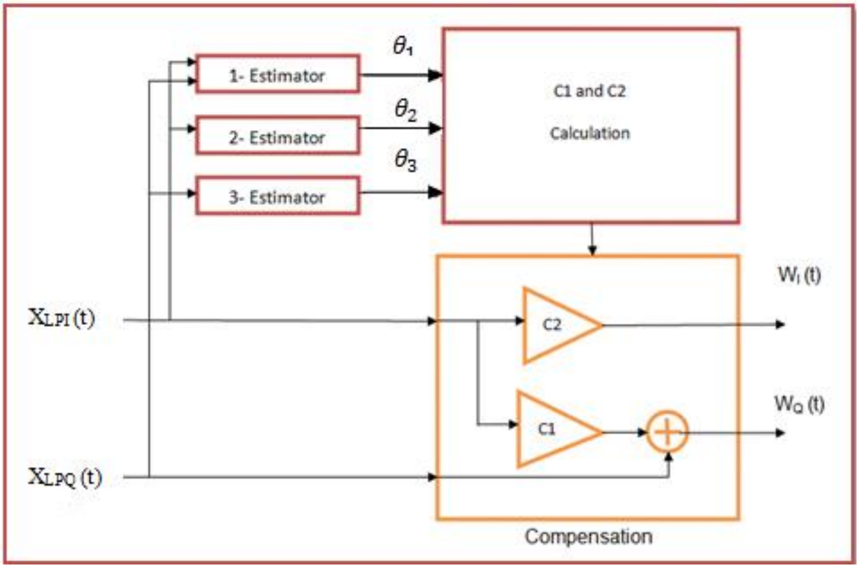

Figure 4. 1: IQ imbalance estimation and compensation implemented according to the algorithm in [5].

The θ–parameters are estimated from the IQ imbalanced signal after low pass filter $x_{LP}(t) = x_{LPI}(t) + jx_{LPQ}(t)$ by using three estimators [5]:

$$\theta_1 = (-1) * \text{mean}(\text{sgn}(x_{LPI}(t))\, x_{LPQ}(t)) \tag{4.1}$$

$$\theta_2 = \text{mean}\,(|x_{LPI}(t)|) \tag{4.2}$$

$$\theta_3 = \text{mean}\,(|x_{LPQ}(t)|) \tag{4.3}$$

Then we get the values of the compensator coefficient c_1 that is calculated from the two θ– parameters θ_1 and θ_2 :

$$c_1 = \frac{\theta_1}{\theta_2} \tag{4.4}$$

And the other compensator coefficient c_2 is calculated from the three θ–parameters θ_1,θ_2, and θ_3 :

$$c_2 = \sqrt{\left(\left({\theta_3}^2 - {\theta_1}^2\right)/{\theta_2}^2\right)} \tag{4.5}$$

Finally, based on the SBIQM, the value of estimated gain is:

$$g = \frac{\theta_3}{\theta_2} \tag{4.6}$$

And the value of estimated phase in:

$$\varphi = \arcsin\left(\frac{\theta_1}{\theta_3}\right) \tag{4.7}$$

After calculating the two compensation coefficients c_1 and c_2, the compensation for the IQ imbalance can be done according to the algorithm as in Figure 4.1. The compensated signals w_I , and w_Q are calculated as:

$$w_I(t) = c_2 * x_{LPI}(t) \tag{4.8}$$

$$w_Q(t) = c_1 * x_{LPI}(t) + x_{LPQ}(t) \tag{4.9}$$

The compensated signal for IQ imbalance $w(t)$ is then divided by c_2 in order to get the same original signal.

$$w(t) = (w_I(t) + j\, w_Q(t))/c_2 \tag{4.10}$$

For more details check Appendix 1.

4.2 Pilot Based Algorithm

The pilot based algorithm is widely used for estimation the channel properties for the system. The transmitter sends known data (pilots) in order to estimate the IQ imbalance of the channel. The pilot based algorithm used to compensate the IQ imbalance in order to get less error and high performance system by using enough training symbol to estimate the IQ imbalance and consequently perform the proper compensation.

As proposed in [4], the preamble is used to estimate the amplitude and phase compensation parameters $K_{est.b}$ and P_{est} as follows:

$$\mathrm{K_{est.b}} = \sqrt{\frac{\sum_{\mathrm{k=1}}^{\mathrm{L}} \mathrm{x_{LPQ}}\,(K)^2}{\sum_{\mathrm{k=1}}^{\mathrm{L}} \mathrm{x_{LPI}}\,(K)^2}} \tag{4.11}$$

$$\mathrm{P_{est}} = \frac{\sum_{\mathrm{k=1}}^{\mathrm{L}} \mathrm{x_{LPI}}\,(K).\mathrm{x_{LPQ}}\,(K)}{\sum_{\mathrm{k=1}}^{\mathrm{L}} \mathrm{x_{LPI}}\,(K)^2} \tag{4.12}$$

where:
L= length of preambles, where 64 is proposed as in [4].

The compensation of the amplitude and the phase is performed as follows:

$$\mathrm{w_I}\,(K) = \mathrm{x_{LPI}}\,(K) \tag{4.13}$$

$$\mathrm{w_Q}\,(K) = \frac{1}{\mathrm{K_{est.b}}\sqrt{1-\mathrm{P_{est}}^{2}}} * \mathrm{x_{LPQ}}\,(K) - \mathrm{P_{est}}\,.\,\mathrm{x_{LPI}}\,(K)] \tag{4.14}$$

For more details check Appendix 1.

CHAPTER 5

5 Performance Evaluation

In this chapter, we will evaluate the performance of two IQ compensation algorithms: a blind algorithm, and a pilot-based algorithm, which have already been described in chapter 4. The comparison will be based on the performance of the two algorithms regarding these two measures:

1) Improving the Signal to Distortion Ratio (SDR), where the SDR will be calculated for the received signal before and after the IQ compensation is applied.
2) Reducing the Bit Error Rates (BER).

In all cases, the IQ imbalance will be modeled using the two models that have already described in chapter 3, i.e, DBIQM, and SBIQM. The values reported in Table 3.1 will be used in this chapter such that the phase and amplitude error that are used for the two models are equivalent.

As explained in chapter 3, the DBIQM and the SBIQM are related via rotation and scaling operations. It was also mentioned in chapter 4 that the derivation of the algorithms under investigation is based on the SBIQM. Therefore, when the actual IQ imbalance is following the DBIQM instead of the SBIQM, the two algorithms should be modified such that they take care of the implicit rotation that exists within the DBIQM. This is done by modifying the two algorithms in a way allowing them to rotate the symbols back, by using the a rotation angle that both of the algorithms are estimating.

To study the effect of the considered compensation algorithms on improving the BER, and the SDR, we simulate the following six cases:

1) The case of having no IQ imbalance, where the cause of the bit errors is coming only due to the AWGN channel. This case will be called the “No IQ Imb.” case.
2) The case of having the IQ imbalance affecting the signals in addition to the AWGN channel. Here we don’t consider any type of IQ compensation. This case will be called the “IQ Imb.” case.

3) The case of having the blind algorithm applied to the received signal in order to compensate for the IQ imbalance. In this case also AWGN channel will be assumed. This case will be called the “Blind Alg.” case.
4) The case of having the pilot-based algorithm applied to the received signal in order to compensate for the IQ imbalance. In this case also AWGN channel will be assumed. This case will be called the “Pilot-Based Alg.” case.
5) The case of having the blind algorithm applied to the received signal combined with re-rotating the signal in order to compensate for the IQ imbalance. In this case also AWGN channel will be assumed. This case will be called the “Blind Alg.+R” case.
6) The case of having the pilot-based algorithm applied to the received signal combined with re-rotating the signal in order to compensate for the IQ imbalance. In this case also AWGN channel will be assumed. This case will be called the “Pilot-Based Alg. +R” case.

5.1 Performance Evaluation when the IQ Imbalance is Modeled based on the DBIQM

In this section, we assume that the received signal suffers from medium IQ imbalance with $\epsilon_R = 0.06$, and $\Delta\varphi_R = 4$ degrees, where the IQ imbalance is modeled according to DBIQM. See Table 3.1. As described earlier, DBIQM implies that the phase and amplitude error due to the IQ imbalance affect both branches. At this medium IQ imbalance, we consider three signal constellations: 4 QAM, 16 QAM, and 64 QAM.

Figures 5.1, 5.2, and 5.3 depict the BER for all the six cases at medium IQ imbalance for the 4 QAM, 16 QAM, and 64 QAM, respectively. From these three figures, it is clear that:

1) The degradation in the BER due to the considered IQ imbalance is not noticeable when the 4 QAM (Figure 5.1) is used, especially at the low SNR region.

2) As the constellation size increases (for example, 16 QAM and 64 QAM), the effect of the IQ imbalance on degrading the BER becomes more pronounced.

3) Both the Blind and the Pilot-Based algorithms improve the BER; however, the Blind algorithm results in better performance.

4) When the rotation back is introduces along with the compensation algorithms, the Pilot-Based algorithm outperform the Blind algorithm. This performance difference indicates that the Pilot-Based algorithm has a better estimation for the rotation phase.

Figure 5.4 and 5.5 illustrate the constellation diagram for the transmitted symbols that have no IQ imbalance, the received symbol with IQ imbalance, and these symbols after applying the compensation algorithms. These figures include the cases of the 4 QAM, 16 QAM, and 64 QAM. It is clear that the effect of the IQ imbalance on degrading the constellation, thus the system performance, becomes more sever with increasing the constellation order. In the case of the 4 QAM, the effect of the IQ imbalance on the constellation is minor (subfigures (a) and (b) in Figure 5.4, and 5.5), which result on minimum effect of the IQ imbalance on the BER (Figure 5.1). However, in the case of the 64 QAM, the degredation in both the constellation and the BER due to the IQ imbalance is more pronounced (subfigure (e), and (f) in Figures 5.4 and 5.5, as well as Figure 5.3).

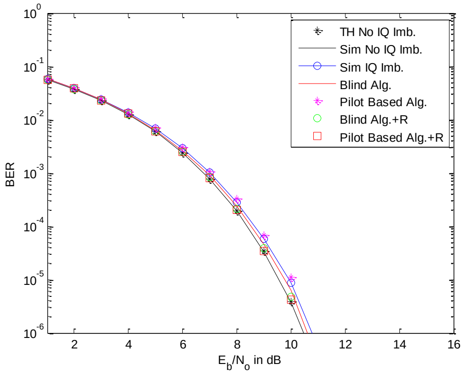

Figure 5.1: BER for 4 QAM of DBIQM.

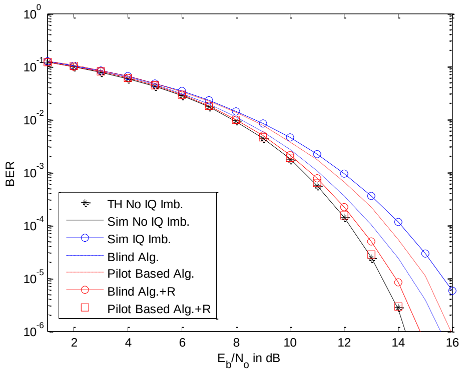

Figure 5.2: BER for 16 QAM of DBIQM.

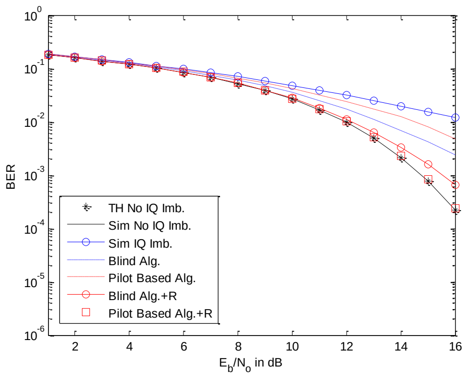

Figure 5.3: BER for 64 QAM of DBIQM.

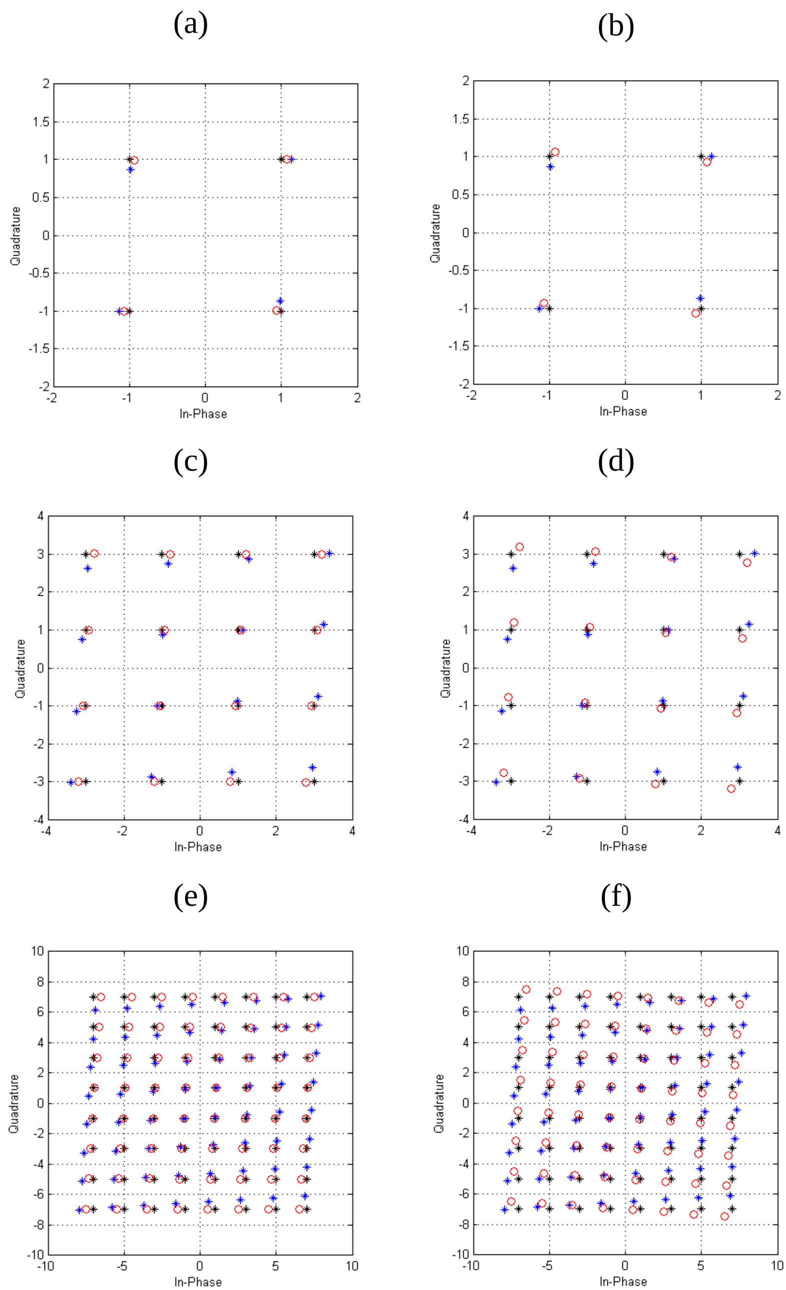

Figure 5.4: Constellation diagram for the transmitted symbols (Black Stars), IQ imbalanced symbols (Blue Stars), and the compensated symbols (Red Stars), using the **DBIQM** for: (a) 4- QAM , Blind Alg., (b) 4- QAM , Pilot Based Alg., (c) 16- QAM , Blind Alg., (d) 16- QAM , Pilot Based Alg., (e) 64- QAM , Blind Alg., (f) 64- QAM , Pilot Based Alg., when gain error=0.06 and phase error=4 degree. **No rotation correction is applied within the algorithms.**

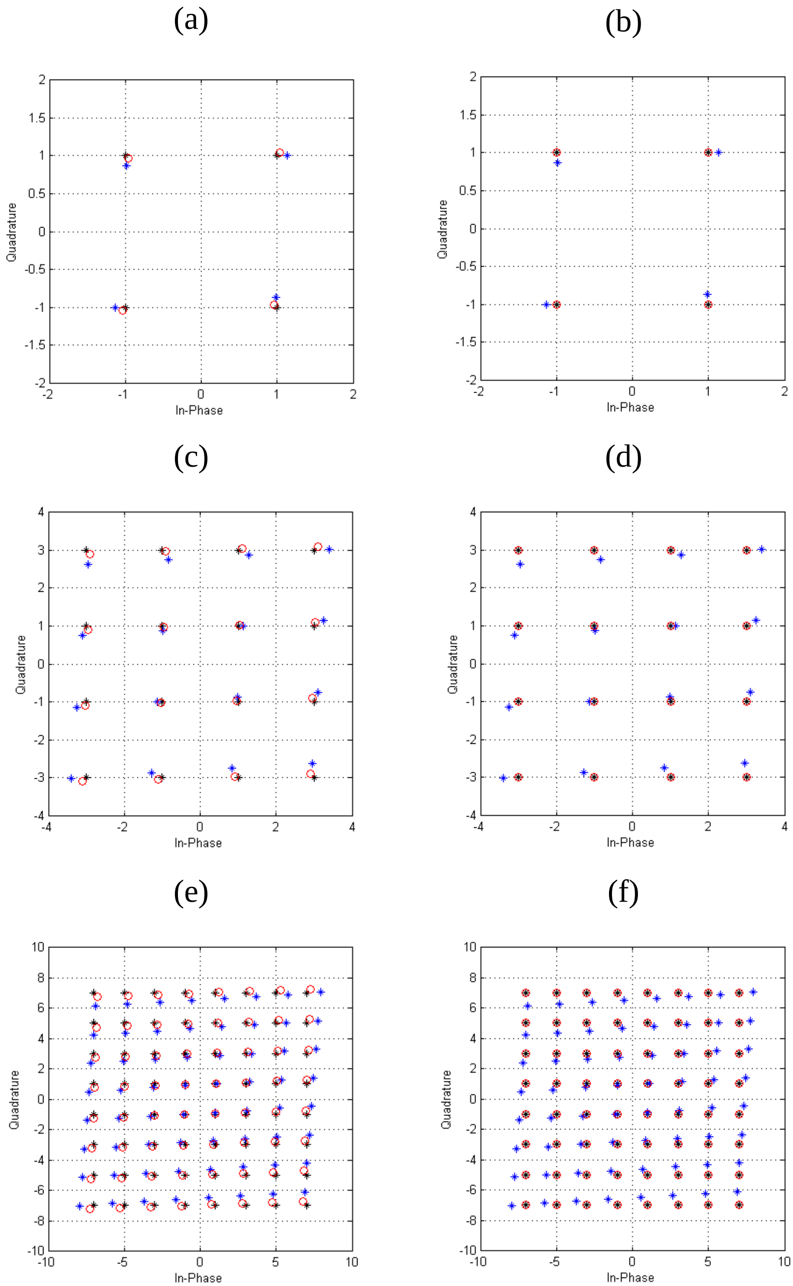

Figure 5.5: Constellation diagram for the transmitted symbols (Black Stars), IQ imbalanced symbols (Blue Stars), and the compensated (Red Stars), using the **DBIQM** for: (a) 4- QAM , Blind Alg., (b) 4- QAM , Pilot Based Alg., (c) 16- QAM , Blind Alg., (d) 16- QAM , Pilot Based Alg., (e) 64- QAM , Blind Alg., (f) 64- QAM , Pilot Based Alg., when gain error=0.06 and phase error=4 degree. **Rotation correction within the algorithms is applied**.

In order to evaluate the performance of the two algorithms in improving the SDR of the received signal, the steps were performed:

1) The considered range of the amplitude-phase error space (i.e., ϵ_R between 0 and 0.1, and $\Delta\varphi_R$ between 0 and 6 degrees), was divided into smaller areas as illustrated in Figure 5.6 - 5.7.

2) In each area (i.e, specific range of ϵ_R and $\Delta\varphi_R$), the improvement in SDR achieved by each of the compensation algorithms are written in different colors.

It is clear from Figure 5.6 that the Blind algorithm (Blue No.) outperforms the Pilot Based algorithm (Red No.) by a minimum of 3 dB. As an example, when the value of the gain error is between 0.06 and 0.07, and the value of the phase error is less than 1 degree, the Blind algorithm gives an SDR gain of 20 dB, compared to 17 dB given by the Pilot Based algorithm. For small values of the gain errors, the SDR gain achieved by the Pilot Based algorithm is as small as 0 dB (i.e., no gain is achieved). By increasing the values of the gain error, the two algorithms ability to compensate the IQ imbalance is increased, for example, SDR gains of about 24 dB and 21 dB are achieved by the Blind and Pilot Based algorithms, respectively, when the value of phase error =1. And by decreasing the values of the phase error, the ability of the two algorithms to improve the SDR increases, but still the Blind algorithm is better than the Pilot Based algorithm.

From Figure 5.7., when the rotation is included with the compensation algorithms, the performance of both algorithms is improved where the minimum SDR is 6 DB throughout the considered values of ϵ_R and $\Delta\varphi_R$. In all cases, the Pilot Based algorithm outperforms the Blind algorithm in terms of the SDR gain.

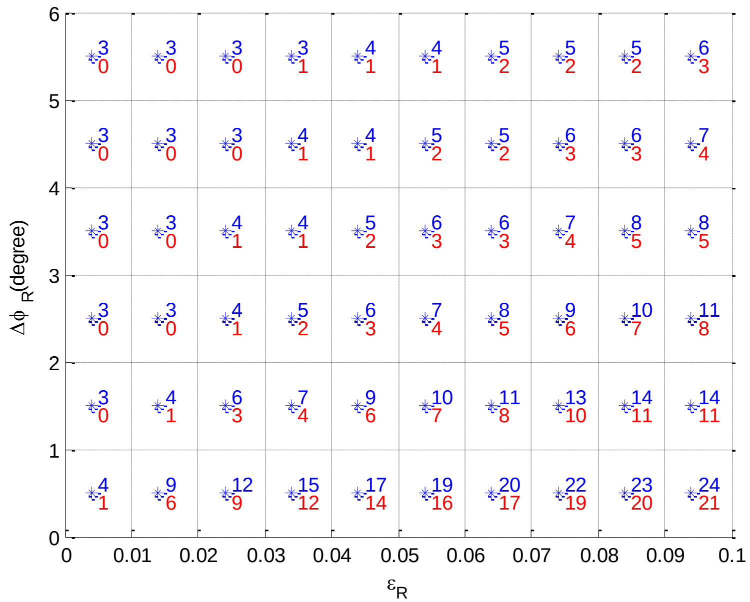

Figure 5. 6: SDR gain after applying the Blind algorithm (Blue no.) and the Pilot Based (Red no.). IQ imbalance is modeled based on the DBIQM. **No rotation correction is applied within the algorithms.**

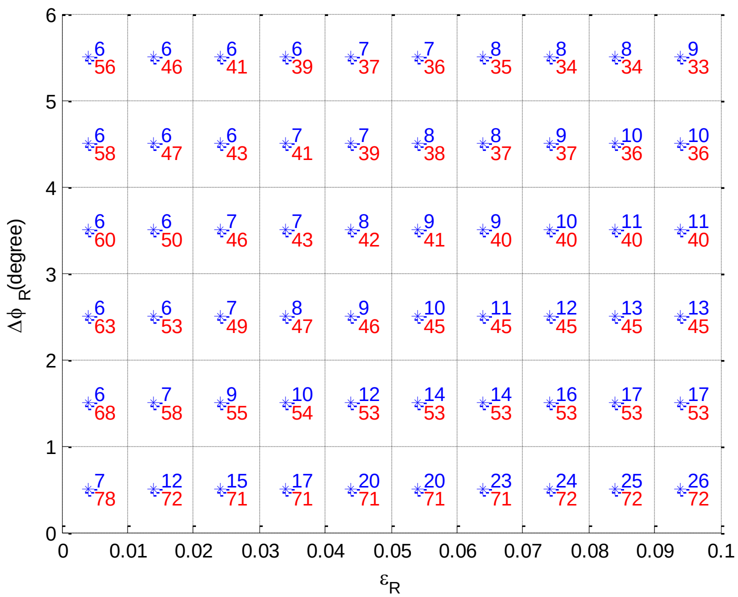

Figure 5. 7: SDR gain after applying the Blind algorithm (Blue no.) and the Pilot Based (Red no.). IQ imbalance is modeled based on the DBIQM. **Rotation correction within the algorithms is applied.**

5.2 Performance Evaluation when the IQ Imbalance is Modeled based on the SBIQM

In this section, the two algorithms (i.e., the blind and the pilot-based) are used to estimate and compensate the IQ imbalance, where the actual IQ imbalance is molded using the SBIQM. We also use a medium level of the IQ imbalance where gain error = 0.1132 and the value of the phase error = -8 degrees. See Table 3.1 for the equivalency of these values between the SBIQM and the DBIQM. Figure 5.8 - 5.10 illustrate the BER for 4 QAM, 16 QAM, and 64 QAM for the different considered cases.

Both algorithms are able to eliminate the IQ imbalance and restore the BER of an imbalanced-free signal. After applying the rotation back, the performance of both algorithms degrade, this is logical since the SBIQM has no rotation component. This result can also be clearly seen by investigating Figures 5.11 and 5.12, where in Figure 5.11 the constellation of the received signal after applying the compensation for IQ imbalance matches the original signal. However, in Figure 5.12, we find that applying the rotation results in distorting the constellation.

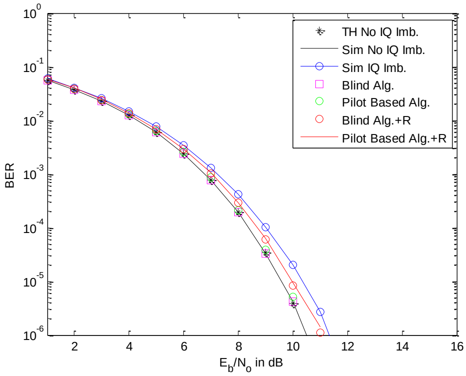

Figure 5.8: BER for 4 QAM of SBIQM.

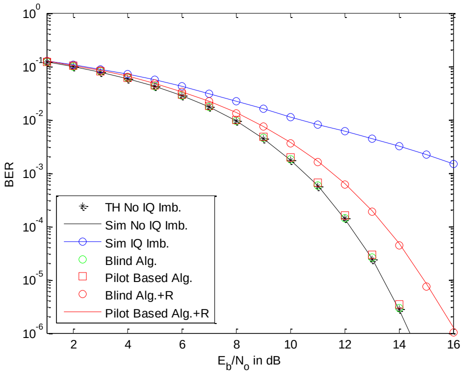

Figure 5.9: BER for 16 QAM of SBIQM.

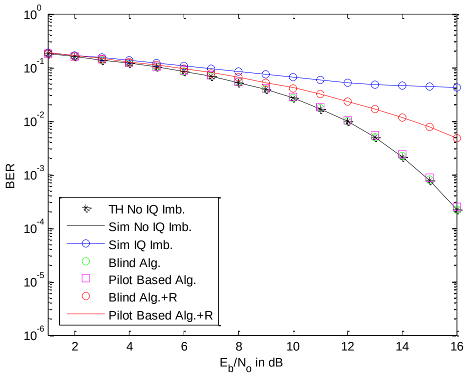

Figure 5.10: BER for 64 QAM of SBIQM.

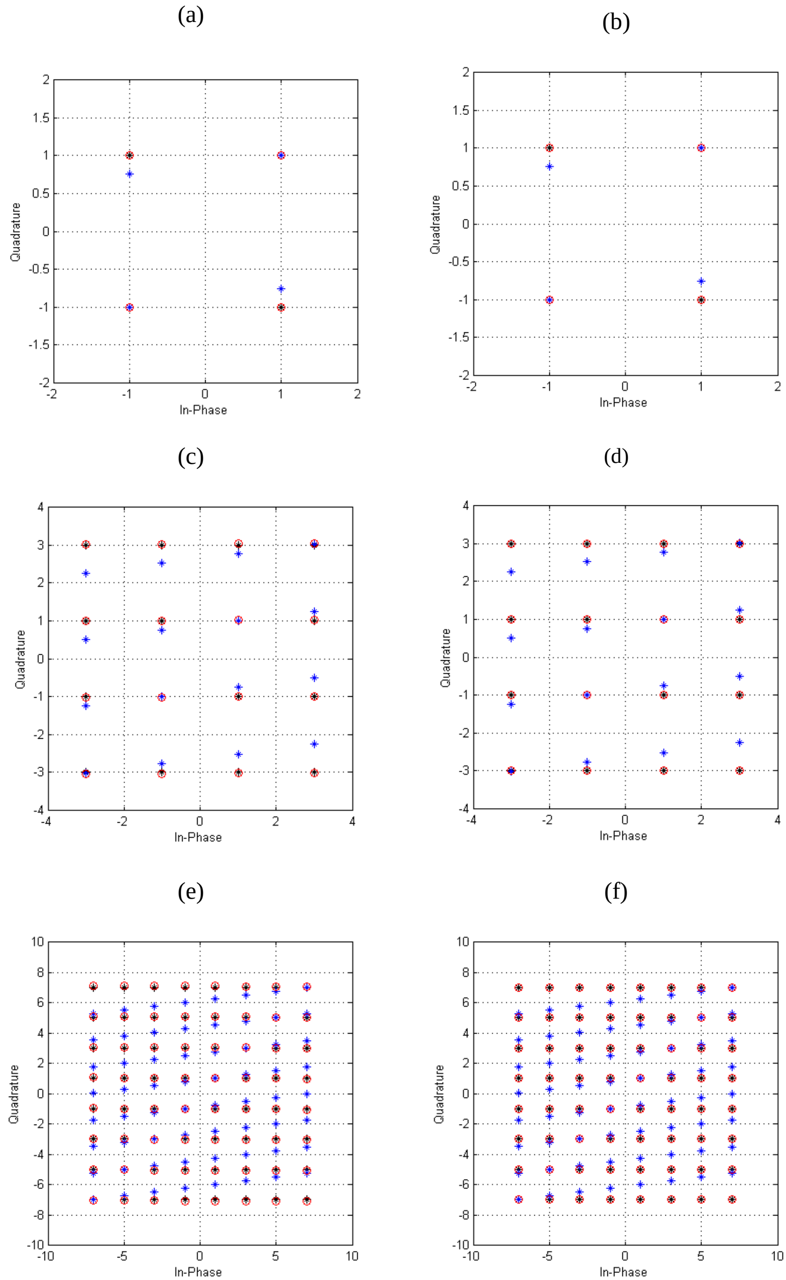

Figure 5.11: Constellation diagram for the transmitted symbols (Black Stars), IQ imbalanced symbols (Blue Stars), and the compensated (Red Stars), using the **SBIQM** for: (a) 4- QAM , Blind Alg., (b) 4- QAM , Pilot Based Alg., (c) 16- QAM , Blind Alg., (d) 16- QAM , Pilot Based Alg., (e) 64- QAM , Blind Alg., (f) 64- QAM , Pilot Based Alg., when gain error=0.1132 and phase error=-8 degrees. **No Rotation correction within the algorithms is applied**.

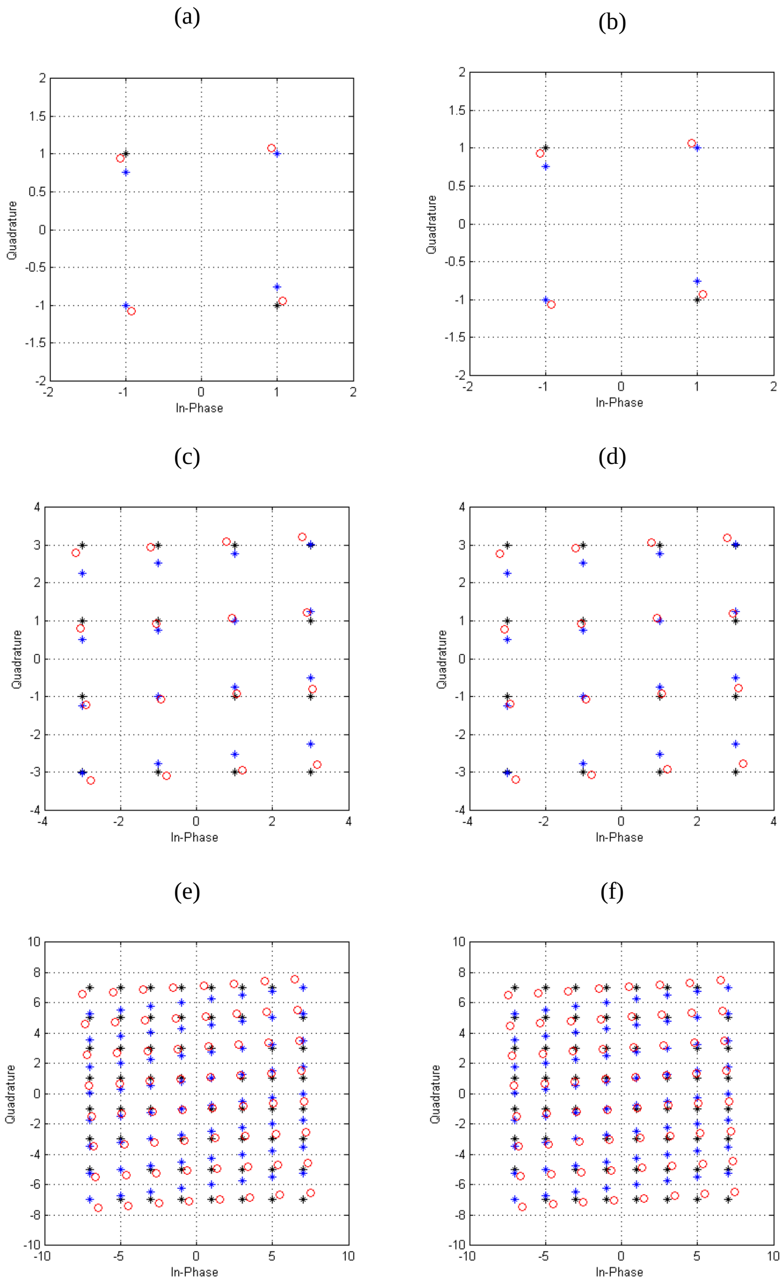

Figure 5.12: Constellation diagram for the transmitted symbols (Black Stars), IQ imbalanced symbols (Blue Stars), and the compensated (Red Stars), using the **SBIQM** for: (a) 4- QAM , Blind Alg., (b) 4- QAM , Pilot Based Alg., (c) 16- QAM , Blind Alg., (d) 16- QAM , Pilot Based Alg., (e) 64- QAM , Blind Alg., (f) 64- QAM , Pilot Based Alg., when gain error=0.1132 and phase error=-8 degrees. **Rotation correction within the algorithms is applied**.

The SDR gain resulting from using the two compensating algorithms is illustrated in Figure 5.13. In all cases, i.e., all the considered values of the gain and phase errors:

1) The SDR gain of both algorithms is more than 23 dB.

2) By increasing the values of the gain error, the Blind algorithm ability to compensate the IQ imbalance is increased, but the Pilot Based algorithm ability to compensate the IQ imbalance is decreased. And by decreasing the values of the phase error, the ability of the two algorithms to improve the SDR increases, but still the Pilot Based is better than the Blind algorithm.

3) The pilot-based algorithm outperforms the blind algorithm. These high SDR gain values come from the fact that both algorithms are using the SBIQM as part of their assumptions.

When rotation is applied within the two algorithms, the performance of both algorithms degrades significantly as expected, due to introducing a none-existing rotation to the signal. See Figure 5.14.

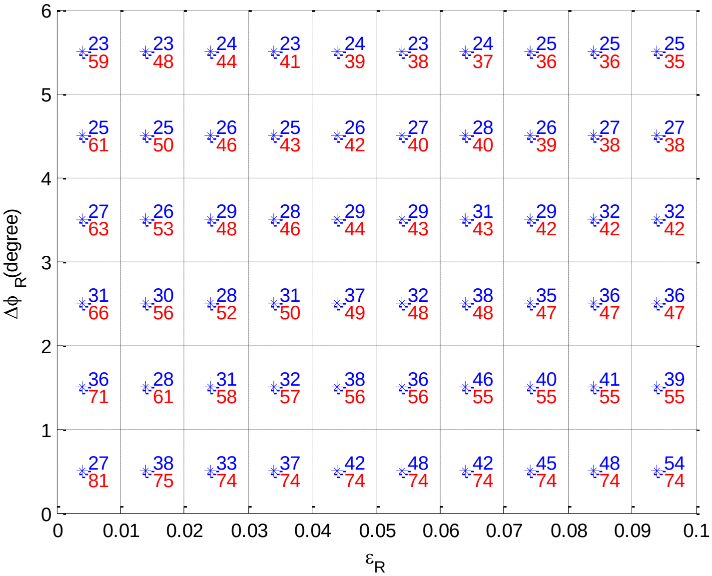

Figure 5. 13: SDR gain after applying the Blind algorithm (Blue no.) and the Pilot Based (Red no.). IQ imbalance is modeled based on the SBIQM. **No rotation correction is applied within the algorithms.**

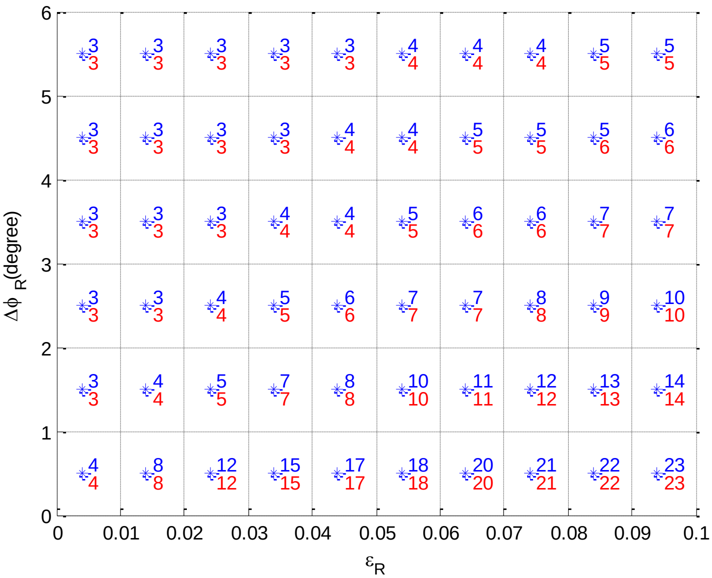

Figure 5. 14: SDR gain after applying the Blind algorithm (Blue no.) and the Pilot Based (Red no.). IQ imbalance is modeled based on the SBIQM. **Rotation correction within the algorithms is applied.**

CHAPTER 6

6 Conclusions

Two models of IQ imbalance are described, the Double-Branch IQ imbalance Model (DBIQM) and the Single-Branch IQ imbalance Model (SBIQM). Even though, these two models vary according to the way of modeling the mismatch between the I component and the Q component, it was found that the two models are related to each other via a simple scaling and rotation operations.

Based on the SBIQM, two algorithms, a blind algorithm and a pilot-based-algorithm, are developed. In this thesis work, the estimation and the compensation steps of these two algorithms are described. Then the performance of these two algorithms is compared under the DBIQM and the SBIQM.

When the SBIQM was used to model the IQ imbalance, it was found that the two algorithms work properly to compensate for the IQ imbalance. The achieved BER for the compensated received signal is very close to the theoretical BER for a signal with no IQ imbalance. The SDR gain achieved by the Pilot-Based compensation algorithm outperforms that which can be achieved by the Blind-algorithm. But the Blind algorithm saves resources, i.e., no need for pilot signal. Both algorithms work properly even with high order QAM modulation.

When the DBIQM was used to model the IQ imbalance, it was found that two algorithms work properly to compensate the IQ imbalance only if a rotation block is implemented within these algorithms. This rotation step is needed in order to compensate for the rotation component that is included in the DBIQM. After adding this rotation modification to both algorithms, the BER of the compensated signal improves significantly, especially for the pilot-based algorithm. The SDR gain obtained after applying the Pilot Based algorithm is also outperforms that of the blind algorithm

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Appendix 1

A Blind and Pilot Based Algorithms

This part describes the two algorithms that used in chapter 4 to estimate and compensate the IQ imbalance in details for SBIQM for Blind and Pilot Based algorithms respectively as in [5] and [4].

A.1 SBIQM

The equation of the single branch IQ imbalance in [5], is illustrated as in:

$$\begin{pmatrix} x_{LPI}(t) \\ x_{LPQ}(t) \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -g\sin(\varphi) & g\cos(\varphi) \end{pmatrix} \begin{pmatrix} x_I(t) \\ x_Q(t) \end{pmatrix} \tag{1.1}$$

$$x_{LPI}(t) = x_I(t) \tag{1.2}$$

$$x_{LPQ}(t) = g\cos(\varphi)\, x_Q(t) - g\sin(\varphi)\, x_I(t) \tag{1.3}$$

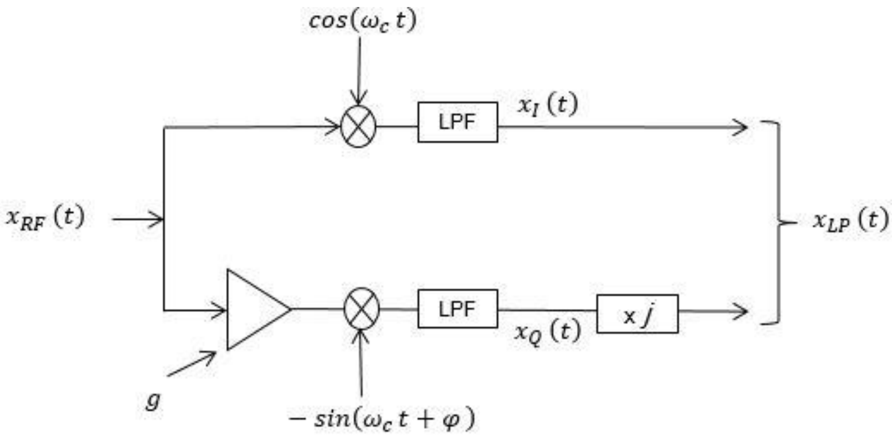

Figure2.1: IQ imbalance: I branch without errors Q branch with gain and phase errors

A.1.1 Estimation of the Blind Algorithm

The estimator parameters are represented by θ–parameters are estimated from the IQ imbalanced signal after low pass filter $x_{LP}(t) = x_{LPI}(t) + jx_{LPQ}(t)$ by using three estimators [5]:

$$\theta_1 = (-1) * \text{mean}(\text{sgn}(x_{LPI}(t))\, x_{LPQ}(t)) \tag{1.4}$$

$$\theta_2 = \text{mean}\,(|x_{LPI}(t)|) \tag{1.5}$$

$$\theta_3 = \text{mean}\,(|x_{LPQ}(t)|) \tag{1.6}$$

Then we get the values of the compensator coefficient c_1 and c_2 as in:

$$c_1 = \frac{\theta_1}{\theta_2} \tag{1.7}$$

$$c_2 = \sqrt{\left(\left({\theta_3}^2 - {\theta_1}^2\right)/{\theta_2}^2\right)} \tag{1.8}$$

Or:

$$c_1 = g \sin(\varphi) \tag{1.9}$$

$$c_2 = g \cos(\varphi) \tag{1.10}$$

The value of estimated gain is:

$$g = \frac{\theta_3}{\theta_2} \tag{1.11}$$

And the value of estimated phase in:

$$\varphi = \arcsin\left(\frac{\theta_1}{\theta_3}\right) \tag{1.12}$$

A.1.2 Compensation of the Blind Algorithm

After calculating the two compensation coefficients c_1 andc_2 as in (1.9) and (1.10) respectively, the compensation for the IQ imbalance can be done according to the algorithm as in:

$$\mathrm{w(t)} = \mathrm{w_I}\,(t) + \mathrm{j}\,\mathrm{w_Q}\,(t) \tag{1.13}$$

Where: $\mathrm{w(t)}$ represented the compensated signal for IQ imbalance, $\mathrm{w_I}\,(\mathrm{t})$ and $\mathrm{w_Q}\,(\mathrm{t})$ represented the real and the imaginary parts of the compensated signal respectively.

$$\mathrm{w_I}\,(\mathrm{t}) = \mathrm{c_2} \ast \mathrm{x_{LPI}}\,(\mathrm{t}) \tag{1.14}$$

$$\mathrm{w_I}\,(\mathrm{t}) = g\,\cos(\varphi) \ast \mathrm{x_{LPI}}\,(\mathrm{t}) \tag{1.15}$$

And:

$$\mathrm{w_Q}\,(\mathrm{t}) = \mathrm{c_1} \ast \mathrm{x_{LPI}}\,(\mathrm{t}) + \mathrm{x_{LPQ}}\,(\mathrm{t}) \tag{1.16}$$

$$\mathrm{w_Q}\,(\mathrm{t}) = g\,\sin(\varphi)\ \mathrm{x_{LPI}}\,(\mathrm{t}) + g\,\cos(\varphi)\,\mathrm{x_{LPQ}}\,(\mathrm{t}) - g\,\sin(\varphi)\,\mathrm{x_{LPI}}\,(\mathrm{t}) \tag{1.17}$$

$$\mathrm{w_Q}\,(\mathrm{t}) = g\,\cos(\varphi) \ast \mathrm{x_{LPQ}}\,(\mathrm{t}) \tag{1.18}$$

So, by applying the equations (1.15) and (2.18) in (1.13), we get:

$$\mathrm{w(t)} = \quad g\,\cos(\varphi)\ \mathrm{x_{LPI}}\,(\mathrm{t}) + \mathrm{j}\,g\,\cos(\varphi)\,\mathrm{x_{LPQ}}\,(\mathrm{t}) \tag{1.18}$$

Then the by dividing the equation (1.19) by $g\,\cos(\varphi)$ in order to get same signal as sent as in:

$$\mathrm{w(t)} = \mathrm{x_{LPI}}\,(\mathrm{t}) + \mathrm{j}\,\mathrm{x_{LPQ}}\,(\mathrm{t}) \tag{1.20}$$

A.2 SBIQM

The equation of the single branch IQ imbalance in [4], is illustrated as in:

$$\begin{pmatrix} x_{LPI}(k) \\ x_{LPQ}(k) \end{pmatrix} = \begin{pmatrix} k_I & 0 \\ -k_Q \sin(\varphi\ ) & k_Q \cos(\varphi_{err}\ ) \end{pmatrix} \begin{pmatrix} x_I(k) \\ x_Q(k) \end{pmatrix} \quad (1.21)$$

$$x_{LPI}(k) = k_I\ x_I(k) \quad (1.22)$$

Where; k_I =1

$$x_{LPQ}(k) = k_Q\ \cos(\varphi_{err})\ x_Q(k) - k_Q\ \sin(\varphi_{err})\ x_I(k) \quad (1.23)$$

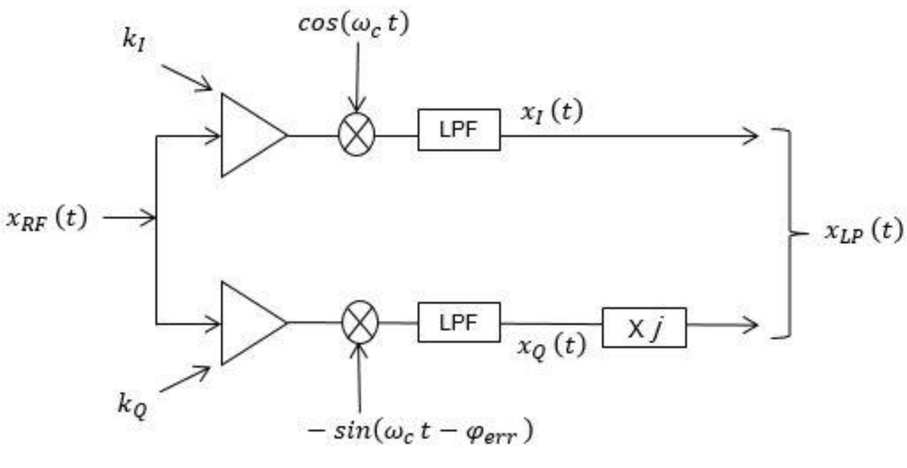

Figure 2.2: IQ imbalance: I branch with gain error=1, and without phase error. Q branch with gain and phase errors

A.2.1 Estimation of the Pilot Based Algorithm

As proposed in [4], the long preamble is used to estimate the amplitude and phase compensation parameters $K_{est.b}$ and P_{est} as follows:

$$\mathrm{K_{est.b}} = \sqrt{\frac{\sum_{k=1}^{L} x_{LPQ}(k)^2}{\sum_{k=1}^{L} x_{LPI}(k)^2}} \tag{1.25}$$

$$\mathrm{P_{est}} = \frac{\sum_{k=1}^{L} x_{LPI}(k).x_{LPQ}(k)}{\sum_{k=1}^{L} x_{LPI}(k)^2} \tag{1.26}$$

Where:
L= long of preambles, where 64 is proposed as in [4].

And:

$$x_{LPQ}(t)^2 = (k_Q \ cos(\varphi_{err}) \ x_Q(k) - k_Q \ sin(\varphi_{err}) \ x_I(k))^2$$

$$= k_Q{}^2 \ (cos(\varphi_{err})^2 \ x_Q(k)^2 + sin(\varphi_{err})^2 \ x_I(k)^2$$

$$-2 \, cos(\varphi_{err}) \ sin(\varphi_{err}) \, x_I(k) x_Q(k))$$

$where$: $\sum_{k=1}^{L} x_I(k)^2 = P$, $\sum_{k=1}^{L} x_Q(k)^2 = P$, *and*

$2 \, cos(\varphi_{err}) \ sin(\varphi_{err}) \, x_I(k) x_{Q\,(k)} = 0.$

$$= k_Q{}^2 \ (cos(\varphi_{err})^2 \ P + sin(\varphi_{err})^2 \ P - 0$$

$$= k_Q{}^2 \ P(cos(\varphi_{err})^2 + sin(\varphi_{err})^2),$$

where:

$cos(\varphi_{err})^2 + sin(\varphi_{err})^2 = 1$

$$= k_Q{}^2 P \quad (1.27)$$

And:

$$\sum_{k=1}^{L} \mathrm{x}_{\mathrm{LPI}}(k)^2 = \sum_{k=1}^{L} x_I(k)^2 = P \quad (1.28)$$

By applying equations (1.27) and (1.28) in to the equation (1.25), the gain estimated as in:

$$K_{est.b} = \sqrt{\frac{k_Q{}^2 P}{P}}$$

$$K_{est.b} = k_Q \quad (1.29)$$

By applying equations (1.22), (1.23) as in:

$$\sum_{k=1}^{L} \mathrm{x}_{\mathrm{LPI}}(k).\mathrm{x}_{\mathrm{LPQ}}(k) = \sum_{k=1}^{L} x_I(k)\, k_Q\, cos(\varphi_{err})\, x_Q(k) - k_Q\, sin(\varphi_{err})\, x_I(k)^2$$

Where: $\mathrm{x_I}\ \mathrm{k_Q}\ \cos(\varphi_{\mathrm{err}})\ \mathrm{x_Q}(k) = 0$, then:

$$= \sum_{k=1}^{L} - k_Q\, sin(\varphi_{err})\, x_I(k)^2$$

$$= - k_Q\, sin(\varphi_{err})\, P \quad (1.30)$$

By applying equations (1.30) and (1.28) in to the equation (1.26), the phase estimated as in:

$$P_{est} = \frac{- k_Q\, sin(\varphi_{err})\, P}{P}$$

$$P_{est} = - k_Q\, sin(\varphi_{err}) \quad (1.31)$$

$$P_{est} = P_{est} / K_{est.b}$$

$$P_{est} = -k_Q \ sin(\varphi_{err})/ k_Q$$

$$P_{est} = - \ sin(\varphi_{err}) \tag{1.32}$$

A.2.2 Compensation of the Pilot Based Algorithm

The compensation for the IQ imbalance can be done according to the algorithm of the amplitude and the phase is performed as in:

$$\mathrm{w}(k) = \mathrm{w_I}(k) + \mathrm{j}\,\mathrm{w_Q}(k) \tag{1.33}$$

$$\mathrm{w_I}(k) = \mathrm{x_{LPI}}(k) \tag{1.34}$$

$$\mathrm{w_Q}(k) = \frac{1}{\mathrm{K_{est.b}}\sqrt{1-\mathrm{P_{est}}^2}} * \mathrm{x_{LPQ}}(k) - \mathrm{P_{est}}\,.\,\mathrm{x_{LPI}}(k)] \tag{1.35}$$

Where: w(k) represented the compensated signal for IQ imbalance, $\mathrm{w_I}$ (t) and $\mathrm{w_Q}$ (k) represented the real and the imaginary parts of the compensated signal respectively.

By applying equation (1.22) in to the equation (2.34), will get:

$$\mathrm{w_I}(k) = x_I(k) \tag{1.36}$$

And: by applying equation (1.22), (1.23), (1.25), (1.31), and (1.32) in to the equation (1.35) as in:

$$\mathrm{w_Q}(K) = \frac{1}{K_{est.b}\sqrt{1-(P_{est}/K_{est.b})^2}} * k_Q \ cos\big(\varphi_{err}\big) \ x_Q(k)$$

$$- k_Q \ sin(\varphi_{err}) \ x_I - \mathrm{P_{est}}\,.\,x_{I\,(k)}$$

$$= \frac{1}{K_{est.b}\sqrt{1-sin(\varphi_{err})^2}} * k_Q \ cos\big(\varphi_{err}\big) \ x_Q(k)$$

$$- k_Q \ sin(\varphi_{err}) \ x_I(k) + k_Q \ sin(\varphi_{err})\,.\,x_I(k)$$

Where: $\sqrt{1 - sin(\varphi_{err})^2} = cos(\varphi_{err})$

$$= \frac{1}{k_Q * cos(\varphi_{err})} * k_Q \; cos(\varphi_{err}) \; x_Q(k)$$

$$= x_Q(k) \tag{1.37}$$

Finally by applying equation (1.36) and (1.37) in to the equation (1.33) will get same signal as sent as in:

$$\text{w(k)} = x_I(k) + \text{j}\, x_Q(k) \tag{1.38}$$

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