Master's Thesis

Dynamic Tone Reservation

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Master Thesis Abstract

Orthogonal Frequency Division Multiplexing (OFDM) is a widely used modulation technique and has several advantages compared with single carrier modulation technique. Many attractive features of OFDM make it the preferred modulation for next generation wireless communication systems.

A major drawback of OFDM is its high *Peak-To-Average Power Ratio* (PAR). Various techniques have been proposed to reduce high PAR. Tone reservation is one of them. In this thesis, *Dynamic Tone Reservation* (DTR), a recently proposed PAR reduction method based on the conventional tone reservation method is investigated.

In *Standard Tone Reservation* (STR), reserved tone locations remain fixed in each OFDM block. The receiver can simply ignore the data of reserved tones. In DTR, reserved tone locations are changing dynamically according to the actual realization of the OFDM symbol. It is impossible to know locations of reserved tones before signal transmission. The challenge of DTR is deciding whether a tone is a reserved tone or a data tone. If we do not remove reserved tones properly, the *Bit Error Rate* (BER) performance will be degraded. A method that can use the standard receiver without knowledge of reserved tone locations is preferred.

In this thesis, modifications to DTR are investigated. The goal of *Modified Dynamic Tone Reservation* (MDTR) is to obtain acceptable BER performance without knowledge of locations of reserved tones.

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Chapter 1

Introduction

Nowadays OFDM is a widely used modulation technique and has multiple advantages compared with single carrier modulation technique. It provides high spectral efficiency while allowing for low-complexity mitigation of time dispersion (or equivalently, frequency selectivity). These attractive features of OFDM make it the preferred modulation technique for next generation wireless communication systems. OFDM is already in use for *Wireless Local Area Network* (WLAN) standards, *Digital Audio Broadcasting* (DAB), and *Digital Video Broadcasting* (DVB).

A major drawback of OFDM is its high *Peak-to-Average Power Ratio* (PAR), which can cause high non-linear distortion. This distortion is caused by driving the power amplifier into its non-linear operating range. Non linear distortion of the transmit signal will degrade performance at the receiver side and is likely to cause unacceptable out-of-band leakage at the transmitter side. One way of avoiding distortion is a larger dynamic range of the *High Power Amplifier* (HPA), which decreases the power efficiency.

Various techniques to reduce PAR have been proposed. In this thesis, a recently proposed tone-reservation-based PAR reduction method is investigated: *Dynamic Tone Reservation* (DTR) [1]. DTR adjusts locations of reserved tones based on the actual transmission data. This adjustment will cause a *Bit Error Rate* (BER) increase since locations of reserved tones change from block to block. A modification of the constellation-point choice at the transmitter side is proposed in order to deal with this drawback of dynamic tone reservation.

1.1 Multicarrier Modulation

1.1.1 Concept of Multicarrier Modulation

The concept of using parallel data transmission and frequency division multiplexing was published in the mid 1960s [2]. In recent years, it became popular because VLSI technology advanced and allowed commercially affordable implementations [3].

Multicarrier Modulation (MCM) divides the available bandwidth into N parallel subcarriers. At same data rate, MCM has a longer symbol time than single carrier modulation. Long symbols make the system less sensitive to time dispersion caused by multipath propagation channels.

Modern communication systems, such as *3GPP Long Term Evolution Advanced* (LTE-Advanced), target very high data rates. Bandwidth is a scarce resource, and a limiting factor for higher data rate. OFDM is a special form of multicarrier modulation, and constitutes a balanced solution between bandwidth utilization and interference. By proper arrangement of N subcarriers, mutual orthogonality is assured. Orthogonal sinusoidal subcarriers have the following property: the correlation of a subcarrier with any of the remaining $N-1$ subcarriers is zero.

1.1.2 OFDM System

In an OFDM system, the transmit data is divided into blocks of N subcarriers. To obtain N time-domain transmit samples, a N-point *Inverse Discrete Fourier Transform* (IDFT) algorithm is proposed. In practical cases, we use the *Inverse Fast Fourier Transform* (IFFT), which is a highly optimized implementation of the IDFT.

The basic structure of an OFDM system is shown in Figure 1.1. A transmit data sequence is mapped to *Quadrature Amplitude Modulation* (QAM) or *Phase-shift keying* (PSK) constellation. After mapping, the modulated data is divided into N parallel sequences. An N-point IFFT transforms N parallel frequency domain symbols to time domain. To mitigate *Inter Symbol Interference* (ISI) and *Inter Carrier Interference* (ICI) caused by channel dispersion, the time domain signal is typically preceded by a *cyclic prefix* (CP) as guard interval. A CP is a copy of the last L samples of a transmit symbol. L is usually the length of the channel impulse response minus 1. The CP makes the linear convolution between the channels and the transmit signal appear as a circular convolution. After the IFFT, parallel to serial conversion yields the transmit symbol.

The *Digital to Analog Converter* (D/A) converts the discrete-time discrete-amplitude signal into a continuous-time continuous-amplitude signal. The analog signal is amplified by a *High Power Amplifier* (HPA) and transmitted over a channel. At the receiver side, as shown in Figure 1.1, the inverse processing compared to the transmitter side is carried out.

1.2 Peak-to-Average Power Ratio in OFDM

Power consumption is the limiting factor determining standby time of devices. A way to extend standby time is to decrease the PAR of the system. High PAR requires a higher power back-off of the HPA to keep it operating in its linear region [4]. For a certain average power level, consuming the lowest possible supply voltage while operating in the linear region is desirable. Decreasing PAR is thus a way to increase power efficiency of mobile devices.

Besides prolonging the standby time of mobile devices, there is another motivation to reduce the PAR level. Large PAR requires a D/A with sufficient dynamic range to accommodate the high peaks of the transmit signal. However, D/As with large dynamic range and a reasonable amount of quantization noise are very expensive. In an OFDM system, subcarriers are loaded with symbols randomly drawn

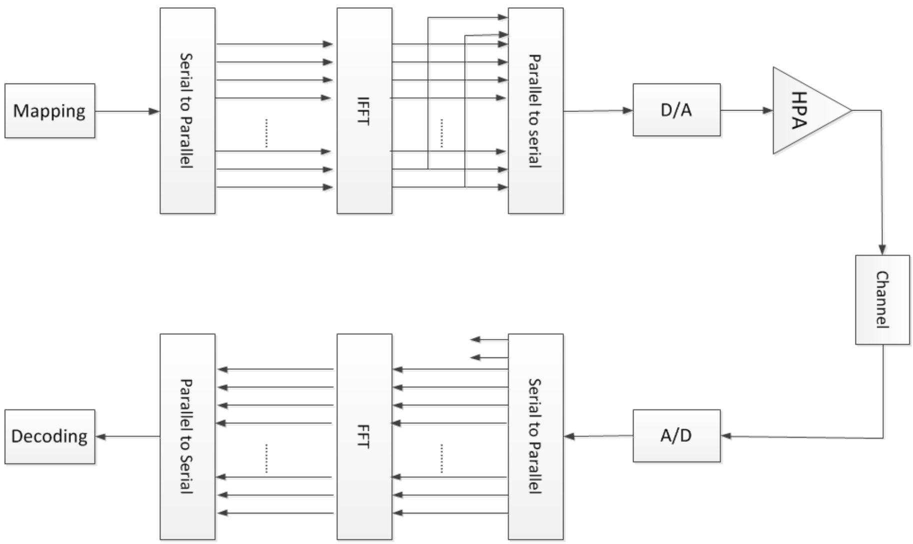


Figure 1.1: Block diagram of OFDM system.

from an alphabet. Due to the *Central Limit Theorem*, the summation of this large number of random sequences, will result in a sequence that is approximately Gaussian distributed. OFDM signals can thus be approximated by sequences of Gaussian-distributed samples.

1.2.1 Definition of PAR

The reason for high PAR of OFDM system lies in the superposition of many complex exponentials with different amplitudes and different phases. The output of the IFFT operation is a sum of N statistically independent terms:

$$x(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{\frac{j2\pi nk}{N}} \tag{1.1}$$

where N is the number of subcarriers of the OFDM system. $X_k, k = \{0, ..., N-1\}$ are symbols drawn from an alphabet.

The instantaneous PAR of an OFDM symbol is defined as:

$$PAR = \frac{\max_n |x(n)|^2}{\frac{1}{N}\sum\limits_{k=0}^{N-1} |x(k)|^2} \tag{1.2}$$

Where $\max_n |x(n)|^2$ is the maximum instantaneous power and $\frac{1}{N}\sum\limits_{k=0}^{N-1} |x(k)|^2$ is the average power of the block.

According to the *Central Limit Theorem*, a linear combination of a large number of independent random terms has Gaussian distribution. Consequently, for an

OFDM system with large N, real part and imaginary part of $x(n)$ have Gaussian distribution: $\Re\{x(n)\} \sim N(0, \frac{\sigma^2}{2})$, $\Im\{x(n)\} \sim N(0, \frac{\sigma^2}{2})$, $E\{\Re\{x(n)\}\Im\{x(n)\}\} = 0$.

Useful probabilistic measures of PAR include the probability that the PAR of a length-N OFDM symbol is below a certain level (CDF) or exceeds a certain level (CCDF). The absolute of $x(n)$ has Rayleigh distribution and its power follows exponential distribution. Under the consumption that real and imaginary part of $x(n)$ have Gaussian distribution, the probability that the amplitude of $x(n)$ exceeds a certain value p is given by the equation below:

$$Pr\{|x(n)| > p\} = e^{-(\frac{p}{\sigma})^2} \tag{1.3}$$

1.2.2 Power Amplifier

For convenience, a simple static *Solid State Power Amplifier* (SSPA) as HPA model is used to describe the motivation for PAR reduction. Let $s(t) = A(t) \exp j\theta(t)$ denote the signal after PAR reduction, where t is the time index, A is the amplitude of the transmitted signal and $\theta(t)$ is the phase. The amplified output signal of the SSPA can be modeled as:

$$v(t) = g(|A(t)|) \exp j(\theta(t) + \Phi(|A(t)|)) \tag{1.4}$$

$g(.)$ and $\Phi(.)$ are the AM/AM and the AM/PM conversions of the nonlinear amplifier, respectively. AM and PM stand for amplitude modulation and phase modulation, respectively. The AM/PM conversion can be neglected when AM/AM is approximated by [5]:

$$g(A) = \frac{A}{(1 + (\frac{A}{A_0})^{2p})^{\frac{1}{2p}}} \tag{1.5}$$

In Equation (1.5), A_0 is the saturation amplitude and p controls the smoothness of the transition from the linear region to the saturation region. A_0 and p are both positive values. If p is smaller, the SSPA will have worse linear performance. If p is infinite, the SSPA is an ideal clipping amplifier [6].

When the HPA reaches the saturation amplitude level, it can not deliver more power, no matter how much the input amplitude is increased. Amplifiers are rated at their 1dB compression point, which is the point at which the output power becomes nonlinear.

In order to avoid nonlinear distortion of the SSPA, a power back-off has to be applied to leave headroom for the fluctuation of the signal's amplitude. *Input Back-off* (IBO) is defined as:

$$IBO = 10\log_{10}(\frac{A_0^2}{E\{|s(t)|^2\}}) \tag{1.6}$$

The *Output Back-off* (OBO) is defined as:

$$OBO = 10\log_{10}(\frac{E\{|v(t)|^2\}}{A_0^2}) \tag{1.7}$$

In Equation (1.7), IBO 0dB means that the average power of the input signal is equal to the saturation power. When the amplifier operates around the saturation region, it will cause in-band distortion and *out-of-band* (OOB) distortion. In-band distortion degrades the BER performance. OOB distortion leads to high interference. The IBO value affects the CCDF of PAR of the HPA output signal and can be adjusted such that the clipping probability stays below a certain value. There is a tradeoff between the efficiency of the HPA and the level of distortion. The lower the PAR of the OFDM signal, the lower the IBO value and thus the higher the power efficiency.

1.2.3 Peak Regrowth in OFDM Systems

There are many methods to deal with high PAR of OFDM systems. However, reducing PAR of the discrete time signal $x(n)$ to a certain level does not guarantee the same PAR for the continuous time signal $x(t)$. As shown in Figure 1.2, peaks of $x(t)$ may grow after passing through the D/A. To truly capture the continuous time peaks, the discrete time signal $x(n)$ is over-sampled by a factor ϵ, usually $\epsilon \geq 4$ is sufficient [7].

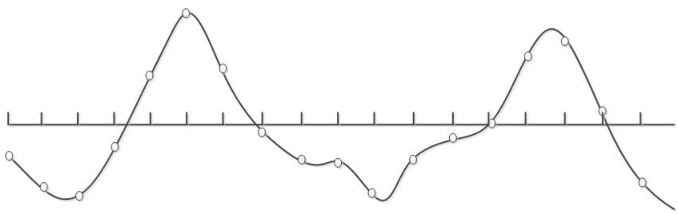


Figure 1.2: Peak re-growth of a continuous-time signal

1.3 PAR Reduction

1.3.1 Criteria for Selection of PAR Reduction Technique

There are many factors that need to be considered when choosing a specific technique to reduce high PAR [8]. The first factor is PAR reduction capability, which is most important. The second factor is spectral efficiency. The third factor that needs to be considered is the BER increasing at the receiver side after applying PAR reduction. There are some other considerations, like loss of data rate, computational complexity and so on.

1.3.2 PAR Reduction Methods

Reduction techniques always have some harmful effects. Careful consideration of system requirements is necessary. In this section we will briefly review some PAR reduction techniques.

Amplitude Clipping and Filtering

The most straightforward way to reduce PAR is clipping the parts that lie outside the allowed amplitude region [9]. Let us denote the clipping level A. A usually is the HPA saturation level. x_{out} is the output signal after clipping,

$$x_{\text{out}}(n) = \begin{cases} x_{\text{out}}(n), & \text{when } |x_{\text{out}}(n)| < A \\ A\ \texttt{sign}(x_{\text{out}}(n)), & \text{when } |x_{\text{out}}(n)| > A \end{cases} \tag{1.8}$$

There are several different kinds of clipping techniques. Usually, clipping is performed at the transmitter side, in digital domain after the IFFT. At the receiver side we need to estimate the clipping location. Clipping causes in-band distortion and *OOB* distortion. In-band distortion results in an increase of BER. *OOB* distortion can be reduced by filtering. However, filtering causes peak re-growth.

Coding

The main idea of PAR reduction through coding is to select those code words that can reduce the PAR for transmission [10]. The basic idea is to design codes whose codewords have lower PAR and to avoid transmit sequences which have high PAR. Coding needs to find the best codes and store large lookup tables for encoding and decoding [8].

Active Constellation Extension Technique

Active constellation extension extends some outer constellation points in order to minimize the PAR of an OFDM symbol [11]. As shown in Figure 1.3, for *Quaternary Phase Shift Keying* (QPSK) in [11], there are four alternative constellation points, real and imaginary axes are decision boundaries. Errors occur when noise translates the received point into one of the other three quadrants. Points that lie on the constellation boundary offer degrees of freedom for PAR reduction. It guarantees a lower BER performance. For this reason, we can modify constellation points within the quarter-plane as shown in Figure 1.3 with no degradation in BER performance. There is no data rate loss and no side information needed.

However, this method will increase transmit power. For large constellations, only a small part of the constellation points can be moved.

Tone Injection

The idea of tone injection [12] is to extend the constellation so that each of the points in the original basic constellation can be mapped onto several equivalent points in the extended constellation. This degree of freedom can be exploited to reduce PAR.

Assume that $M-ary$ *square quadrature amplitude modulation* (QAM) is used as a modulation scheme, where d is the minimum distance between constellation points. In this case, the real and imaginary parts of each symbol can take values in the set $\{\pm\frac{d}{2}, \pm\frac{3d}{2}, ..., \pm\frac{d(\sqrt{M}-1)}{2}\}$, where $\sqrt{M}$ is the number of levels per dimension. To reduce PAR, the tone injection method maps each input symbol onto

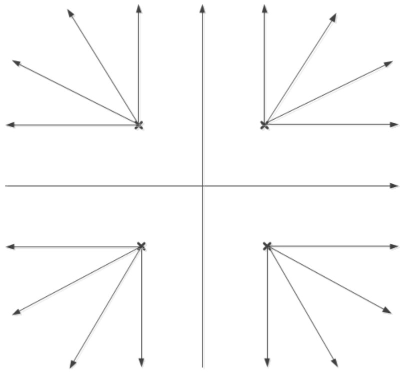


Figure 1.3: The ACE technique for QPSK modulation.

several equivalent points in the extended constellation. Mathematically, the tone injection method modifies the input symbol X by adding variables as follows:

$$X = X + pD + jqD \tag{1.9}$$

Where p and q are integer values, and D is a positive real number. In order not to decrease BER, D should be $D \geq d\sqrt{M}$. Thus, the receiver can perform a modulo-D operation to remove $(p + jq)D$ and map points back onto the original constellation.

An optimal tone injection scheme needs to search over all combinations of p and q and demands extra IFFT operators. The computational complexity is thus significant.

Tone Reservation

Like tone injection, tone reservation [13] is based on adding a time domain signal to the original signal to reduce PAR. Tone reservation selects some tones which do not transmit payload data but are modulated with a complex numbers chosen such that the PAR decreases. Since subcarriers are orthogonal in OFDM, modification of these reservation tones will not affect the data.

Figure 1.4 illustrates a block diagram of tone reservation. $\mathbf{Q}$ denotes the IFFT matrix. The transmit signal after tone reservation is given by $\mathbf{a} = \mathbf{Q}(\mathbf{X} + \mathbf{C}) = \mathbf{x} + \mathbf{c}$. The goal of tone reservation is to minimize the PAR of $\mathbf{a}$. $\mathbf{X}$ and $\mathbf{C}$ are frequency domain signals, and $\mathbf{x}$ and $\mathbf{c}$ are time domain signals. $\mathbf{X}$ and $\mathbf{C}$ are disjoint in frequency. In tone reservation, the reserved tones will use transmit power and decrease data rate. However, one of the advantages of tone reservation is that it does not require side information.

1.4 Convex Optimization

Convex optimization has been used in signal processing for a long time to solve problems off-line [14]. In [15] it is revealed that choosing values of reserved tones

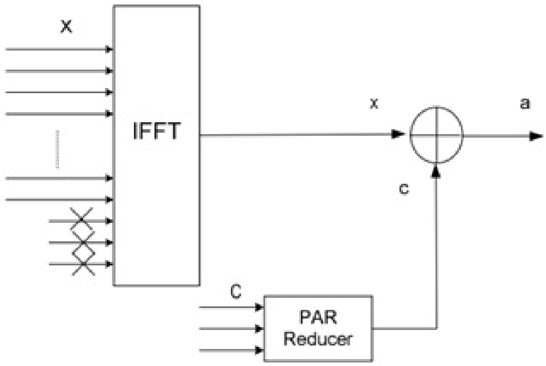


Figure 1.4: Tone reservation.

to minimize PAR can be cast as convex optimization problem.

1.4.1 Definition of Convex Optimization

A general optimization problem can be formulated as

$$\begin{array}{ll}\text{minimize} & f_0(\mathbf{x}) \\ \text{subject to} & f_i(\mathbf{x}) \leq b_i \quad i = 1, ..., m\end{array} \tag{1.10}$$

The vector $\mathbf{x}$ contains the optimization variables $x_1, ..., x_n$. $f_0 : \mathbf{R^n} \to \mathbf{R}$ is the objective function. The functions $f_i : \mathbf{R^n} \to \mathbf{R}, i = 1, ..., m$ are the constraint functions and the constants $b_1, ..., b_m$ are the limits for the constraints. A vector $\mathbf{x}^*$ is called optimal, or a solution of the Equation (1.10), if it has the smallest objective value among all vectors that satisfy the constraints: for any $\mathbf{z}$ with $f_1(\mathbf{z}) \leq b_1, ..., f_m(\mathbf{z}) \leq b_m$, we have $f_0(\mathbf{z}) \geq f_0(\mathbf{x}^*)$ [16].

There are some important functions and techniques for verifying convexity [17]. A function $f : \mathbf{R^n} \to \mathbf{R}$ is convex if its domain $\mathbf{dom}f$ is convex and the following inequality holds for all $\mathbf{x}, \mathbf{y} \in \mathbf{dom}f$, $\theta \in [0, 1]$:

$$f(\theta\mathbf{x} + (1-\theta)\mathbf{y}) \leq \theta f(\mathbf{x}) + (1-\theta)f(\mathbf{y}) \tag{1.11}$$

If f is concave then $-f$ is convex.

Figure 1.5 illustrates two examples of convex sets. The left set is a hexagon, which includes its boundary (shown darker), and it is convex. The right set is kidney shaped set and it is not convex, since the line segment between the two points in the set shown as dots is not contained in the set [16].

Convex optimization includes a broad class of optimization problem, for example least-squares linear programming (LP), quadratic programming (QP), second-

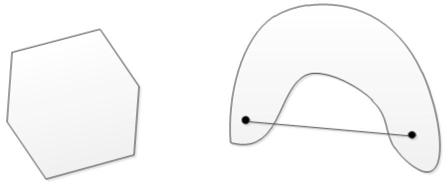


Figure 1.5: Convex definition

order cone programming (SOCP), semi-definite programming (SDP), and the l_1 minimization at the core of compressed sensing [14].

Convex problems can be solved using dedicated numerical solvers together with MATLAB interfaces, such as YALMIP [18], CVX [19], or CVXMOD [20], to name just a few recently developed parser-solvers.

1.4.2 Norms

Assume,

$$\mathbf{a} = \begin{bmatrix} a_1 & a_2 & \cdots & a_N \end{bmatrix} \tag{1.12}$$

The l_1- norm of $\mathbf{a}$ is

$$\|\mathbf{a}\|_1 = \sum_{i=1}^{N} |a_i| \tag{1.13}$$

If $p \geq 1$, the l_p- norm of $\mathbf{a}$ is

$$\|\mathbf{a}\|_p = (\sum_{i=1}^{N} |a_i|^p)^{\frac{1}{p}} \tag{1.14}$$

Chapter 2

Standard Tone Reservation

Tone reservation was proposed by Gatherer and Polley [22], Tellado and Cioffi [13]. A subset of tones is chosen as reserved tones. Those reserved tones are added to the original signal to cancel the peaks of the OFDM signal. Because there is orthogonality between OFDM subcarriers, reserved tones are not adding distortion to data subcarriers. In this chapter, the original tone reservation is introduced referred hereinafter as *Standard Tone Reservation* (STR). STR is the basis for the *Dynamic Tone Reservation* (DTR) algorithm which is discussed in the next chapter.

The chapter is organized as follows. In Section 2.1, we show the definition of STR. Section 2.2 deals with using convex optimization to solve the STR problem. Section 2.3 investigates the reserved tone allocation strategies and power re-growth properties of STR.

2.1 Definition Of Standard Tone Reservation

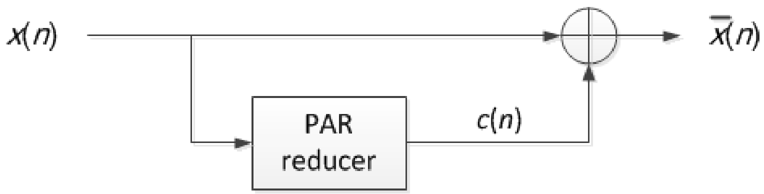


Figure 2.1: A block diagram of tone reservation.

OFDM system with N subcarriers $S \subset \{0, 1, 2, 3, \ldots, N-1\}$. STR reduces the signal peaks by sacrificing a set of data tones S_r as reserved tones. The set S_r with size N_{rt} is used for synthesizing a PAR reduction signal which is then added to the original OFDM signal. The remaining set of tones S_d is refilled as *Data Tone* set with size N_d. The output in time domain after peak reduction is tone reservation can be expressed as:

$$\begin{aligned}\bar{x}(n) &= x(n) + c(n) \\ &= \text{IFFT}(X_k + C_k) \\ &= \tfrac{1}{\sqrt{N}} \textstyle\sum_{k=0}^{N-1} (X_k + C_k) e^{\frac{j2\pi kn}{N}}\end{aligned} \tag{2.1}$$

$x(n)$ and $c(n)$ are time domain signals, X_k and C_k are frequency domain signals. Using vector notation, the frequency domain vector $\mathbf{C} = [C_0, C_1, ..., C_{N-1}]^T$ is added to the original data vector $\mathbf{X} = [X_0, X_1, ..., X_{N-1}]^T$. The set of reserved tones and the set of data tones are two disjoint sets, $S_r \cup S_d = S$, $N_d + N_{rt} = N$, which can be formulated as:

$$X_k + C_k = \begin{cases} C_k & \text{if } k \subset S_r, \\ X_k & \text{if } k \subset S_d \end{cases} \tag{2.2}$$

After tone reservation, the PAR is given by:

$$PAR = \frac{\max_n |x(n) + c(n)|^2}{E[|x(n)^2|]} \tag{2.3}$$

The reduction signal $c(n)$ can be constructed using different methods, such as convex programming algorithms, or with low-complex heuristics aiming at reducing a single peak [23]. The advantage of STR is that it does not need side information. Reserved tones can be removed easily at the receiver side. STR does not degrade the BER performance.

In order to deal with peak re-growth and to capture the continuous-time peaks as shown in Figure 1.2, the continuous-time signal is approximated by an over-sampled version of the discrete-time signal. After applying an over-sampled IDFT, (2.1) can be written as:

$$\bar{x}(n) = \frac{1}{\sqrt{N\epsilon}} \sum_{k=0}^{N\epsilon-1} (X_k + C_k) e^{\frac{j2\pi(k-1)(n-1)}{N\epsilon}} \tag{2.4}$$

for $k = 1, ..., N$, $n = 1, ..., \epsilon N$.

The discrete time signal $\bar{x}(n)$ is over-sampled by a factor $\epsilon = 4$.

2.2 Solution of STR using Convex Optimization

For real-valued transmit signals, a linear programming method can be used to find optimized solution of reserved tones [13]. This method is suitable for baseband systems (DMT). Tellado [13] also proposed an iterative sub-optimal algorithm based on gradients that has a lower computational complexity.

The tone reservation is shown in Equation (2.1), adding a frequency domain signal $\mathbf{C}$ to original signal $\mathbf{X}$, to minimize the maximum peak value of $\bar{x}(n)$,

$$\min_{\mathbf{C}} \max |\mathbf{Q}_\epsilon(\mathbf{X} + \mathbf{C})|_\infty \tag{2.5}$$

where $\bar{\mathbf{x}} = \mathbf{x} + \mathbf{c} = \mathbf{Q}_\epsilon(\mathbf{X}+\mathbf{C})$. $\mathbf{Q}_\epsilon$ is the IFFT operation, $\mathbf{Q}_\epsilon[k,n] = \frac{1}{\sqrt{\epsilon N}} e^{j2\pi \frac{(k-1)(n-1)}{\epsilon N}}$, for $k = 1, ..., N$, $n = 1, ..., \epsilon N$. So the PAR minimization problem can be written in convex form,

$$\begin{aligned} &\min_{\mathbf{C}} \quad t \\ &\text{subject to} \quad |\mathbf{Q}_\epsilon(\mathbf{X}+\mathbf{C})|_\infty \le t \\ &C_k = 0 \quad \text{if } k \subset S_d \end{aligned} \tag{2.6}$$

where t is the peak of OFDM symbol. Problem (2.6) has to be solved for every OFDM symbol.

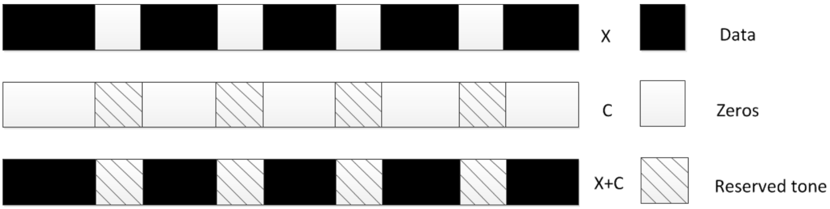


Figure 2.2: Example of reserved tone locations to reduce PAR.

Figure 2.2 illustrates how reserved tones and data tones form an OFDM symbol when added together. The algorithm can be summarized as follows:

Step 1. Choose S_r.

Step 2. Load tones in S_d with data while tones in S_r are set to zero.

Step 3. Minimize maximum peak of $|\mathbf{Q}_\epsilon(\mathbf{X}+\mathbf{C})|_\infty$ by solving (2.6) to find the values of $\mathbf{C}$. Set locations in $\mathbf{C}$ corresponding to S_d to zero.

Step 4. Add $\mathbf{X}$ and $\mathbf{C}$ together and perform IFFT.

2.3 Optimal Solution of Standard Tone Reservation

For STR, the set S_r is fixed before transmission. Since S_r is known, reserved tones can be removed directly at receiver side without side information. But finding the optimal choice of reserved tones in OFDM is a hard problem. Furthermore, a power constraint for reserved tones is required.

2.3.1 Location of Reserved Tones

In [24], Petersson proposed some ways to locate reserved tone, such as blocking the highest tones, equal-spaced tones or random tones. Blocking of highest tones means to place the reserved tone set at the border of the available frequency band. Equal-spaced tones means that there is equal space between reserved tones. Random tones means to generate a random set S_r for each OFDM symbol.

In the following the three schemes are compared for uncoded 16QAM modulated OFDM symbol. Among the three reserved tone placement schemes, random

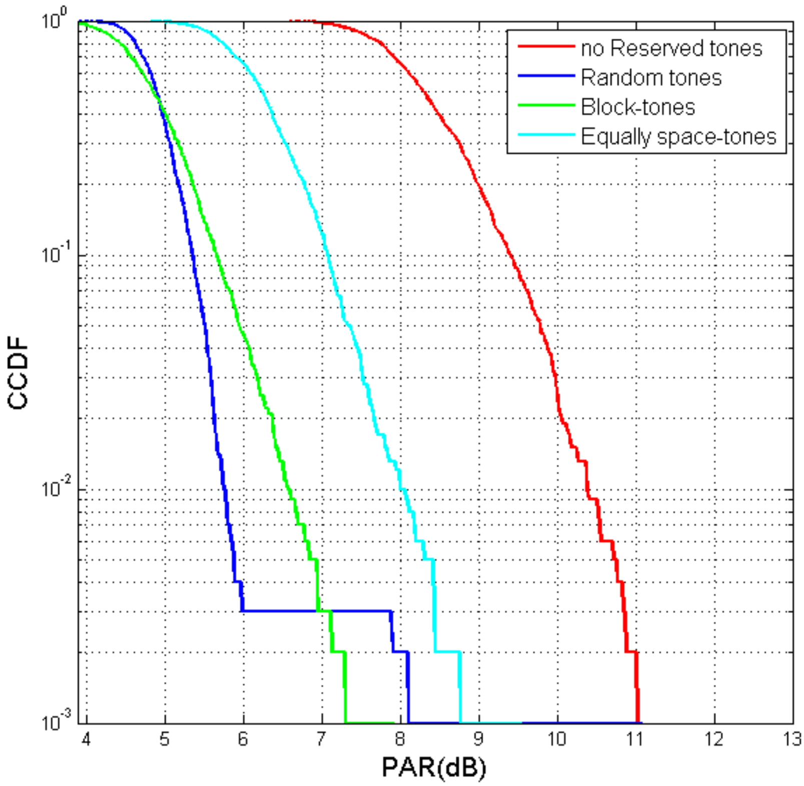


Figure 2.3: Comparison of different location schemes of reserved tones: no reserved tone, random tones, equal-spaces tones and block of highest tones. Number of OFDM subcarriers is 256; number of reserved tones is 8. Modulated with 16QAM.

tones have best CCDF of PAR. The simulation result is shown in Figure 2.3. $N_{rt} = 8$, S_r is chosen among 256 subcarriers as follows:

Random Tones S_r is generated by a random vector from $S \subset \{0, 1, 2, 3, \ldots, 255\}$.

Equal Tones $S_r = \{32, 64, 96, 128, 160, 192, 224, 255\}$

Block Tones $S_r = \{248, 249, 250, 251, 252, 253, 254, 255\}$

The random tones choice has about 1.5dB PAR gain over equal-spaced tones at 10^{-3}. In the low probabilities range, the random tones choice tends to perform the same as the other two placement schemes.

A simple model for a class-A power amplifier [25] is an ideal linear HPA model, where linear amplification is achieved to the saturation point, η is the HPA efficiency and it is defined as

$$\eta = \frac{P_{out,avg}}{P_{DC}} \tag{2.7}$$

where $P_{out,avg}$ is the average output power and P_{DC} is the bias power consumed by the amplifier regardless of the input power. For class A amplifiers, $P_{DC} = 2P_{out,max}$. Thus, (2.7) yields

$$\eta = \frac{0.5}{PAR} \tag{2.8}$$

As shown in Figure 2.3, in order to guarantee that the probability of clipping is less than 10^{-3} without peak reduction (that means, guarantee that on average no more than 1 in 1000 symbols are clipped), an *IBO* equal to the PAR level at probability 10^{-3} is required ($PAR = 11$dB). Thus the efficiency of the HPA is $\eta = \frac{0.5}{10^{1.1}} \approx 3.9\%$ to amplify an OFDM signal with 256 subcarriers using a class-A power amplifier. After tone reservation, using a randomly chosen set of 8 reserved tones, $\eta = \frac{0.5}{10^{0,7}} \approx 10\%$ to amplify an OFDM signal with 256 subcarriers. Increased efficiency is the strong reason why applied tone reservation in an OFDM system.

2.3.2 Power Increase of Reserved Tone

A challenge of tone reservation is the power increase on reserved tone. The average power increase can be written as

$$\Delta E = 10\log_{10}\left(\frac{E\{\|\mathbf{x}+\mathbf{c}\|^2\}}{E\{\|\mathbf{x}\|^2\}}\right) \tag{2.9}$$

where $E\{\|\mathbf{x}+\mathbf{c}\|^2\}$ is average power of time domain signal after using tone reservation, $E\{\|\mathbf{x}\|^2\}$ is average power of the original time domain signal. PAR reduction is achieved by adding power to all tones. The PAR gain and ΔE comparison of three reserved tone placement schemes compared in Figure 2.3 is presented in Table 2.1.

Table 2.1: Power Increase and PAR Gain Comparison of STR

	Random	Equal	Block
ΔE	0.66dB	0.30dB	0.81dB
PAR gain	4dB	2.5dB	3dB

In Table 2.1, the PAR gain is the PAR level difference compared at probability 10^{-3}. The power increase of the block-tone STR is higher than the equal-tone STR. But the power increase of the random-tone STR is lower than block-tone STR, the PAR gain of the random tones is still 1dB better compared to block-tone STR. So adding more power to reserved tone and the locations of reserved tones are both important for PAR reduction performance of tone reservation. In practice, it is impossible to add very high power to reserved tones. The question is how to allocate reserved tone and how much power to add to them in order to reach the required PAR level. In [26], the trade-off between target PAR level and allowed reduction magnitude of reserved tones is discussed.

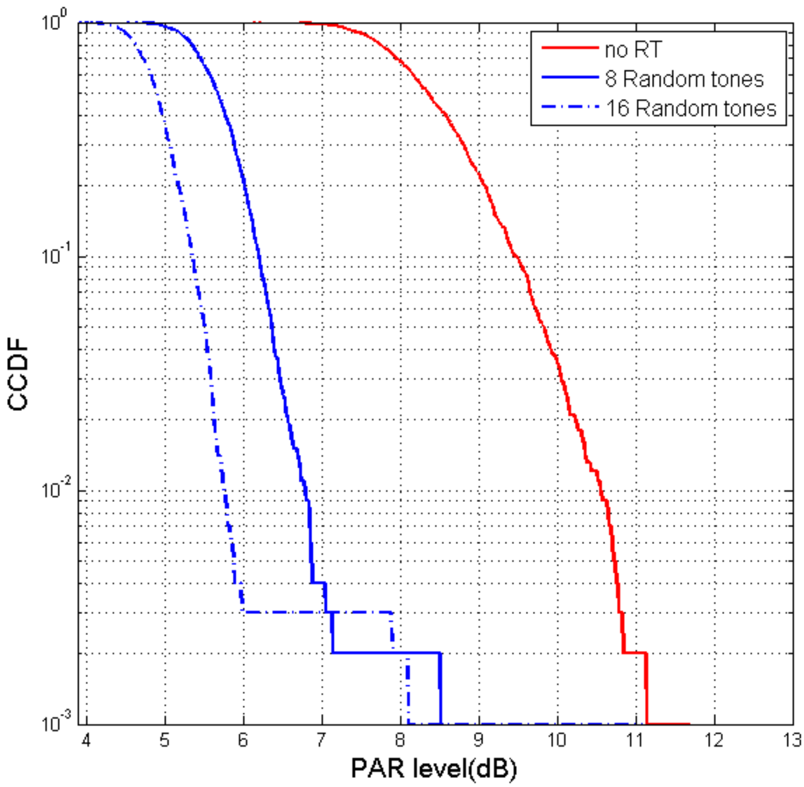


Figure 2.4: Number of reserved tones is 8 and 16. CCDF of PAR performance comparison.

The performance of PAR reduction gain of tone reservation also depends on the number of reserved tones. Increasing the number of reserved tones, is another way to increase the power to achieve the desired PAR level. In Figure 2.5, the same parameter as in Figure 2.4 are used, but the number of reserved tones is increased from 8 to 16. As shown, the CCDF of PAR is about 1dB better. There is a trade-off between PAR reduction performance and system throughput. Reserving tones for PAR reduction means that those tones are not available for carrying data.

Chapter 3

Dynamic Tone Reservation

As discussed in Section 2.3.1, a random choice of S_r yield the best PAR performance compared to other strategies, such as equally spaced tones STR and block tones STR. For all those strategies, S_r is selected before transmission.

A static choice of S_r (that is, same for all symbols) is clearly suboptimal in the sense of achieving the lowest PAR for a given N_{rt}. In this chapter, a new S_r is chosen for each symbol based on its actual realization—an idea referred to as *Dynamic Tone Reservation* (DTR) [1].

In Section 3.1, the DTR algorithm is introduced. Simulation results of the DTR algorithm are presented in Section 3.2. Section 3.3 presents a conclusion based on the simulation results.

3.1 DTR Algorithm For Finding S_r

Let $\mathbf{X}$ and $\mathbf{C}$ denote two vectors of same length N. The following convex problem is solved to find S_r:

$$\begin{aligned}&\min_{\mathbf{C}}\ t\\&\text{subject to } |\mathbf{Q}_\epsilon(\mathbf{X}+\mathbf{C})|_\infty \le t\\&|\mathbf{C}|_1 \le N_{rt}\sqrt{\tau}\sigma\end{aligned} \tag{3.1}$$

where τ is a power level constraint to control the total power of reserved tones. N_{rt} is the number of reserved tones. σ^2 is the average power of data tones in an OFDM symbol. Γ is the power constraint in dB domain. The relationship between τ and Γ is $\tau = 10^{\frac{\Gamma}{10}}$. $|\mathbf{C}|_1$ denotes the l_1 norm of the reserved tone vector (cf. (1.13)). The idea of the l_1 norm constraint is to obtain a $\mathbf{c}$ with only a few significant values, whose positions indicate S_r. The average power of data tones is calculated symbol by symbol. For each OFDM symbol, (3.1) yields a set S_r to minimize the PAR. At the same time, the power of reserved tones is kept below a defined value.

As shown in Figure 3.1, after obtaining S_r by solving Equation (3.1), Equation (2.6) is solved to find the values of reserved tones. The DTR algorithm can be summarized as follows:

Step 1. Find the optimal value of $\mathbf{C}$ to minimize PAR ($|\mathbf{Q}_\epsilon(\mathbf{X}+\mathbf{C})|_\infty$) of the OFDM symbol. $|\mathbf{C}|_1$ is kept below $N_{rt}\sqrt{\tau}\sigma$, which is the total power level of reserved tones.

Step 2. The positions of the N_{rt} largest elements of $|\mathbf{C}|$ yield S_r.

Step 3. Set the values of $\mathbf{X}$ at locations included in S_r to zero (cf. Figure 2.2).

Step 4. Solve Equation (3.1) for the $\mathbf{X}$ obtained in Step 3.

Step 5. Adding $\mathbf{X}$ and $\mathbf{C}$ yields the OFDM symbol to transmit.

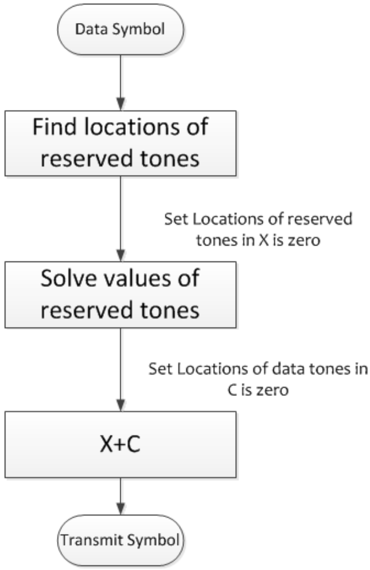


Figure 3.1: Flow chart of Dynamic Tone Reservation.

3.2 Simulation Result of Dynamic Tone Reservation

This section presents simulation results of DTR and the comparison of the PAR reduction performance between DTR and STR using uncoded 16QAM modulation on all 128 subcarriers.

In Figure 3.2, the CCDF of PAR of STR with random tone set, DTR, and original OFDM signal are compared. A Γ is used as power constraint $\Gamma = 3$dB for both DTR and STR. The simulation results show that DTR has more than 1.5dB PAR reduction gain compared to STR at probability 10^{-3}. Compared to STR with random tone set without a power constraint, DTR with a 3dB power constraint has about 0.5dB better PAR reduction performance at probability 10^{-3} as shown in Figure 3.2.

In Figure 3.3, a power constraint level of 6dB is used for both STR and DTR. The PAR level difference between STR and DTR is a bit smaller compared to Figure 3.2 but still larger than 1dB at 10^{-3}. In Figure 3.4, the power constraint is increased to 10dB. The CCDF of PAR of DTR compared to STR is still 0.5dB at

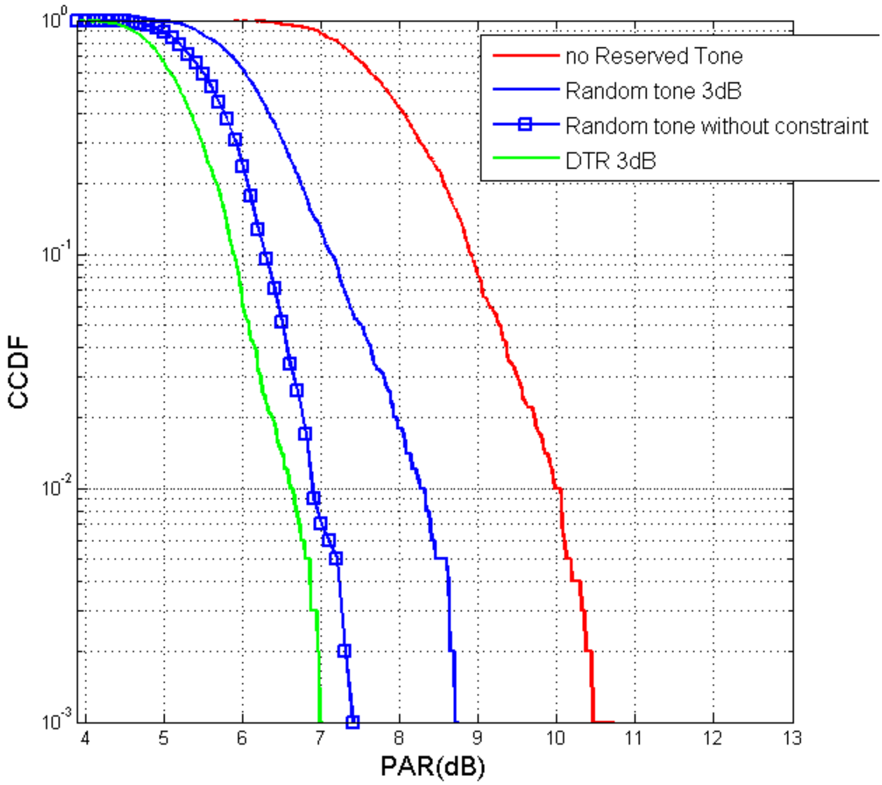


Figure 3.2: 16QAM, $N = 128$. $N_{rt} = 4$. Comparison of CCDF of PAR of DTR, the random tones STR and the original OFDM signal. Power constraint Γ is 3dB.

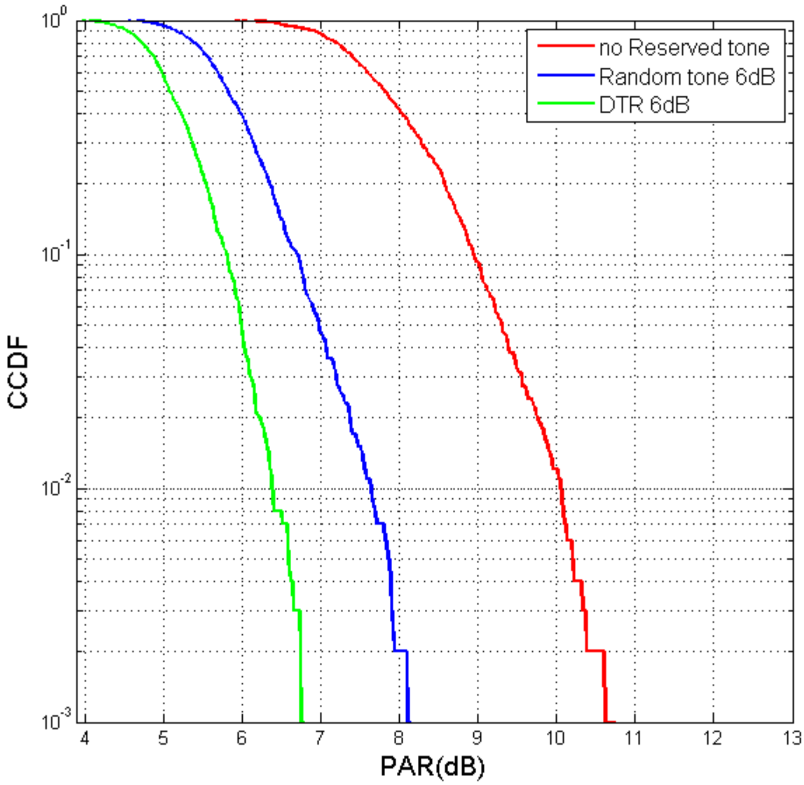


Figure 3.3: 16QAM, $N = 128$. $N_{rt} = 4$. Power constraint Γ is 6dB. CCDF of PAR comparison between DTR, the random tones STR and the original OFDM signal.

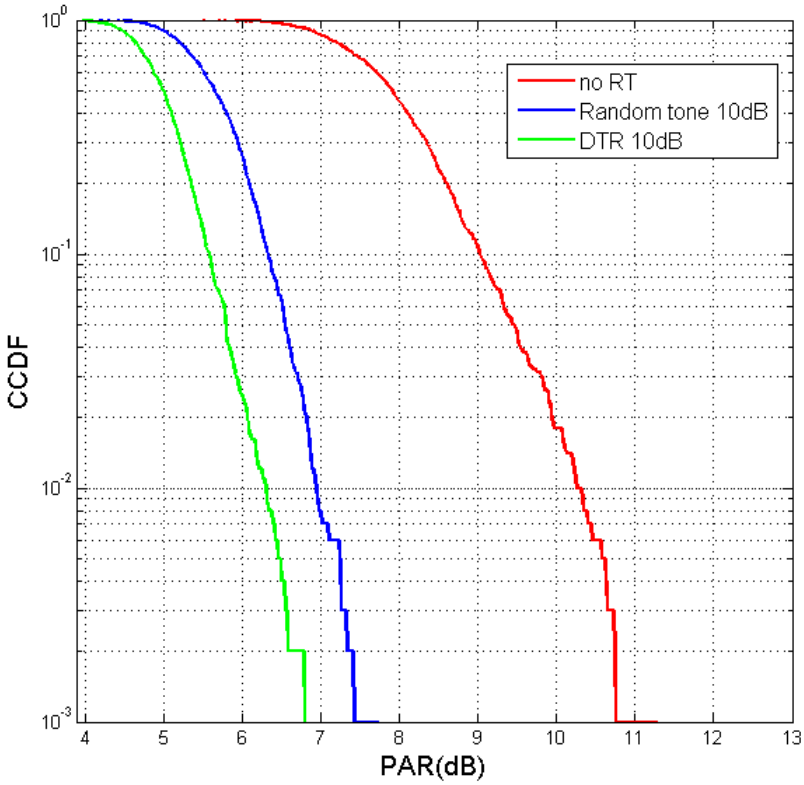


Figure 3.4: 16QAM, $N = 128$. $N_{rt} = 4$. Power constraint Γ is 10dB. CCDF of PAR comparison between DTR, the random tones STR and the original OFDM signal.

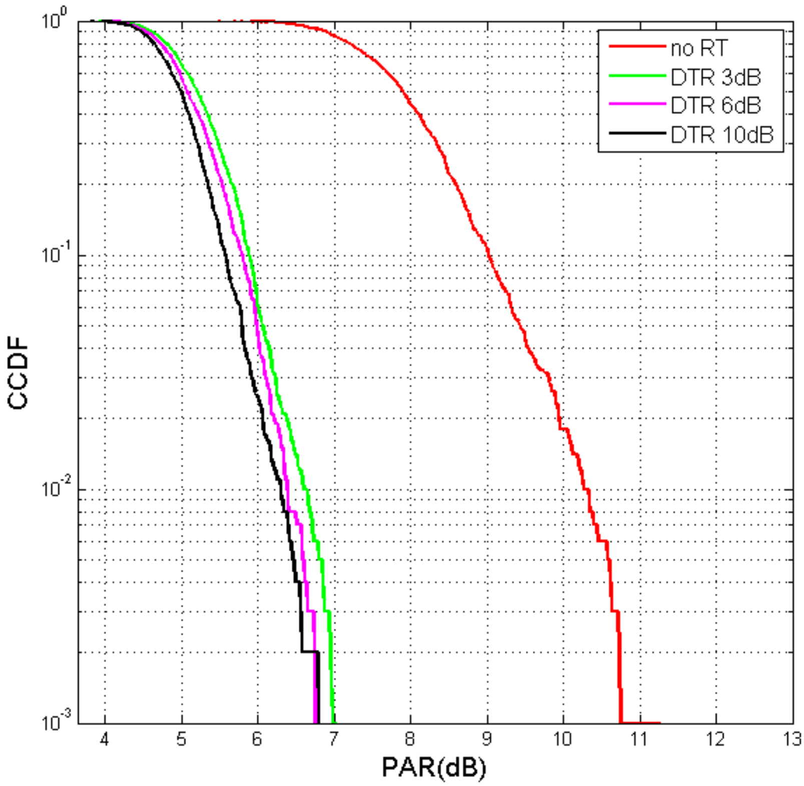


Figure 3.5: 16QAM, $N = 128$. $N_{rt} = 4$. Comparison the CCDF of PAR of NTR and DTR with different power constraints.

10^{-3}. When Γ is 10dB the CCDF of STR is almost the same as for STR without power constraint (Figure 3.2).

Figure 3.5 shows how the power constraint level influences the performance of DTR. Γ is 3dB, 6dB and 10dB respectively. The CCDF of PAR does not improve significantly with Γ. When Γ is 10dB the CCDF of PAR is only about 0.3dB better than with Γ 3dB at probability 10^{-3}.

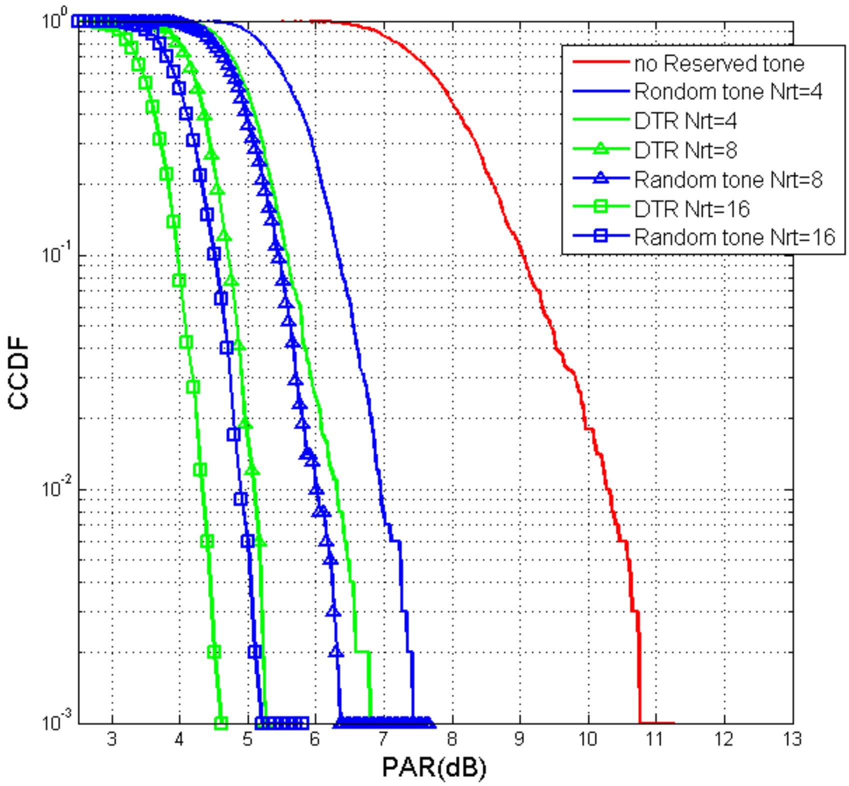


Figure 3.6: 16QAM, $N = 128$. Comparison the CCDF of PAR of NTR and DTR with 16 reserved tones, 8 reserved tones and 4 reserved tones. Power constraint Γ is 10dB

In Figure 3.6, the number of reserved tones is increased from 4 to 8 and 16. Γ is 10dB. When increasing the number of reserved tones, the PAR level of DTR compared to STR does not show a clear improvement. But the PAR level of DTR compared to the original OFDM signal, is decreased by about 5dB to 6.5dB at 10^{-3}.

3.3 Conclusion of Simulation Results

This section summarizes the simulation results above. Table 3.1 compares the power increase of tones after DTR and STR. The PAR reduction gain for three power constraint levels is also shown in Table 3.1. In Section 3.3, PAR gain is defined as PAR difference between the original OFDM signal and the proposed peak reduction method at probability 10^{-3}.

All results are average values over 1000 OFDM symbols. When increasing the power constraint from 3dB to 10dB, the PAR gain of DTR increases slightly (from 3.5dB to 4dB). The power increase ΔE is around 0.25dB. For STR, along with increasing the power constraint level, ΔE is increasing from 0.22dB to 0.67dB. The PAR gain is increasing from 1.7dB to 3.2dB at the same time.

Although ΔE of STR is increasing by a larger margin, the PAR gain of DTR is always better than STR. Considering Figure 3.2 and Figure 3.4, when Γ of STR is 10dB, the CCDF of PAR is almost as same as without a power constraint for STR. So even without a power constraint for STR, when Γ of STR and DTR is 10dB, the PAR gain of STR is still worse than DTR (0.8dB).

Table 3.1: Power Increasing and PAR Gain of STR and DTR

	STR			DTR		
Γ	3dB	6dB	10dB	3dB	6dB	10dB
ΔE	0.22dB	0.42dB	0.67dB	0.27dB	0.24dB	0.21dB
PAR gain	1.7dB	2.4dB	3.2dB	3.5dB	4dB	4dB

Table 3.2 shows the relation between PAR gain and *Tone Reservation Ratio* (R). R is defined as

$$R = \frac{N_{rt}}{N} \tag{3.2}$$

where N is the number of tones per OFDM symbol and, N_{rt} is the number of reserved tones per OFDM symbol. In Table 3.2, N is 128, N_{rt} is 4, 8, and 16. The power constraint is 10dB for all cases. All symbols are modulated with uncoded 16QAM. We average the results every 1000 OFDM symbols.

When N_{rt} is increased from 4 to 8 and 16, the PAR gain of DTR is always around 1dB better than STR. For both DTR and STR, when N_{rt} is increasing, the PAR gain is increasing. At same time, ΔE is decreased along with N_{rt}. Especially for DTR, the decline of ΔE is severe. When the number of reserved tones is 16 for DTR, ΔE is decreased by -0.38dB. DTR always has better PAR reduction performance than STR regardless of R.

In conclusion, both in terms of PAR reduction performance and power increase, DTR is superior to STR. When R increased, we have better PAR reduction performance, but spectral efficiency is decreased.

The purpose of PAR is to increase the HPA efficiency η. Every 3dB PAR reduction will double the HPA efficiency. Table 3.3 summarizes HPA efficiency results for using uncoded 16QAM, 128 subcarriers, 8 reserved tones, and power

Table 3.2: PAR Performance For Different N_{rt}

	STR			DTR		
N_{rt}	4	8	16	4	8	16
ΔE	0.67dB	0.60dB	0.43dB	0.21dB	0.05dB	-0.38dB
PAR gain	3.2dB	4.6dB	5.6dB	4dB	5.6dB	6.2dB
R	0.03	0.06	0.13	0.03	0.06	0.13

constraint level $\Gamma = 10$dB. DTR has more than three percent better HPA efficiency than STR.

Table 3.3: HPA Efficiency Comparison STR and DTR

	STR	DTR
PAR level	6dB	5dB
η	12.5%	15.8%

Chapter 4

Modified Dynamic Tone Reservation

In previous chapters, we have analyzed *Dynamic Tone Reservation* and compared its PAR reduction performance to *Standard Tone Reservation*. DTR has better PAR reduction performance than STR. But the receiver of DTR signals has the problem of deciding which tone is a reserved tone. If we do not remove reserved tones properly, the BER performance will be degraded. In STR, S_r remains fixed for every OFDM block. The receiver can simply ignore the data on reserved tones and correctly decode the received symbols. When applying DTR, S_r is changing dynamically according to the actual realization of the OFDM symbol. We do not know S_r before signal transmission. For this reason, DTR requires a strategy that can be used by the standard receiver to obtain acceptable BER performance without knowledge of S_r. The strategy is hereinafter referred to as *Modified Dynamic Tone Reservation* (MDTR).

The principle of MDTR is introduced in Section 4.1. The proposed schemes of modification are described in Section 4.2. The receiver of MDTR is presented in Section 4.3. Simulation results and conclusions presented in Section 4.4.

4.1 Principle of Modified Dynamic Tone Reservation

The basic idea of MDTR is that if a point lies far outside the constellation boundary, then its associated tone is with high probability a reserved tone. Points of reserved tones are moved in a special fashion at the transmitter side so that they have sufficient distance to help the receiver make a decision.

The idea of MDTR is to modify the values on reserved tone such that the receiver benefits from the additional distance information. Like in DTR, in MDTR Equations (2.6) and (3.1) are solved to find values and locations of reserved tones, respectively. Subsequently, MDTR modifies the value of a reserved tone if the distance between the point and any nominal constellation points is smaller than a certain distance constraint. Real part and imaginary part of the complex-baseband signal are dealt with separately.

4.2 Schemes of Modified Dynamic Tone Reservation

Figure 4.1 sketches a single OFDM symbol in constellation domain after applying the DTR algorithm ($N = 64$ tones, $N_{rt} = 8$ reserved tones, uncoded 16QAM on

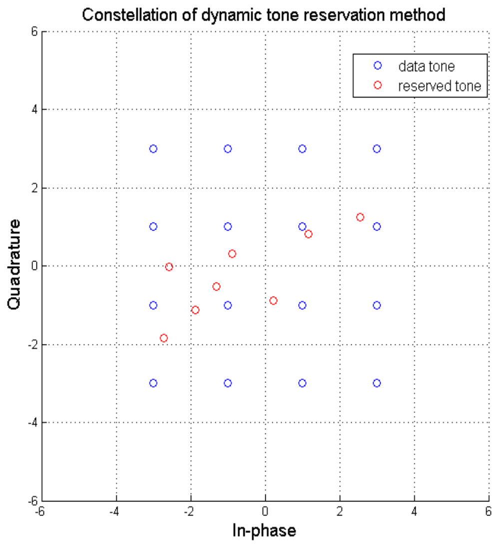


Figure 4.1: 16QAM, $N = 64$. $N_{rt} = 8$. Constellation of dynamic tone reservation before modification.

all data tones). The blue points are data tones, and the red points are reserved tones. If the receiver does not know the locations of reserved tones, it is impossible to decode correctly even in the high SNR region. In Figure 4.1, some reserved tones (red points) are in the same decision areas as data tones (blue points). Dotted lines in Figure 4.1 are decision boundaries. In order to provide additional distance information, MDTR tries to move reserved tones far away from nominal constellation points.

Three different schemes to modify OFDM symbols after applying DTR are investigated in terms of CCDF of PAR and BER performance: moving points to the decision boundary, moving points to the corner of the decision boundary and moving points outside the constellation boundary. The three schemes will be explained in following sections.

4.2.1 Moving Points to the Decision Boundary (Scheme 1)

The first scheme moves points on reserved tones to decision boundaries along the shortest path. Compared with Figure 4.1, in Figure 4.2 the points on reserved tones are moved to the nearest decision boundaries.

Hereinafter, 16QAM is used. The in-phase component and quadrature component are encoded by assigning information bit points 01, 00, 10, 11 to the levels 3d, d, -d, -3d, respectively. In our simulation, d is 1. The minimum distance between constellation points is 2. The distance constraint in Scheme 1 is set to 1. The Euclidean distance is used as distance measure.

X_n, $n \subset S_r$ are points on reserved tones. The nominal constellation point is $P(k) \subset \{\pm 1 \pm j, \pm 1 \pm 3j, \pm 3 \pm j, \pm 3 \pm 3j\}$, $k = 1, 2, ..., 2^M$. M is the number of

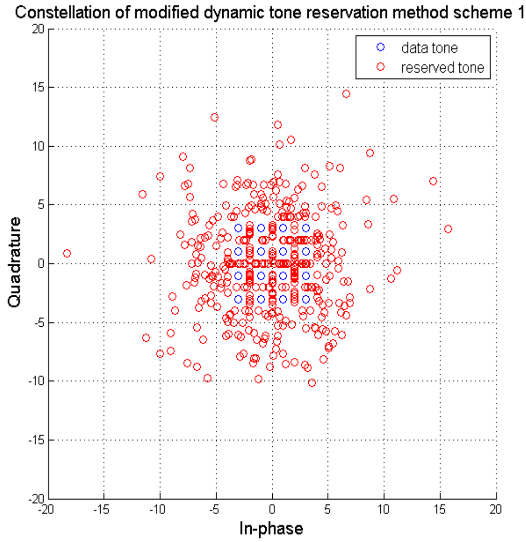


Figure 4.2: 16QAM, $N = 64$. $N_{rt} = 8$. 100 OFDM symbols. Constellation of modified dynamic tone reservation. Moving points to decision boundary along shortest path.

bits per symbol. For QPSK, 16QAM and 64QAM, M is 2, 4 and 6, respectively. The points are moved as follows:

Step 1. Consider one reserved tone X_n. Calculate the distances from $\Re\{X_n\}$ to the real parts of all nominal constellation points.

$$D_r(k) = |\Re\{X_n\} - \Re\{P(k)\}|, \quad k = 1, 2, ..., 2^M \tag{4.1}$$

Calculate the distances from $\Im\{X_n\}$ to the imaginary parts of all nominal constellation points.

$$D_i(k) = |\Im\{X_n\} - \Im\{P(k)\}|, \quad k = 1, 2, ..., 2^M \tag{4.2}$$

Step 2. $D(k) = \max\{D_r(k), D_i(k)\}$, $k = 1, 2, ..., 2^M$. Find the minimum distance,

$$D = \min_k D(k) \tag{4.3}$$

The index of nearest point is

$$k_{\min} = \arg\min_k D(k) \tag{4.4}$$

Step 3. If $D(k)$ is smaller than 1, the reserved tone X_n should be moved to the nearest decision boundary. The nearest constellation point to the reserved tone X_n is $P(k_{\min})$.

Step 4. Calculate the distance from $P(k_{\min})$ to X_n:

$$d_r = \Re\{P(k_{\min})\} - \Re\{X_n\} \tag{4.5}$$

$$d_i = \Im\{P(k_{\min})\} - \Im\{X_n\} \tag{4.6}$$

MDTR always moves points on reserved tones along the shortest path. If $|d_r| < |d_i|$, the real part of the new moved reserved point X_k^{new} is $\Re\{X_n\}$. The imaginary part of X_k^{new} is,

$$\Im\{X_k^{\text{new}}\} = \begin{cases} \Im\{P(k_{\min})\} - 1 & \text{if } d_i > 0, \\ \Im\{P(k_{\min})\} + 1 & \text{if } d_i < 0 \end{cases} \tag{4.7}$$

If $|d_r| \geq |d_i|$, the imaginary part of the new moved reserved point X_k^{new} is $\Im\{X_n\}$. The real part of X_k^{new} is,

$$\Re\{X_k^{\text{new}}\} = \begin{cases} \Re\{P(k_{\min})\} - 1 & \text{if } d_r > 0, \\ \Re\{P(k_{\min})\} + 1 & \text{if } d_r < 0 \end{cases} \tag{4.8}$$

Step 5. Apply Step 1 to Step 4 to all the reserved tones.

Moving reserved tones to decision boundaries along the shortest paths ensures that MDTR modifies the original values of reserved tones as little as possible. As shown in Figure 4.2, it sketches the constellation diagram after moving reserved tones to the decision boundary after 100 OFDM symbols. If $D(k)$ is larger than 1, the reserved tone will not be moved. Many reserved tones outside the constellation are thus kept at their original place.

4.2.2 Moving points to the corner of decision boundary (Scheme 2)

Scheme 2 moves reserved tones to the corners of decision boundaries. Compared to Figure 4.1, in Figure 4.3 (a) red points with a distance $D(k) \leq 10$ are moved to the corners of decision boundaries. That means not only reserved tones inside the constellation boundary but also the reserved tones near to the boundary plane will be moved. Reserved tones far away from constellation boundary are not modified.

The points are moved as follows:

Step 1. Carry out Step 1 and Step 2 of Scheme 1 as described in Section 4.2.1.

Step 2. If D given by Equation (4.3) is smaller than 10, the point on reserved tone X_n will be moved. $P(k_{\min})$ is the nearest constellation point to the reserved tone.

Step 3. Calculate the distance from X_n to $P(k_{\min})$,

$$d = X_n - P(k_{\min}) \tag{4.9}$$

Depending on the distance from d to $\{1+j, 1-j, -1+j, -1-j\}$, determine the new point:

$$X_k^{\text{new}} = \begin{cases} P(k_{\min}) + (-1-j) & \text{if } |d-(-1-j)| \text{ is smallest,} \\ P(k_{\min}) + (-1+j) & \text{if } |d-(-1+j)| \text{ is smallest,} \\ P(k_{\min}) + (1+j) & \text{if } |d-(1+j)| \text{ is smallest,} \\ P(k_{\min}) + (1-j) & \text{if } |d-(1-j)| \text{ is smallest,} \end{cases} \tag{4.10}$$

Step 4. Carry out Step 1 to Step 3 for all the reserved tones in an OFDM symbol.

Figure 4.3 (a) shows the constellation points of 100 OFDM symbols after modification. Points on reserved tones with $D(k) > 10$ are moved to the corners of decision boundaries.

4.2.3 Moving Points Outside the Constellation Plane (Scheme 3)

Scheme 3 moves points on reserved tones outside the constellation boundary. Figure 4.4 shows the constellation points for 100 OFDM symbols after modification. The distance constraint is $2\sqrt{2}$. Scheme 3 results in larger modification of points on reserved tones than Scheme 1 (moving points to the decision boundary). The steps of this scheme are described as following.

In the following, 16QAM is used. In-phase and quadrature phase are selected from $\{-3d, -d, d, 3d\}$, where $d = 1$. The largest absolute value on each dimension (real or imaginary) of the 16QAM constellation is thus $P_{\max} = 3$.

Step 1. Carry out Step 1 and Step 2 of Scheme 1 as described in Section 4.2.1. If D is smaller than distance constraint $2\sqrt{2}$, then X_n should be moved to outside the constellation boundary.

Step 2. Compute:

$$d_{r1} = |-\Re\{X_n\} + P_{\max}| \tag{4.11}$$

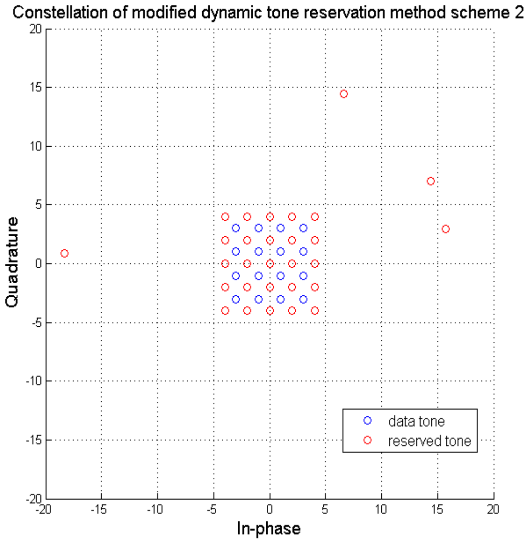
$$d_{r2} = |-\Re\{X_n\} - P_{\max}| \tag{4.12}$$

$$d_{i1} = |-\Im\{X_n\} + P_{\max}| \tag{4.13}$$

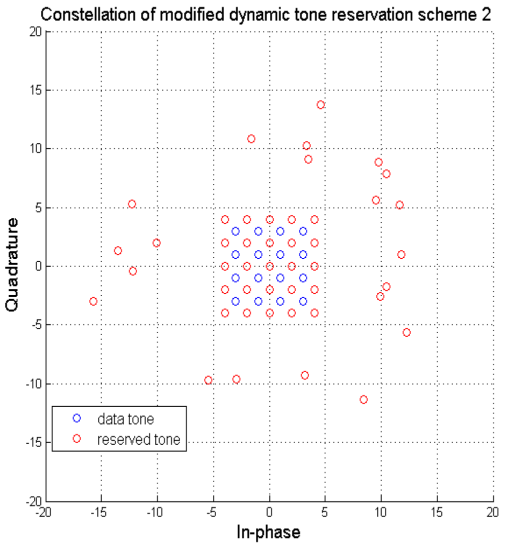
$$d_{i2} = |-\Im\{X_n\} - P_{\max}| \tag{4.14}$$

Step 3. Find the minimum value among d_{r1}, d_{r2}, d_{i1}, d_{i2}. The new point on the reserved tone is,

$$X_k^{\text{new}} = \begin{cases} \Im\{X_n\}j + P_{\max} + 2\sqrt{2} & \text{if } d_{r1} \text{ is smallest,} \\ \Im\{X_n\}j - (P_{\max} + 2\sqrt{2}) & \text{if } d_{r2} \text{ is smallest,} \\ \Re\{X_n\} + (P_{\max} + 2\sqrt{2})j & \text{if } d_{i1} \text{ is smallest,} \\ \Re\{X_n\} - (P_{\max} + 2\sqrt{2})j & \text{if } d_{i2} \text{ is smallest,} \end{cases} \tag{4.15}$$



(a) 100 OFDM symbols. Distance constraint is 10.



(b) 100 OFDM symbols. Distance constraint is 6.

Figure 4.3: 16QAM. $N = 128$. $N_{rt} = 4$. Constellation of modified dynamic tone reservation. Moving points to the corner of decision boundary.

As shown in Figure 4.4, points on reserved tones that are inside the constellation boundary are moved to the boundary. Step 3 ensures that the reserved

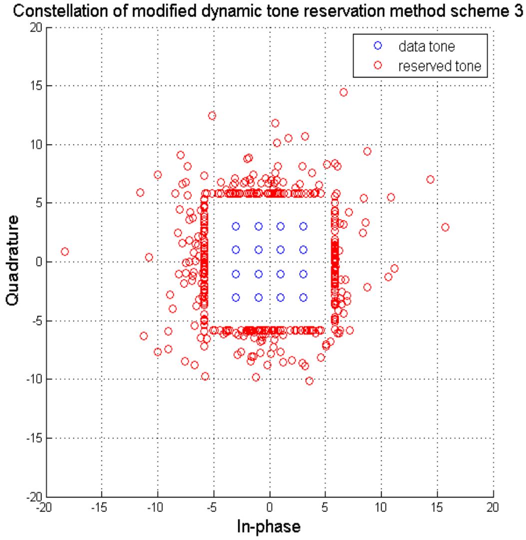


Figure 4.4: 16QAM, $N = 64$. $N_{rt} = 8$. 100 OFDM symbols. Constellation of modified dynamic tone reservation. Moving points outside constellation plane.

tones are moved along shortest paths. Moving along shortest paths minimizes the performance degradation of MDTR compared to DTR.

4.3 Receiver for Modified Dynamic Tone Reservation

At receiver side, received points on reserved tones appear "to be pushed away" from original constellation positions. In order to minimize the degradation of BER at the receiver side, the points on reserved tones must be discarded before decoding. Let $\mathbf{Z}$ denote a received OFDM symbol and let $Z(n)$ denote the received point on subcarrier No. n, $n \subset 1, 2, ..., N$. The receiver performs the following steps:

Step 1. For each OFDM symbol, calculate the absolute distances from the received point $Z(n)$ to all nominal constellation points $P(k)$, $k = 1, 2, 3, ...2^M$ of the constellation.

$$d_k(n) = |Z(n) - P(k)|, \ k = 1, 2, 3, ...2^M \tag{4.16}$$

Step 2. Find the smallest distance $Z_{\min}(n) = \min_k d_k(n)$.

Step 3. Carry out Step 1 and Step 2 for all points of a received OFDM symbol yields a distance information vector $\mathbf{Z}_{\min} = \{Z_{\min}(1), ..., Z_{\min}(N)\}$.

Step 4. Sort values in $\mathbf{Z}_{\min}$ in descending order. Tones corresponding to the N_{rt} largest distances are reserved tones.

Because locations of reserved tones are changing dynamically, the MDTR algorithm needs to be applied to each symbol. For each OFDM symbol, points on reserved tones must be removed.

4.4 Simulation Results and Conclusion

In this section, different MDTR schemes are compared in terms of CCDF of PAR and BER performance by means of simulation.

4.4.1 CCDF of PAR of Different MDTR Schemes

Figure 4.5 (a) shows the CCDF of PAR of the three MDTR schemes for QPSK, 16QAM and 64QAM. The power constraint Γ, as defined in Equation (3.1), is set to 3dB. In general, modulation types are not that important for the performance of STR compared to ACE (Section 1.3.2).

As shown in Figure 4.5 (a), the modulation type has almost no impact on the CCDF of PAR except for Scheme 2.

Let ρ denote the *Modification Ratio* defined as the number of modified reserved tones divides by the total number of reserved tones:

$$\rho = \frac{\text{Number of Modified Tones}}{\text{Number of Reserved Tones } N_{rt}} \tag{4.17}$$

Table 4.1: Comparison of PAR Reduction Performance and ρ

	Scheme 1		Scheme 2		Scheme 3	
Modulation type	QPSK	64QAM	QPSK	64QAM	QPSK	64QAM
ρ	0.532	0.452	0.995	0.897	0.912	0.582
PAR level	7dB	7dB	8.8dB	7.8dB	7.5dB	7.2dB

Table 4.1 compares ρ and PAR levels for the three modified schemes. $N_{rt} = 4$ for all modulation types. For Scheme 1, the PAR level is essentially the same for QPSK and for 64QAM. There is no significant dependence of ρ on the modulation type. In Scheme 1, reserved tones are modified in a relatively small area compared with the other two schemes.

In Scheme 2, CCDF of PAR of 64QAM is about 1dB improvement compared with the performance of QPSK at probability 10^{-3}. As shown in Figure 4.7 (a), Scheme 2 with QPSK moves all reserved tones to the corners of the decision boundaries. When modulated with 64QAM, there are reserved tones whose $D(k)$ is larger than 10 and are thus not moved. Thus CCDF of PAR performance of 64QAM is better than QPSK of Scheme 2 in Figure 4.5 (a).

In Figure 4.5 (a), 64QAM CCDF of PAR performance is better than QPSK and 16QAM when distance constraint is 10. The CCDF of PAR performance of 16QAM in Figure 4.5 (b) with distance constraint equal to 6 is closer to the performance of 64QAM in Figure 4.5 (a) with distance constraint equal to 10. As shown in Figure 4.10 (b), the BER performance for 16QAM is almost the same for both distance constraint 6 and distance constraint 10.

For Scheme 3, the ρ-values for QPSK and 64QAM are 0.912 and 0.582, respectively. The PAR improvement is only 0.3dB. The constellation diagram after 100 OFDM symbols is shown in Figure 4.8 (a) and 4.8 (b) for QPSK and 64QAM, respectively. For QPSK, more points on reserved tones are pushed away from their original locations.

The biggest gap of ρ does not necessarily lead to the biggest difference in PAR performance (cf. Scheme 2).

Table 4.2: Power increase Comparison

	Scheme 1		Scheme 2		Scheme 3	
Modulation type	QPSK	64QAM	QPSK	64QAM	QPSK	64QAM
ΔE(dB)	0.367	0.285	0.176	0.171	1.169	0.424

Table 4.2 summarizes the power increase ΔE for three schemes. Scheme 2 exhibits the lowest ΔE and the worst PAR reduction ability of the three schemes. The PAR performance of Scheme 2 depends strongly on the distance constraint. Scheme 1 has the best PAR performance and its ΔE is slightly higher than Scheme 2.

Scheme 3 with QPSK yields a ΔE of 1.169dB. ΔE is decreasing to 0.424dB when modulated with 64QAM. Although Scheme 3 has worst PAR reduction than Scheme 2, ΔE of Scheme 3 is higher than of Scheme 2.

Figure 4.9 shows that an increase in the number of reserved tones is beneficial to PAR reduction performance of MDTR.

4.4.2 BER Performances of MDTR Schemes

The motivation of MDTR is to obtain a lower BER than with DTR. For $d = 1$, the minimum distance $D_{\min}$ between two constellation points is 2. The transmitted energy per bit (Eb) is defined as

$$E_b = \frac{D_{\min}^2}{2} \frac{M-1}{3\log_2 M} \tag{4.18}$$

M is the number of bits per symbol. For QPSK, 16QAM and 64QAM, M is 2, 4 and 6 , respectively. The SNR range is -2dB to 25dB. And $\text{snr} = 10^{\frac{\text{SNR}}{10}}$.

After transmitting the signal through the AWGN channel, the receiver described in Section 4.3 is used to detect and discard reserved tones. The BER simulation results are shown in Figure 4.10 (a). STR has better BER performance than DTR and MDTR because the information of locations of reserved tones is known. MDTR improves the BER performance compared to the DTR algorithm.

Moving points to the decision boundary (Scheme 1) yields the best PAR performance. Scheme 2 results in an overall larger distance to the nominal constellation points compared to Scheme 1. Thus, Scheme 2 has a better BER performance than Scheme 1 for SNRs larger than roughly 10dB. Scheme 3 achieves the best BER performance. For SNRs larger than 5dB, the BER performance of Scheme 3 is essentially equal to the performance of STR.

The BER performance of MDTR using QPSK, 16QAM and 64QAM is shown in Figure 4.11, Figure 4.12, and Figure 4.13, respectively. MDTR is less effective for larger constellations.

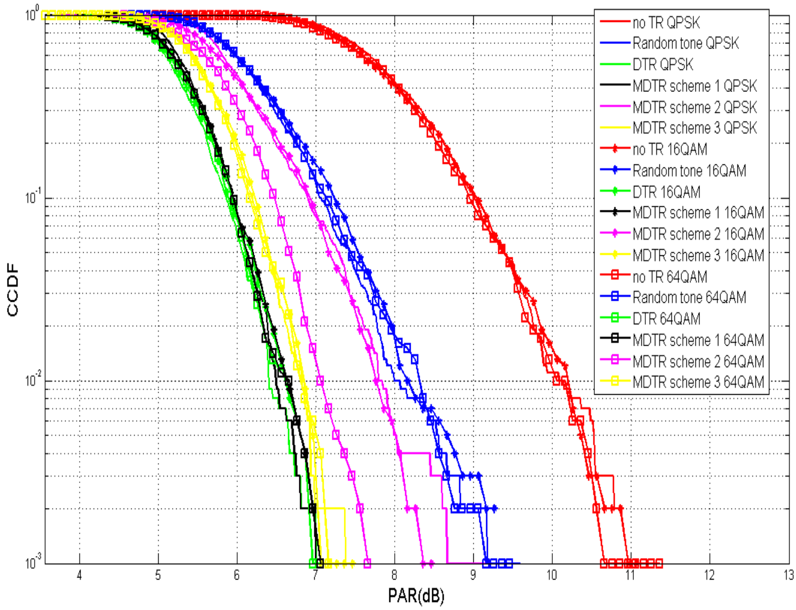
4.4.3 Conclusion

Modified Dynamic Tone Reservation is an effective method to improve the BER performance of *Dynamic Tone Reservation*.Three different approaches to moving points after DTR are investigated. They allow a tradeoff between BER performance and PAR reduction.

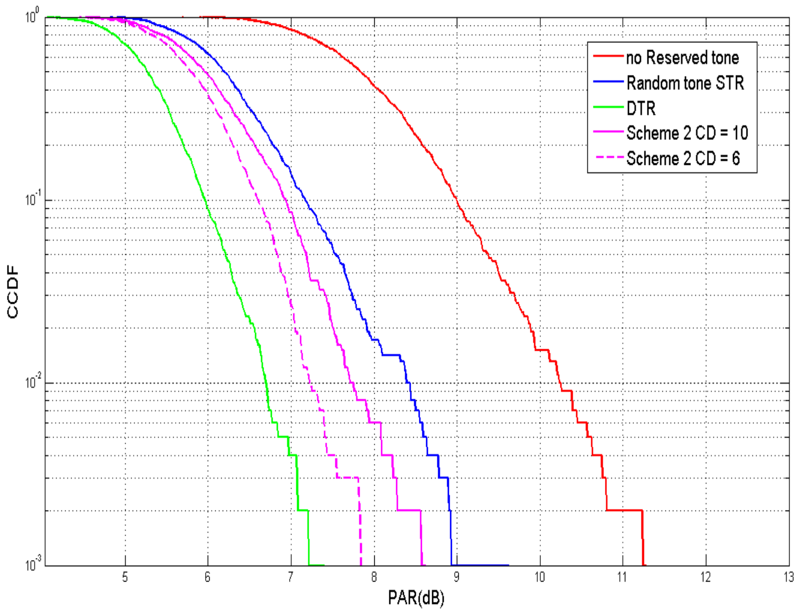
Scheme 1 (moving points to decision boundaries) achieves the best PAR reduction since points are moved in a relatively small area. In terms of BER performance, Scheme 1 is worse than the other two schemes.

The PAR reduction of Scheme 2 (moving points to corners of decision boundaries) depends on the distance constraint and on the modulation type. When choosing a suitable distance constraint, the PAR reduction ability of Scheme 2 is close to Scheme 1 and Scheme 3. At same time, Scheme 2 has lowest power increase of all the schemes. In terms of BER performance, Scheme 2 is worse than Scheme 3 but better than Scheme 1.

Scheme 3 (moving points outside the constellation plane) has the best BER performance of the three schemes. However, it also exhibits the highest power increase especially when modulated with QPSK. In terms of PAR reduction performance, Scheme 3 is a little worse than Scheme 1.

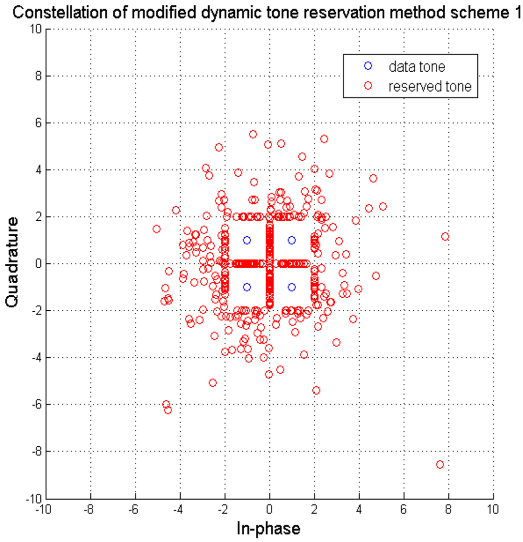


(a) CCDF of PAR comparison of different MDTR schemes been modulated with QPSK, 16QAM, 64QAM.

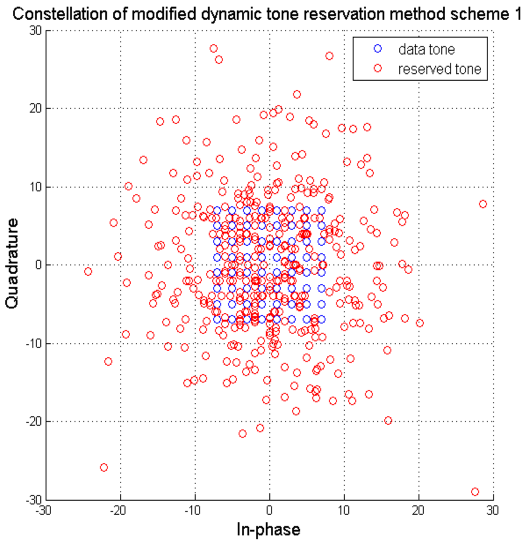


(b) CCDF of PAR comparison different distance constraint for scheme 2 using 16QAM.

Figure 4.5: $N = 128$. $N_{rt} = 4$. CCDF of PAR comparison.

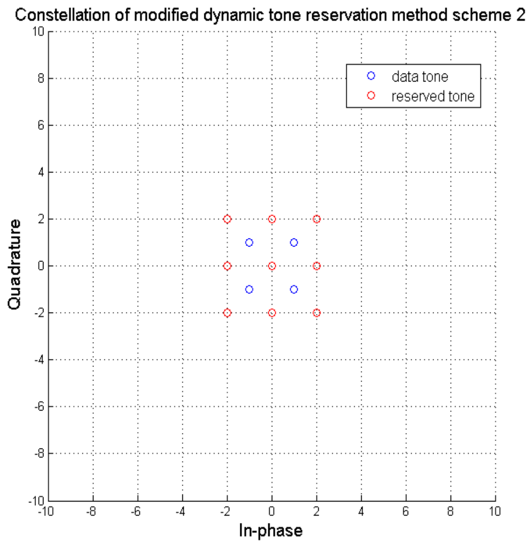


(a) QPSK, $N = 128$. $N_{rt} = 4$. 100 OFDM symbols.

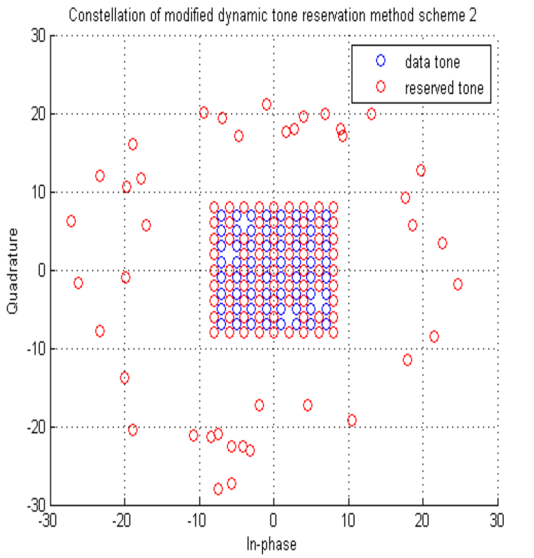


(b) 64QAM, $N = 128$. $N_{rt} = 4$. 100 OFDM symbols.

Figure 4.6: Constellation of modified dynamic tone reservation. Moving points to the decision boundary.

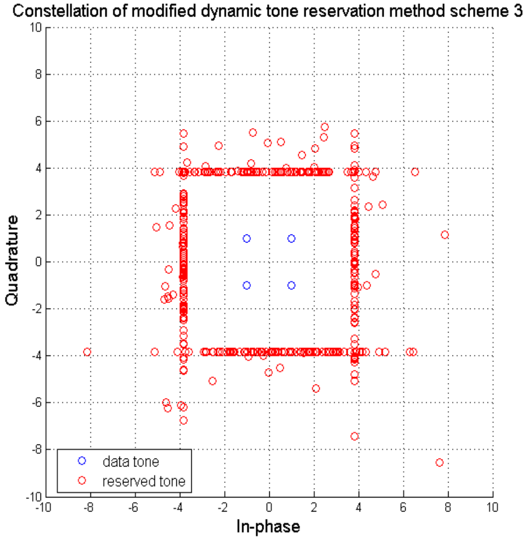


(a) QPSK, $N = 128$. $N_{rt} = 4$. 100 OFDM symbols.

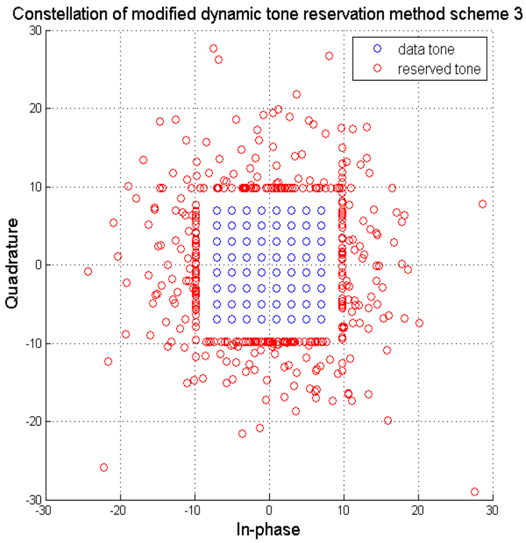


(b) 64QAM, $N = 128$. $N_{rt} = 4$. 100 OFDM symbols.

Figure 4.7: Constellation of modified dynamic tone reservation. Moving points to the corner of decision boundary.



(a) QPSK, $N = 128$. $N_{rt} = 4$. 100 OFDM symbols.



(b) 64QAM, $N = 128$. $N_{rt} = 4$. 100 OFDM symbols.

Figure 4.8: Constellation of modified dynamic tone reservation. Moving points outside the constellation plane.

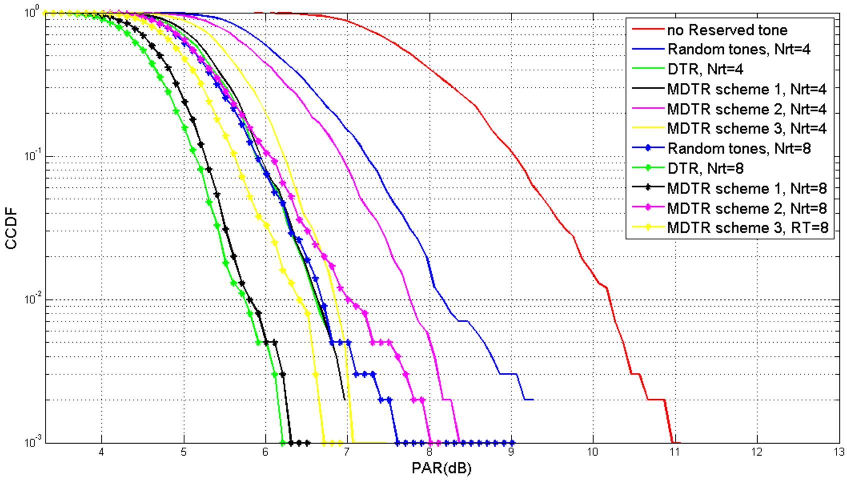
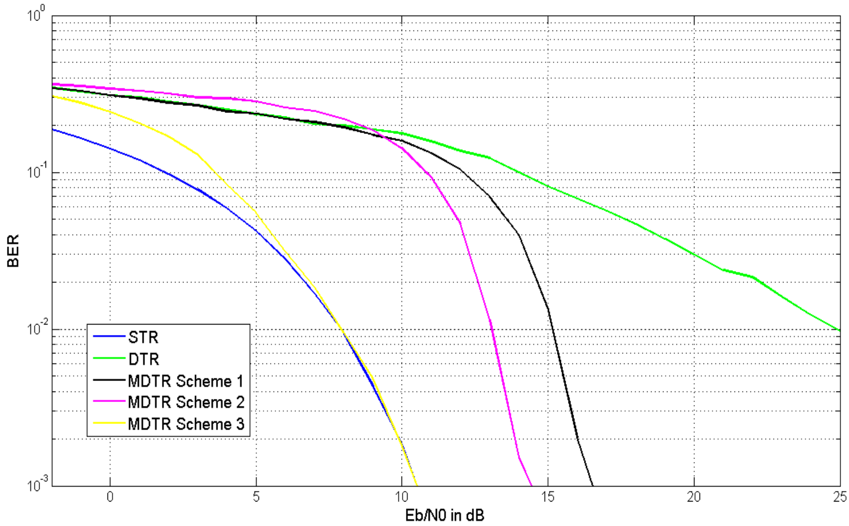
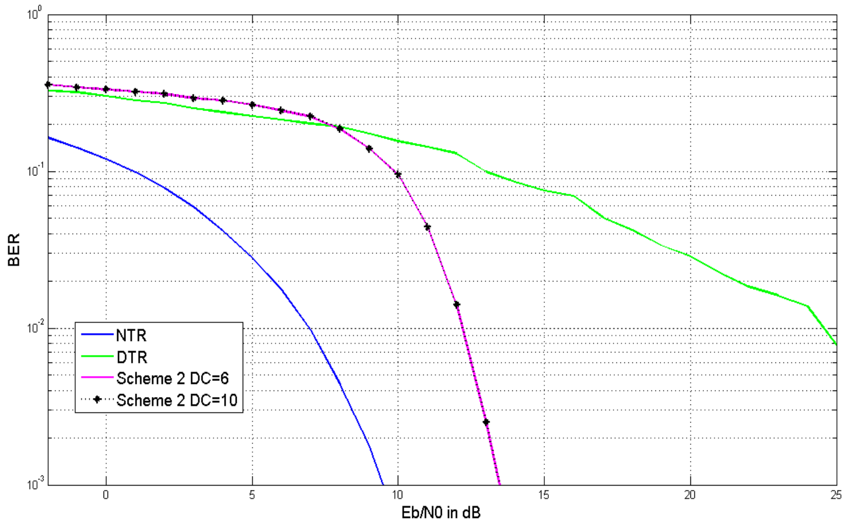


Figure 4.9: $N = 128$, $N_{rt} = 4$ and $N_{rt} = 8$. CCDF of PAR comparison of different MDTR schemes been modulated with 16QAM.



(a) Distance constraint is 10.



(b) BER comparison of different distance constraints for scheme 2.

Figure 4.10: 16QAM, $N = 128$. $N_{rt} = 4$. BER comparison of different MDTR schemes.

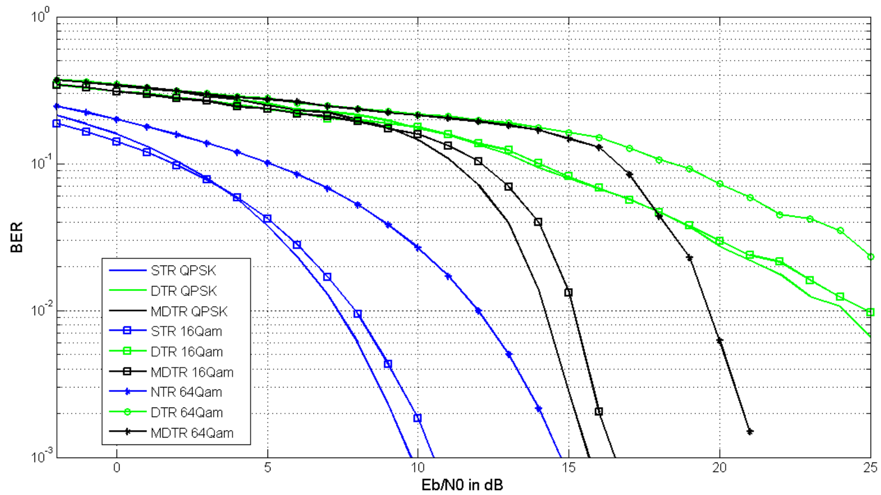


Figure 4.11: BER performance of Schemes 1 when modulated with QPSK, 16QAM, 64QAM.

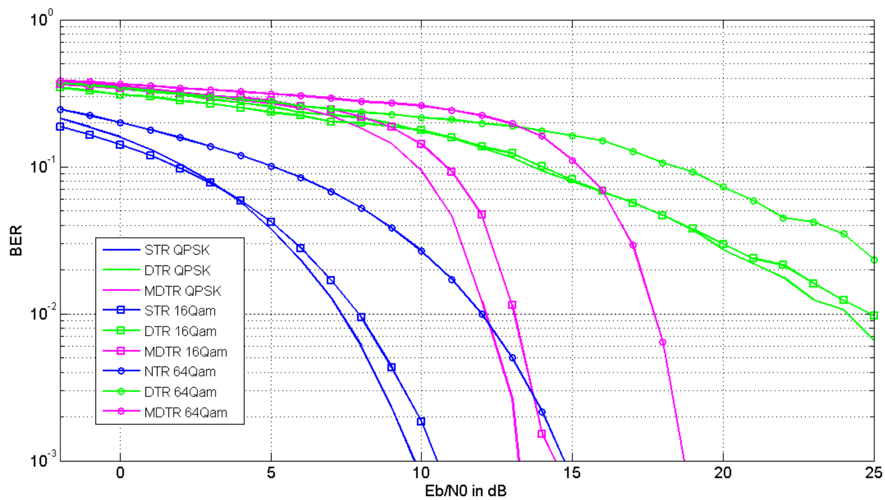


Figure 4.12: BER performance of Schemes 2 when modulated with QPSK, 16QAM, 64QAM.

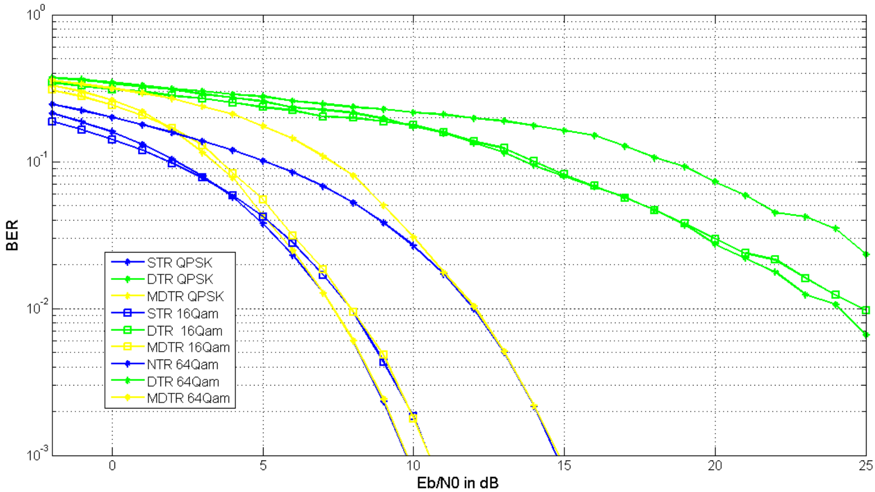


Figure 4.13: BER performance of Schemes 3 when modulated with QPSK, 16QAM, 64QAM.

Chapter 5

Summary and Conclusions

In this thesis, new tone reservation methods to reduce the high PAR of OFDM systems are investigated. The focus is on *Dynamic Tone Reservation* (DTR), where in contrast to *Standard Tone Reservation* (STR), the set of reserved tones changes from symbol to symbol. Simulation results show that DTR outperforms STR in terms of PAR reduction and power increase.

In DTR, the locations of reserved tones are unknown to the receiver. *Modified Dynamic Tone Reservation* (MDTR) attempts to assist the receiver in finding the set of reserved tones. Three modifications to DTR are suggested. BER and CCDF of PAR of those three schemes are compared through simulations.

The schemes move points to decision boundaries (Scheme 1), to the corners of decision boundaries (Scheme 2) and outside the constellation plane (Scheme 3).

Scheme 1 has the best CCDF of PAR among these three schemes, but BER performance is the worst. The BER performance of Scheme 2 is better than of Scheme 1. The CCDF of Scheme 2 depends on the constellation size and the constraint distance. A suitable constraint distance of Scheme 2 will provide better CCDF of PAR (close to Scheme 3). Scheme 2 has the lowest power increase among the three schemes.

Scheme 3 has the best BER performance. In terms of CCDF of PAR, Scheme 3 is worse than Scheme 1 but better than Scheme 2. Scheme 3 has the highest power increase.

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