Master's Thesis

Performance Evaluation of MIMO Systems

By

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August 25, 2011

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Abstract

MIMO is becoming one of the most interesting research topics in the area of telecommunication and wireless technology. With increase in the requirements of data rate and signal to noise ratio due to increase in number of users, one antenna is not the right choice because it will result in low data rates. Therefore, multiple antenna networks are starting to be deployed in areas with high user traffic.

This thesis investigates the behavior of different types of multiple antenna systems namely SIMO, MISO and MIMO and single antenna system namely SISO, which was used as a reference. Three modulation schemes were considered namely 2-PAM, 4-QAM, and 16-QAM because they are common nowadays. The symbol error rate of these modulation schemes was compared using the aforementioned antenna systems in different environment namely Rayleigh, AWGN, and Ricean at different signal to noise ratio.

Simulation results were compared with theoretical upper bound for each of the modulation schemes in Rayleigh channel. In all the simulations frequency flat fading was assumed.

Acknowledgments

This Master's thesis would not exist without the support and guidance of supervisor, Göran Lindell and some other professors at the LTH. Many important concepts and analytical points that the supervisor has provided has greatly improved my concepts about the thesis work.

I would also like to thank my friends and family for supporting me and providing me with some assistance.

Hassan Mamoor

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CHAPTER 1

INTRODUCTION

This chapter gives a detailed description of theoretical portion of MIMO and also theoretical analysis of modulation schemes being employed.

MIMO is becoming a significant area for the engineers to carry out their advanced research. It has also gained interest in industry. MIMO has the tendency to achieve high communication performances like high bit rates, smaller error rates, mitigation of co-channel interference, and improvement in spectral efficiency, robustness and increase in communication range by means of diversity, spatial multiplexing and beamforming techniques. These techniques are discussed in section 1.1. Given the bit rate of wireless system increases and keeping transmitting power and the spectrum allocated constant, symbol error rate of MIMO systems will decrease. Section 1.2 discusses history of MIMO. MIMO has different antenna configurations related to it for example SIMO and MISO which are discussed in Section 1.3. In section 1.4, different applications are discussed which are commonly starting to use MIMO technology. Section 1.5 gives brief description about thesis problem.

1.1 Functions of MIMO

Diversity is an antenna technique, which is used to enhance the signal diversity at transmitter or receiver by exploiting multipath fading. Here the receiver receives different and independent copies of signal from the transmitter. Therefore, the copies of signal have the likelihood to fade independently and the receiver can reliably decode the transmitted signal from these copies [1][2].An example of diversity scheme is Alamouti's transmit diversity scheme. ***Spatial multiplexing*** is used to enhance data rates and channel capacity by dividing serial high rate signal to parallel low rate signals and transmitting them over multiple antennas and if these signals arrive at receiver with different spatial signatures, receiver can distinguish between most of them thus increasing data rate without increasing the bandwidth or transmission power [1]. Since low rate signals are superimposed on each other, it is possible to separate them on the receiver side using ML receiver. An example of multiplexing scheme is BLAST. ***Beamforming*** is used to increase received signal gain by directing the power of the transmitted signal towards intended receiver whereas direction of interference can be suppressed. Another benefit of MIMO is that usage of multiple antennas will also save battery power in the base stations since high transmit power is not cost effective [1].

1.2 History

The earliest idea of MIMO technology was made in early 1970s. In 1984 several papers regarding concept of beamforming were published in 1984 and 1986. The idea of spatial multiplexing technology, another wide topic within MIMO, was proposed in 1993 to use them in wireless broadcasts. In 1996, main idea to use multiple antennas at one transmitter was proposed to improve the link throughput. In 1998, concept of spatial multiplexing was demonstrated to improve performance of wireless communication systems. During the same year, space-time trellis codes were introduced, which employed multiple transmit antennas in order to get both diversity and spatial multiplexing gain, improve capacity performance and increase data rates. During 90's, it was also shown that the capacity of a MIMO system with certain transmit antennas and receive antennas grows linearly with help of those transmit and receive antennas provided that the links undergo independent fading. BLAST scheme was introduced to accomplish bit rates as high as 40bits/s/Hz and also increase spectral efficiencies. In 2001, the first MIMO system was developed and combined with OFDM technology by Iospan Wireless Inc...Iospan technology supported both diversity and spatial multiplexing. In 2003, it was shown that for a particular MIMO scheme namely space-time transmission scheme there is a tradeoff between multiplexing and diversity gain. From 2005 onwards, many major international companies like Samsung, Intel etc. have started using MIMO-OFDM for Wi-Fi and IEEE 802.16e WiMAX broadband mobile standard. Currently Ericsson is carrying out research for deployment of MIMO in 4G LTE systems using 4×4 antenna configuration, meaning four transmit and four receive antennas, to significantly improve overall data rates. Tests in realistic environments have shown that with the above configuration, data rates have increased significantly and there has been improvement in network capacity as compared to when MIMO initially started, which shows that the future of MIMO technology is very interesting and bright [4][5].Some third generation systems like CDMA use Alamouti's transmit diversity scheme for certain transmission modes or channels. Interestingly, MIMO technology is also employed in IEEE 802.20 mobile broadband wireless access system.

1.3 Forms of MIMO

a) ***SISO***

It is a wireless antenna system in which transmitter and receiver each have single antenna. It is the simplest form of antenna technology but susceptible to frequency fading and multipath effects. The susceptibility to fading of received signal thus results in low SNR. Finally this results in degrading of the symbol error rate performances because of receiver's lack of ability to recover the message information in the signal. Also there is reduction in data speed due to scattering and reflection on to many obstacles within the communication link.

b) ***SIMO***

This wireless antenna system uses single antenna at the transmitter and multiple antennas, M_r at receiver. It is also known as receiver diversity since multipath components arrive at receiver with M_r copies of the signal. The performance is improved since the receiver has the capability to choose stronger signal from the most efficient antenna or even combine the signals from all the available antennas in order to maximize the SNR.

c) ***MISO***

MISO uses M_t antennas at transmitter and single antenna at receiver. It is also called transmit diversity since the signals are transmitted from M_t antennas. An antenna technique known as Alamouti STC can be applied at the transmitter. With Alamouti scheme, the signal is transmitted in space i.e. through M_t different antennas and in time with M_t different times simultaneously. It is used for improved data speed and reduces problems caused by multipath fading and minimize symbol error rate.

d) ***MIMO***

MIMO technology uses M_t antennas at transmitter and M_r antennas at receiver. It is used to improve overall throughput of the wireless link.

1.4 Applications

MIMO is significantly used in number of wireless technologies like Wi-Fi, 4G, LTE and WiMAX, IEEE 802.16e to name some few important ones. In fact, it has become one of the most acknowledged technologies in replacing wired network systems like Ethernet with Wi-Fi. It also improves the robustness to fading and thus is able to provide highly efficient wireless services for e.g. wireless multimedia applications including high speed broadband access, HDTV video etc. to the end-users. MIMO technology started to be deployed in WLAN applications such as wireless HDTV video streaming by means of spatial multiplexing to support high throughput and reasonable spectral efficiency [6]. Some MIMO techniques like space time coding and BLAST techniques have also been introduced in HSDPA systems, which are an evolution of UMTS standard, to achieve higher bit rates [7]. MIMO is also being used with different modulation schemes e.g. with OFDM and OFDMA, MIMO has played a crucial part to handle the problems caused by multipath channel and also to provide higher data rates over longer distances thus improving the range. In wireless wide area network, Iospan Wireless successfully developed a MIMO wireless system using OFDM to build 1-Gbps NLOS broadband wireless systems, which support high throughput and deliver very high spectral efficiency. Efforts have been made to define MIMO layer for IEEE 802.11 standard for WLANs under newly formed Wireless Next Generation group [2].

1.5 Problem Description

Symbol error rate simulations are very useful procedure to compare communication performance of a MIMO communication system. These simulations can be used to compare bit error rate or symbol error rate performances of different modulation schemes. The modulation schemes discussed in the thesis are lower order schemes namely 2-PAM, which is also known as BPSK and lower order QAM (e.g. are 4-QAM and 16-QAM).The focus on this thesis was the catalog of results for different values of transmit antennas (M_T), receive antennas (M_R) and the constellation size, M. Modified version of these QAM schemes are widely used to transmit digital signals such as digital cable TV and cable internet service [7]. The symbol error rate performances were performed under different channel conditions namely AWGN, Rayleigh flat fading and Ricean channel by varying SNR and by using different antenna system configurations namely SISO, SIMO, MISO and MIMO.

It has also been assumed that transmitted signal energy per bit is normalized to one and also we want to compare symbol error performances for different modulation schemes mentioned above.

ML receiver detection scheme has been used to make mapped decision of the transmitted message. Since the numbers of bits generated are equally probable, so ML receiver acts as MAP receiver. Gray coding has been used for the symbol error rate simulations because of better symbol error rate performance.

CHAPTER 2

TRANSMITTER

This chapter gives a detailed description about signal constellation diagram of three modulation schemes, their respective signal energies are also calculated and a very brief description of gray coding is also given.

2.1 2-PAM

In order to find out, how bit error probability of certain scheme is obtained, signal space is one of those methods used to describe the waveforms as a point in signal constellation [9]. Each point can thus represent one symbol in constellation diagram. These constellations are used to build basis functions, which have dimensions of N dimensional vectors of the signal alternatives of different modulation schemes. To make reliable decisions on the bits, we divide the constellation space into suitable decision regions. These decision regions differ for orthogonal and antipodal schemes. The figure below shows a constellation space diagram for 2-PAM, an antipodal scheme.

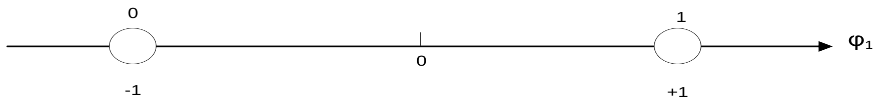

Fig.1: 2-PAM constellation diagram

The decision region of 2-PAM is drawn in a following way:

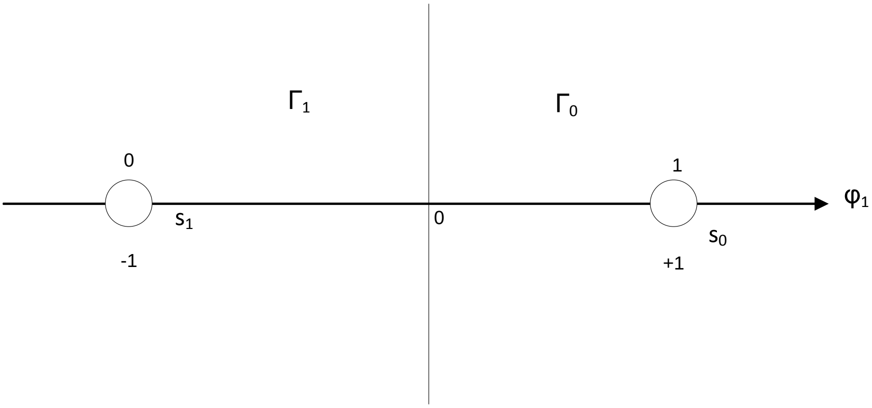

Fig.2: Γ_0 and Γ_1 are decision regions

From Fig.2, since there are two symbols, each symbol carry one bit namely 0 and 1. If the received sequence is in the decision region Γ_0, then the corresponding message or the bit is +1. If it is in the region Γ_1, then the decision is -1. Note the horizontal axis in 2-PAM case denotes the basis function. Applying the same decision threshold technique to M-ary PAM, we will have M different decision regions. Following section shows how to calculate the average bit energy of this antipodal scheme.

2.1.1 2-PAM: Average transmitted signal energy calculation

From Fig.2, we can find the energy in each symbol, s_0 and s_1, which is given by square of the amplitude.

$E_0=A_0^2=1$
$E_1=A_1^2=1$

k=1, since there are two bits per symbol

$P_i = \frac{1}{2}$ since equally likely probability of each symbol is 1 and there are two symbols

Now using the expression for average transmitted signal energy

$$E_s = \sum_{i=0}^{M-1} P_i E_i \quad (1)$$

we get,

$E_s = P_i(E_0 + E_1) = 1$

In order to calculate average transmitted bit energy

$$E_b = \frac{E_s}{k} \tag{2}$$

$\therefore E_b = 1$

2.2 4-QAM

For two dimensional modulation schemes like 4-QAM, there are two orthogonal basis function. Inphase and quadrature components of noise are independent from each other. Two 2^n PAM signals are transmitted on two different channels, one is in phase and the other is quadrature with each channel carrying n bits [9]. In this case, n=2. Fig. 3 shows the constellation diagram for 4-QAM modulation scheme and decision regions.

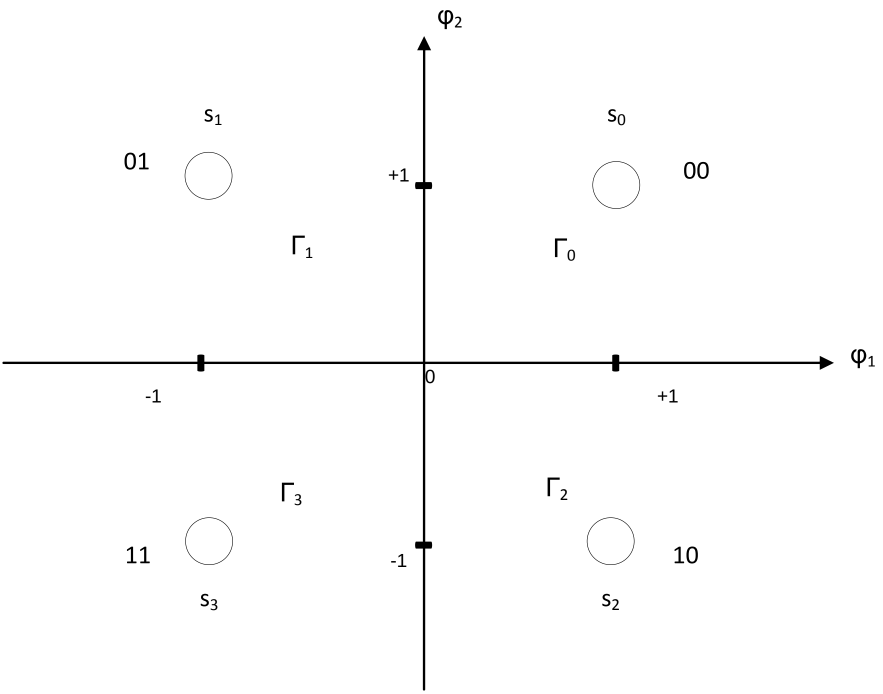

Fig.3: 4-QAM constellation diagram and the decision region

2.2.1 4-QAM: Average transmitted signal energy calculation

From Fig.3, we can find the energy in each symbol, $s_{0,}$ $s_{1,}$ s_2 and $s_{3,}$ which is given by square of the amplitude.

$E_0=A_0^2=2$
$E_1=A_1^2=2$
$E_2=A_3^2=2$
$E_3=A_3^2=2$

k=2, since there are two bits per symbol

$P_i = \frac{1}{4}$ since equally likely probability of each symbol is 1 and there are four symbols.

By using (1), we get

$E_s=P_i\ (E_0+E_1+ E_2+ E_3) =2$

The average transmitted bit energy is calculated similarly by using (2)

$\therefore E_b = 1$

2.3 16-QAM

Since we have four symbols, we have four decision regions. In order to calculate bit error rate for this case, we have to map bits to symbols. Since there are four symbols, each symbol carries two bits each namely 00, 01, 10 and 11, which are also shown Fig. 3. If the received message is in the region $r_{0,}$ then the decision for the bit will be 00. If the received message is in the region $r_{1,}$ then the message will be mapped to 01. Same procedure is carried out for other two remaining regions. For 16-QAM modulation scheme, there are also two orthogonal basis functions. Two 2^n PAM signals are transmitted on two different channels, one is inphase and the other is quadrature with each channel carrying n bits, where n=3. Fig. 4 shows the constellation diagram for 16-QAM modulation scheme and decision regions.

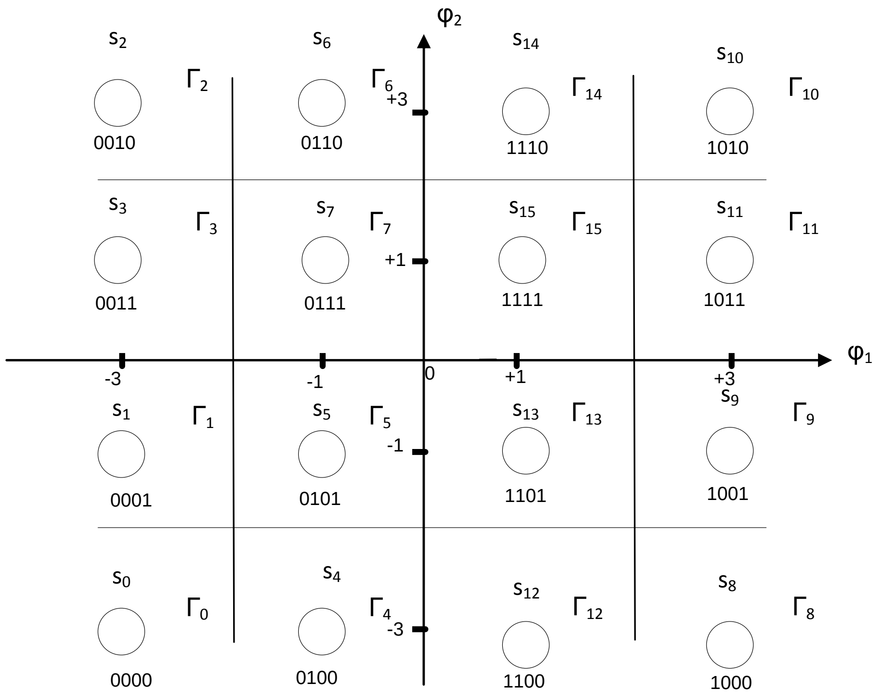

Fig.4: 16-QAM constellation diagram and the decision region

2.3.1 16-QAM: Average transmitted signal energy calculation

From Fig. 4, we can find the energy in each symbol, s_0, s_1, s_2, s_3 till s_{15} which is given by square of amplitude. We know that s_0, s_8, s_{10} and s_2 are equidistant from origin so they will have equal energy. We also know that $s_1, s_3, s_9, s_{11}, s_4, s_{12}, s_6$ and s_{14} are equidistant from origin; therefore their corresponding energies are also equal. Similarly, s_5, s_7, s_{13} and s_{15} have equal energies.

$E_0=A_0^2=18 \therefore E_8= E_{10}= E_2=18$
$E_1=A_1^2=10 \therefore E_3= E_9= E_{11}= E_4= E_{12}= E_6= E_{14}=10$
$E_5=A_2^2=1 \quad \therefore E_7= E_{13}= E_{15}=2$

k=4, since there are four bits per symbol

$P_i = \frac{1}{16}$ since equally likely probability of each symbol is 1 and there are sixteen symbols

By using (1),

$E_s = P_i(4E_0 + 8E_1 + 4E_5) = 10$

Similarly using (2), we get average transmitted bit energy to be 2.5.

Note that the average transmitted bit energy is 2.5, but energy of 1 is used in the computer simulations, therefore, the constellation diagram in Fig. 4 is shrunk or reduced with a factor $\sqrt{\frac{1}{2.5}}$ in order to make the energy 1.

2.4 Gray Coding

In every communication system, good bit error rate performance is one of the most important criterions, which has to be taken into consideration if your system is to work as efficiently as possible. In Fig. 3 and 4, 11 and 10 has been swapped to show the concept of Gray coding. Since we want to minimize our bit errors, we introduce Gray coding. Here, each neighboring bit differs by only one bit, thus resulting in only one bit error. This will reduce the Hamming distance and improve our system performance with higher bit rate for these particular modulations schemes.

2.5 Bit Rate

From 2.1-2.4, a symbol was transmitted through one antenna. Now, the discussion is on bit rates and transmission of symbols through multiple antennas. Transmitter model is given by

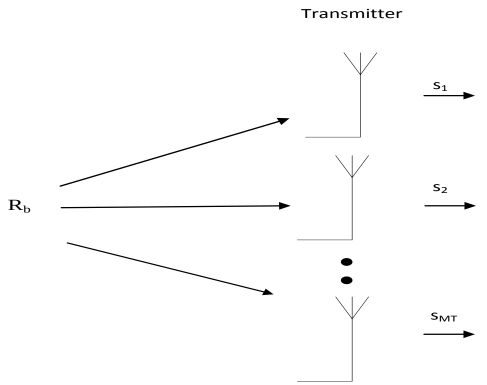

Fig.5

The transmitted symbol is given by $s = \begin{bmatrix} s_1 \\ s_2 \\ \vdots \\ s_{M_T} \end{bmatrix}$. $s_1$carry $\log_2(M)$ bits from antenna 1. Similarly, s_2 also carry $\log_2(M)$ bits from second antenna with each antenna having M-ary constellation. Now bit rate is defined [9] as

$$R_b = \frac{1}{\mathrm{T_b}} \tag{3}$$

We know that new symbol is sent after every T_s, which is constant whereas bit rate is varying. All the MIMO schemes have equal symbol rate since for SISO case one symbol is transmitted after one symbol time T_s and second symbol after second time interval etc. Similarly, MIMO scheme with two antennas at transmitter and receiver transmit two symbols after one symbol time T_s and then another two in the second time interval and so on. Same interpretation can be made to SIMO and MISO.

$$\mathrm{T_s} = \mathrm{kT_b} \tag{4}$$

Since k=$\log_2(M)$, number of bits, therefore,

$$\mathrm{T_s} = \log_2(M)\,\mathrm{T_b}$$

For M_T number of antennas,

$$\mathrm{T_s} = M_T \log_2(M)\,\mathrm{T_b} \tag{5}$$

We know that T_S is constant so W, the bandwidth with which each symbol is transmitted is also constant. This bandwidth is given [9] by

$$W = \frac{\mathrm{c}}{\mathrm{T_s}} \tag{6}$$

where c is a scalar constant.

Substituting (4) and (6) into (5), we get

$$R_b = \frac{M_T \log_2(M)}{T_S} = M_T \log_2(M)\,\frac{\mathrm{W}}{\mathrm{c}} \tag{7}$$

CHAPTER 3

CHANNEL

In this chapter, AWGN channel and antenna system model is discussed in Section 3.1. In Section 3.2, background about Rayleigh channel is discussed and in Section 3.3, Ricean channel is considered for discussion. All three channels are followed by their respective schematic diagram showing the concepts.

3.1 AWGN Channel and Antenna System Model

3.1.1Background

In AWGN channel, bits transmitted from antenna on transmitter side arrive at the receiver with addition of only noise but there is no attenuation and phase rotation. AWGN channel does not take into account frequency selective, flat fading or interference. The amplitude of AWGN noise is Gaussian distribution with mean 0 and variance σ^2, where

$$\sigma^2 = \frac{N_0}{2} \tag{8}$$

Definitions of antenna system model for AWGN channel can be represented as

$$y = As+n \tag{9}$$

where y is the received vector, n is AWGN, s is a transmitted vector and A is a complex matrix.

3.1.2 SISO

i) Schematic Description for SISO

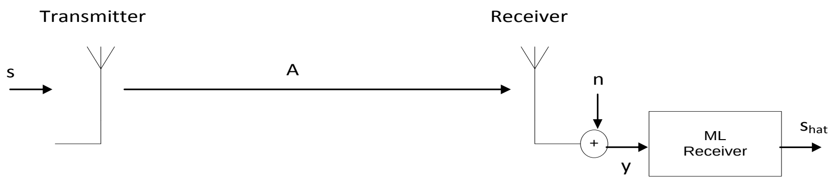

Fig.6: General SISO system

For SISO model, A is a scalar complex also known as AWGN channel, y is the received vector of dimension M_Tx1, s and n are also M_Tx1, s_{hat} is basically same as $\hat{s}$, the decision output symbols, which has same dimension as s, where M_T=M_R=1 because of one input and one output.

ii) Mathematical representation of SISO

$$y = As+n$$

3.1.3 SIMO

i) Schematic Description for SIMO

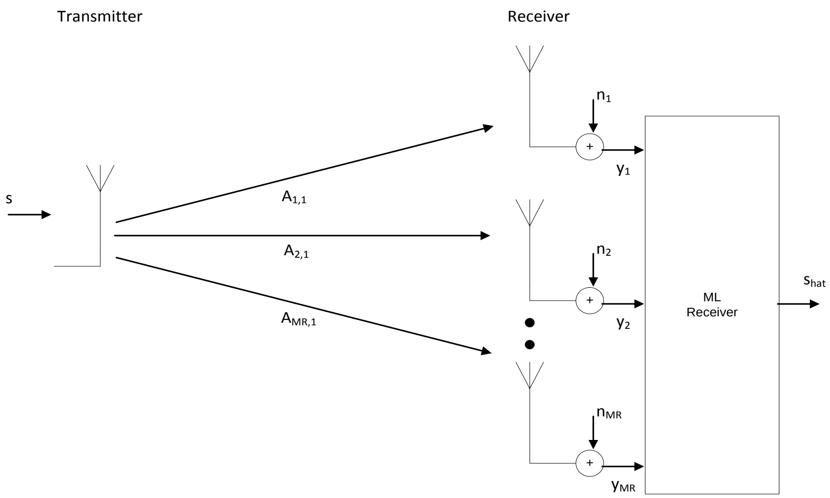

Fig.7: General SIMO system

ii) Mathematical representation of SIMO

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_{M_R} \end{bmatrix} = \begin{bmatrix} A_{1,1} \\ A_{2,1} \\ \vdots \\ A_{M_R,1} \end{bmatrix} s + \begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_{M_R} \end{bmatrix} \qquad (10)$$

SIMO model is represented by the expression given in (10) with *y* and *n* having dimension M_Rx1 and *s* has dimension M_Tx1, *A* is a complex column vector of dimension M_Tx1. $\hat{s}$ has the same dimension as *s* where M_T=1because there is only single input corresponding to single transmitted antenna.

3.1.4 MISO

i) Schematic Description for MISO

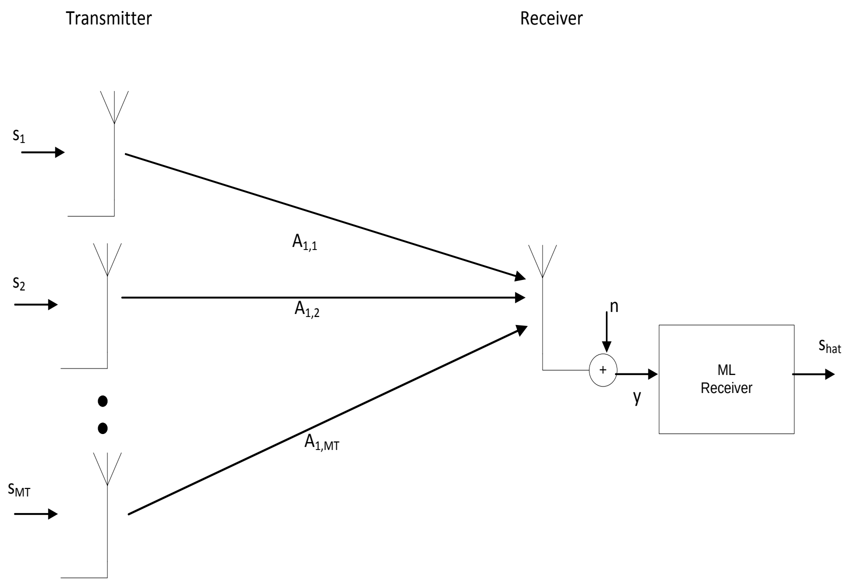

Fig.8: General MISO system

ii) Mathematical representation of MISO

$$y = [A_{1,1} \quad A_{1,2} \quad \cdots \quad A_{1,M_T}] \begin{bmatrix} s_1 \\ s_2 \\ \vdots \\ s_{M_T} \end{bmatrix} + n \qquad (11)$$

MISO model is represented by the expression given in (11) with *s* having dimensions of M_Tx1 and *y* and *n* having dimension M_Rx1, *A* is a row vector having a dimension of $1xM_T$. $\hat{s}$ has the same dimension as *s*, where M_R=1 because there is only single output corresponding to single received antenna.

3.1.5 MIMO

i) Schematic Description for MIMO

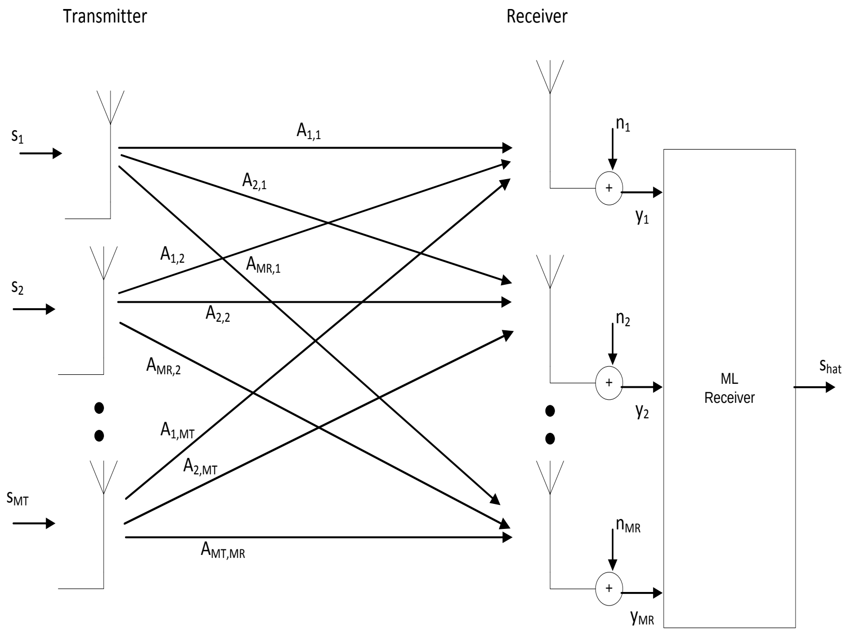

Fig.9: General MIMO systems

ii) Mathematical representation of MIMO

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_{M_R} \end{bmatrix} = \begin{bmatrix} A_{1,1} & A_{1,2} & \cdots & A_{1,M_T} \\ A_{2,1} & A_{2,2} & \cdots & A_{2,M_T} \\ \vdots & \vdots & \ddots & \vdots \\ A_{M_R,1} & A_{M_R,2} & \cdots & A_{M_R,M_T} \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \\ \vdots \\ s_{M_T} \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_{M_R} \end{bmatrix} \quad (12)$$

MIMO model is given by the expression in (12) with *s* having dimensions of M_Tx1, *y* and *n* having dimension of M_Rx1. *A* has the dimension of M_RxM_T. Again, $\hat{s}$ has the same dimension as *s*.

3.2 Rayleigh Channel and Antenna System Model

3.2.1 Background

In Rayleigh fading channel, the bits to be transmitted arrive at receiver with some attenuation and also phase rotation because of the multiplicative factor introduced in the channel. This multiplicative factor is a channel amplitude gain and is a Gaussian distributed complex random variable. The amplitude of AWGN noise is Gaussian distributed with mean 0 and variance σ^2exactly similar to (8).

Antenna system model for Rayleigh channel can be mathematically represented similar to (12) with the only difference is the channel matrix *A* replaced by A_{Ray} suggesting Rayleigh channel.

$$y = A_{Ray}s + n \tag{13}$$

It can be distributed into A_1 and A_Q, where A_1 is the inphase component or the real component of the Rayleigh channel, and A_Q is a quadrature component or the imaginary component, each of them are independent complex Gaussian distributed. Therefore mathematically Rayleigh channel can be represented as

$$A_{Ray} = A_{RayI} + jA_{RayQ} \tag{14}$$

with both real and imaginary channel components having variance $\sigma_h^2 = 1$.

A_{Ray} is a matrix of complex numbers also known as Rayleigh channel, *y* is the received vector, *n* is AWGN and *s* is a transmitted vector. In case of M-PAM, A_{Ray}, *y*, and *n* are all complex whereas *s* is a column vector with s_i ∈ {±1, ±3, ±(M-1)}where i=1,…,M_T. In case of 4-QAM and 16-QAM, the variables are complex numbers with *s* having real, s_1 and imaginary components, s_Q, with $s_1 \in \left\{-\sqrt{\mathrm{M}}+1+2i\right\}_{i=0}^{\sqrt{\mathrm{M}}-1}$ and $s_Q \in \left\{-\sqrt{\mathrm{M}}+1+2i\right\}_{i=0}^{\sqrt{\mathrm{M}}-1}$, where M is the constellation size.

3.2.2 Schematic Diagrams and Mathematical representations

The only difference between Rayleigh and AWGN schematic diagrams and mathematical expressions is that *A* is replaced by A_{Ray} from 3.1.2 - 3.1.5.

3.3 Ricean Channel and Antenna System Model

3.3.1 Background

In a Ricean channel, similar to Rayleigh case bits or symbols arrive with attenuation and rotation. The only difference is the presence of a LOS component along with the NLOS multipath components. The expression for Ricean channel will be

$$y = \left(A_{Ray} + \mathrm{L}\right)s + n \tag{15}$$

where the matrix L is the only factor added, which is basically the non-random complex LOS component matrix having same dimension as A_{Ray} and y, s, n and A_{Ray} are same as defined in section 3.2.1.

3.3.2 SISO

i) Schematic Description for SISO

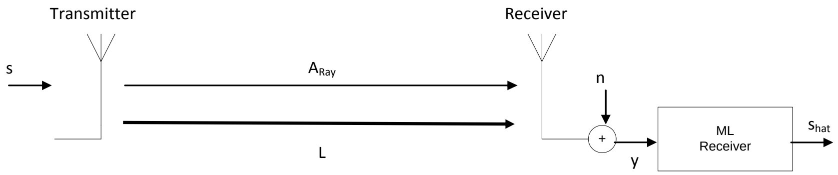

Fig.10: General SISO system

ii) Mathematical representation of SISO

The mathematical representation is same as (15).

3.3.3 SIMO

i) Schematic Description for SIMO

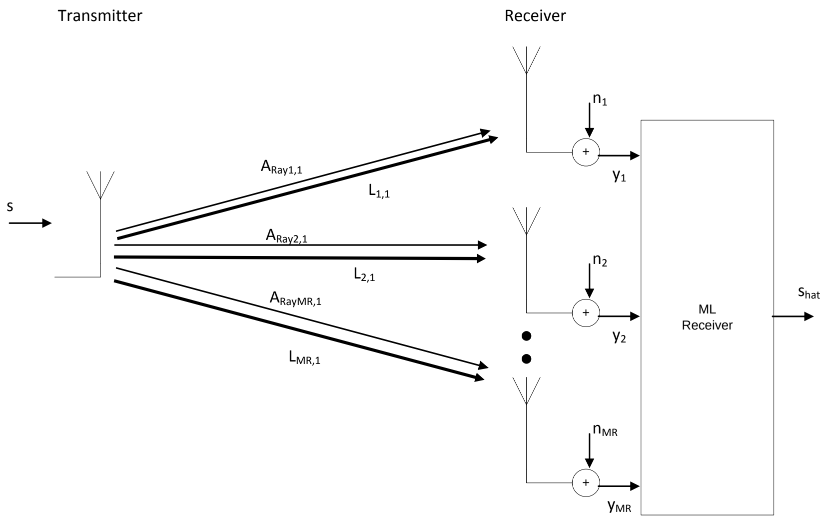

Fig.11: General SIMO system

ii) Mathematical representation of SIMO

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_{M_R} \end{bmatrix} = \begin{bmatrix} A_{Ray_{1,1}} + L_{1,1} \\ A_{Ray_{2,1}} + L_{2,1} \\ \vdots \\ A_{Ray_{M_R,1}} + L_{M_R,1} \end{bmatrix} s + \begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_{M_R} \end{bmatrix} \tag{16}$$

3.3.4 MISO and MIMO

For Ricean MISO, schematic diagram and mathematical expression are similar to Fig.8 and (11) respectively. The only changes made to Fig.8 and (11) are that A is replaced by A_{Ray} and L is added to A_{Ray} as also illustrated in Fig.10 and 11. Refer to (11), (15) and (16). Similarly for Ricean MIMO, in Fig.9 A is replaced by A_{Ray} and L is added as illustrated in Fig.10 and 11. In mathematical representation, we see $\left(A_{Ray} + \mathrm{L}\right)$ instead of A. Refer to (12), (15) and (16).

Consider received average bit energy, ϵ_b where

$$\epsilon_b = \frac{E\{\|Hs\|^2\}}{M_T \log_2(M)} \tag{17}$$

$$Hs = z = \begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ \vdots \\ z_{M_R} \end{pmatrix}$$

$$\|Hs\|^2 = \sum_{i=1}^{M_R} |z_i|^2$$

$$z_j = \sum_{l=1}^{M_T} h_{j,l}\, s_l$$

$$\left|z_j\right|^2 = z_j\, z_j^{\,*} = \sum_{l=1}^{M_T} h_{j,l}\, s_l \sum_{k=1}^{M_T} h_{j,k}^{\ \ *} s_k^{\,*}$$

$$= \sum_{l=1}^{M_T} \sum_{k=1}^{M_T} h_{j,l} \ \ h_{j,k}^{\ \ *}\, s_l\, s_k^{\,*}$$

$$E\{\|Hs\|^2\} = E\left\{\sum_{i=1}^{M_R} \sum_{l=1}^{M_T} \sum_{k=1}^{M_T} h_{i,l} \ \ h_{i,k}^{\ \ *}\, s_l\, s_k^{\,*}\right\}$$

$$= \sum_{i=1}^{M_R} \sum_{l=1}^{M_T} \sum_{k=1}^{M_T} E\,\{h_{i,l} \ \ h_{i,k}^{\ \ *}\, s_l\, s_k^{\,*}\}$$

$$= \sum_{i=1}^{M_R} \sum_{l=1}^{M_T} \sum_{k=1}^{M_T} E\,\{h_{i,l} \ \ h_{i,k}^{\ \ *}\}\, E\{s_l\, s_k^{\,*}\}$$

$$= \sum_{i=1}^{M_R} \ \sum_{l=1}^{M_T} E\left\{h_{i,l} h_{i,l}^{\ \ *}\right\} E\,\{|s_l|^2\}$$

$$= \sum_{i=1}^{M_R} \ \sum_{l=1}^{M_T} E\left\{\left|h_{i,l}\right|^2\right\} E_{s,l}$$

$$= E_{s/antenna} \sum_{i=1}^{M_R} \sum_{l=1}^{M_T} E\left\{\left|\mathrm{L_{i,l}} + \mathrm{A_{i,l}}\right|^2\right\}$$

$$= E_{s/antenna} \sum_{i=1}^{M_R} \sum_{l=1}^{M_T} E\{(L_{i,l} + A_{i,l})(L_{i,l} + A_{i,l})^*\}$$

$$= E_{s/antenna} \sum_{i=1}^{M_R} \sum_{l=1}^{M_T} E\left\{(L_{i,l} + A_{i,l})(L_{i,l}{}^* + A_{i,l}{}^*)\right\}$$

$$= E_{s/antenna} \sum_{i=1}^{M_R} \sum_{l=1}^{M_T} E\left\{\left|L_{i,l}\right|^2 + L_{i,l} A_{i,l}{}^* + A_{i,l} L_{i,l}{}^* + \left|A_{i,l}\right|^2\right\}$$

$$= E_{s/antenna} \sum_{i=1}^{M_R} \sum_{l=1}^{M_T} \left(\left|L_{i,l}\right|^2 + 0 + 0 + E\left\{\left|A_{i,l}\right|^2\right\}\right)$$

The real and imaginary part of vector A is Gaussian distributed with mean 0 and variance ${\sigma_h}^2$

$$\therefore E\{\|Hs\|^2\} = E_{s/antenna} \sum_{i=1}^{M_R} \sum_{l=1}^{M_T} \left(\left|L_{i,l}\right|^2 + 2{\sigma_h}^2\right)$$

$$= E_{s/antenna} \left(\sum_{i=1}^{M_R} \sum_{l=1}^{M_T} \left|L_{i,l}\right|^2 + M_R M_T 2{\sigma_h}^2\right) \tag{18}$$

$$E_{s,Tot} = M_T E_{s/antenna} \tag{19}$$

$$E_{b,sent} = \frac{E_{s,Tot}}{M_T \log_2(M)} \tag{20}$$

By substituting (19) into (20), we get

$$E_{b,sent} = \frac{E_{s/antenna}}{\log_2(M)} \tag{21}$$

By substituting (21) into (18), we get

$$E\{\|Hs\|^2\} = E_{b,sent} \log_2(M) \left(\sum_{i=1}^{M_R} \sum_{l=1}^{M_T} \left|L_{i,l}\right|^2 + M_R M_T 2{\sigma_h}^2\right) \tag{22}$$

By substituting (22) into (17), we get

$$\epsilon_b = \frac{E_{b,sent} \log_2(M) \left(\sum_{i=1}^{M_R} \sum_{l=1}^{M_T} \left|L_{i,l}\right|^2 + M_R M_T 2{\sigma_h}^2\right)}{M_T \log_2(M)}$$

$$= \frac{E_{b,sent} \left(\sum_{i=1}^{M_R} \sum_{l=1}^{M_T} \left|L_{i,l}\right|^2 + M_R M_T 2{\sigma_h}^2\right)}{M_T}$$

$$= \left(\frac{\sum_{\mathrm{i=1}}^{\mathrm{M_R}} \sum_{\mathrm{l=1}}^{\mathrm{M_T}} \left| \mathrm{L_{i,l}} \right|^2}{\mathrm{M_T}} + \mathrm{M_R} 2{\sigma_h}^2 \right) E_{b,sent}$$

$$\epsilon_{\mathrm{b}} = \underbrace{\left(\frac{\|L\|^2}{M_T M_R 2{\sigma_h}^2} + 1 \right) M_R 2{\sigma_h}^2}_{<1} E_{b,sent} \tag{23}$$

For ϵ_{b} to be smaller than $E_{b,sent}$, the expression above the brackets has to be less than 1.

$$\therefore \left(\frac{\|L\|^2}{M_T M_R 2{\sigma_h}^2} + 1 \right) M_R 2{\sigma_h}^2 < 1$$

We know that L is a complex deterministic matrix and therefore, has deterministic components. We here assume that

$$L = \begin{pmatrix} l & l & \cdots & l \\ l & l & \cdots & l \\ \vdots & \vdots & \ddots & \vdots \\ l & l & \ldots & l \end{pmatrix}$$

$$\|L\|^2 = \sum_{i=1}^{M_R} \sum_{j=1}^{M_T} \left| l_{i,j} \right|^2 = M_R M_T |l|^2 \tag{24}$$

$$\therefore \left(\frac{M_R M_T |l|^2}{M_T M_R 2{\sigma_h}^2} + 1 \right) M_R 2{\sigma_h}^2 \leq 1$$

$$\frac{M_R M_T |l|^2}{M_T} + M_R 2{\sigma_h}^2 \leq 1$$

$$M_R M_T |l|^2 + M_T M_R 2{\sigma_h}^2 \leq M_T$$

$$M_R |l|^2 + M_R 2{\sigma_h}^2 \leq 1$$

$$|l|^2 \leq \frac{1 - M_R 2{\sigma_h}^2}{M_R}$$

The Ricean factor K is given by (with ${\sigma_h}^2 = \frac{1}{20}$)

$$\mathrm{K} = \frac{\|L\|^2}{M_T M_R 2{\sigma_h}^2} = \frac{|l|^2}{2{\sigma_h}^2} = 10|m|^2 \tag{25}$$

CHAPTER 4

ML RECEIVER

In this chapter, the theory of ML receiver is briefly discussed. Also consequences of symbols passing through Rayleigh channel are discussed through the properties of this channel. ML expressions for the four antenna configurations of MIMO are also derived here.

4.1 ML Receiver

According to [9], this is the ideal and optimal receiver if transmitted bits are equally likely and is used to minimize symbol error probability as much as possible thus enhancing the communication system performance. It is the most difficult receiver to be implemented because of its complexity. This is because with increase in number of constellation and for higher modulation schemes like 64-QAM and many other higher modulation schemes, many matched filters and correlators are required. Also they are not cost effective for very large systems. As mentioned in section 1.3.4, the ML receiver equals the MAP receiver when symbols are equally likely. The figure below shows general diagram of MAP receiver.

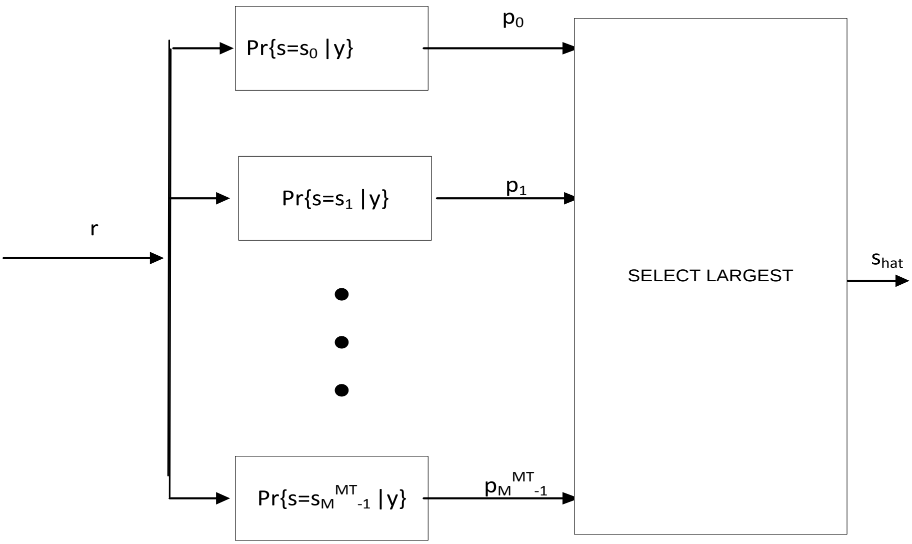

Fig.12: General MAP symbol receiver

The receiver selects the most likely symbol, $\tilde{s}$, to have been transmitted, given by the largest probability (p_0, p_1,…, $p_{M^{MT}-1}$) of that symbol or bit according to Fig.12 or based on minimum Euclidean distance as shown later. This means that if for e.g. p_0 is the largest probability, then $\tilde{s}_0$ is the decision made, which means that s_0 was the most likely transmitted symbol or bits.

ML receiver assumes following situation for making symbol decisions [9].

1) The receiver knows about the synchronization of signaling of the symbols i.e. when the symbols are received from the source.
2) These sequence of symbols transmitted are independent, equally likely identically distributed, which means that the each transmitted symbol has no dependence on the past symbol or even future symbols.
3) There is a systematic non-overlap of each (e.g. SISO, SIMO) or group of symbols (MISO, MIMO) to be transmitted with restriction on symbol interval, which means when one symbol s_0 is transmitted and then s_1, s_1 will be transmitted after s_0 without interference from s_0.
4) ML receiver has knowledge of the channel and thus the received signals.
5) The AWGN is a zero-mean white Gaussian random process as stated under section 3.1.1.

This receiver selects the most likely signal sent based on Fig.12 or minimum Euclidean distance. In other words, let us consider a signal space constellation shown in Fig.13. If a received symbol denoted by y is located as shown in Fig.13, we calculate the squared Euclidean distance of all possible symbols with the received vector in the signal space diagram and then we take the minimum of all those combination. That minimum suggests that the corresponding transmitted symbols were sent. In order to illustrate the scenario discussed in previous sentences, we first consider simplifying ML expression given by

$$\min_{\tilde{s}} \underbrace{\|y - A\tilde{s}\|}^2 \tag{26}$$

where $\tilde{s}$ is possible transmitted symbol.

The general expression indicated in (26) can be simplified generally to

$$\min_{\tilde{s}}(||y - A\widetilde{s_0}||^2, ||y - A\widetilde{s_1}||^2, ||y - A\widetilde{s_2}||^2, \dots, ||y - A\widetilde{s_{\mathrm{M}^{\mathrm{M_T}}-1}}||^2\ ) \quad (27)$$

where M is the constellation size.

(27) can be rewritten as

$$\min_{\tilde{s}}(||y - z_0||^2, ||y - z_1||^2, ||y - z_2||^2, \dots, ||y - \mathrm{z}_{\mathrm{M}^{\mathrm{M_T}}-1}\ ||^2\ ) \quad (28)$$

where $z{=}A\tilde{s}$

(28) apply to SISO, SIMO, MISO and MIMO.

By considering the coefficients of parameters *y*, *s*, *A* and *n* for each of four cases, we obtain specific derivations for each case which are briefly discussed below.

The specific expression of ML decoding for SIMO is given by

$$\begin{aligned}\min\Big(\Big(\big|y_1 - A_{1,1}\widetilde{s_0}\big|^{\mathbf{2}} + \big|y_2 - A_{2,1}\widetilde{s_0}\big|^{\mathbf{2}} + \cdots \\ + \big|y_{M_R} - A_{M_R,1}\widetilde{s_0}\big|^{\mathbf{2}}\Big), \Big(\big|y_1 - A_{1,1}\tilde{s}_1\big|^{\mathbf{2}} + \big|y_2 - A_{2,1}\tilde{s}_1\big|^{\mathbf{2}} + \cdots \\ + \big|y_{M_R} - A_{M_R,1}\tilde{s}_1\big|^{\mathbf{2}}\Big), \dots, \Big(\big|y_1 - A_{1,1}\tilde{s}_{M-1}\big|^{\mathbf{2}} \\ + \big|y_2 - A_{2,1}\tilde{s}_{M-1}\big|^{\mathbf{2}} + \cdots + \big|y_{M_R} - A_{M_R,1}\tilde{s}_{M-1}\big|^{\mathbf{2}}\Big)\Big)\end{aligned} \quad (29)$$

where $\widetilde{s_0}$, $\widetilde{s_1}$,.... $\tilde{s}_{M-1}$here are possible transmitted symbol vectors each having a dimension of 1x1.

Using (11), the expression for MISO is given by

$$\min\left(\left(\left|y - A_{1,1}\, s_1{}^0 - A_{1,2}\, s_2{}^0 - \cdots - A_{1,M_T}\, s_{M_T}{}^0\right|^2, \left|y - A_{1,1}\, s_1{}^1 - A_{1,2}\, s_2{}^1 - \cdots - A_{1,M_T}\, s_{M_T}{}^1\right|^2, \dots, \left|y - A_{1,1}\, s_1{}^{M^{M_T}-1} - A_{1,2}\, s_2{}^{M^{M_T}-1} - \cdots - A_{1,M_T}\, s_{M_T}{}^{M^{M_T}-1}\right|^2\right)\right) \tag{30}$$

where $\widetilde{s_0} = \begin{bmatrix} s_1{}^0 \\ s_2{}^0 \\ \vdots \\ s_{M_T}{}^0 \end{bmatrix}$, $\widetilde{s_1} = \begin{bmatrix} s_1{}^1 \\ s_2{}^1 \\ \vdots \\ s_{M_T}{}^1 \end{bmatrix}$, ..., $s_{\widetilde{M^{M_T}-1}} = \begin{bmatrix} s_1{}^{M^{M_T}-1} \\ s_2{}^{M^{M_T}-1} \\ \vdots \\ s_{M_T}{}^{M^{M_T}-1} \end{bmatrix}$

and $\widetilde{s_0}$, $\widetilde{s_1}$,... , $s_{\widetilde{M^{M_T}-1}}$ each having a dimension of M_Tx1.

By considering (12), the expression for MIMO is illustrated below

$$\min\left(\left(\left|y_1 - A_{1,1}\, s_1{}^0 - A_{1,2}\, s_2{}^0 - \cdots - A_{1,M_T}\, s_{M_T}{}^0\right|^2 + \left|y_2 - A_{2,1}\, s_1{}^0 - A_{2,2}\, s_2{}^0 - \cdots - A_{2,M_T}\, s_{M_T}{}^0\right|^2 + \cdots + \left|y_{M_R} - A_{M_R,1}\, s_1{}^0 - A_{M_R,2}\, s_2{}^0 - \cdots - A_{M_R,M_T}\, s_{M_T}{}^0\right|^2\right), \dots, \left(\left|y_1 - A_{1,1}s_1{}^{M^{M_T}-1} - A_{1,2}s_2{}^{M^{M_T}-1} - \cdots - A_{1,M_T}s_{M_T}{}^{M^{M_T}-1}\right|^2 + \left|y_2 - A_{2,1}s_1{}^{M^{M_T}-1} - A_{2,2}s_2{}^{M^{M_T}-1} - \cdots - A_{2,M_T}s_{M_T}{}^{M^{M_T}-1}\right|^2 + \cdots + \left|y_{M_R} - A_{M_R,1}\, s_1{}^{M^{M_T}-1} - A_{M_R,2}\, s_2{}^{M^{M_T}-1} - \cdots - A_{M_R,M_T}\, s_{M_T}{}^{M^{M_T}-1}\right|^2\right)\right) \tag{31}$$

where $\widetilde{s_0} = \begin{bmatrix} s_1{}^0 \\ s_2{}^0 \\ \vdots \\ s_{M_T}{}^0 \end{bmatrix}$, $\widetilde{s_1} = \begin{bmatrix} s_1{}^1 \\ s_2{}^1 \\ \vdots \\ s_{M_T}{}^1 \end{bmatrix}$, ..., $s_{\widetilde{M^{M_T}-1}} = \begin{bmatrix} s_1{}^{M^{M_T}-1} \\ s_2{}^{M^{M_T}-1} \\ \vdots \\ s_{M_T}{}^{M^{M_T}-1} \end{bmatrix}$

and similar to MISO $\widetilde{s_0}$, $\widetilde{s_1}$,... , $s_{\widetilde{M^{M_T}-1}}$ each have a dimension of M_Tx1.

Considering a simple case with SISO system using 2-PAM as the modulation scheme and Rayleigh channel as shown in Fig.13 to illustrate the ML decoding, we receive an arbitrary signal denoted by y. Due to properties of rotation and attenuation characterized by Rayleigh channel, there is possibility that z_0 and z_1 could replace each other and both z_0 and z_1 can be attenuated with values less than -1 and 1 respectively. In the figure below, -0.8 and 0.8 are taken as arbitrary values to explain the concept. Also 0 and 1 has been rotated 180° and attenuation is given by 0.8. We find the Euclidean distance from y to z_0 i.e. $D^2_{y,z0}$ and then similarly $D^2_{y,z1}$ and we take the minimum of these two distances. It is obvious from the figure that Euclidean distance is minimum from y to z_1, therefore the decision is that s_1 was transmitted.

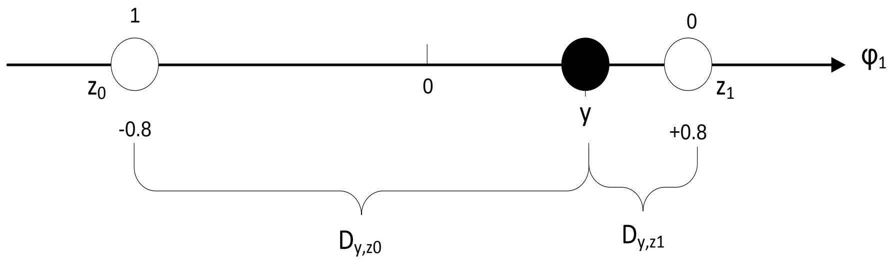

Fig.13: ML decoding with 2-PAM

Now we take another simple case where SISO system is used but now with 4-QAM and Rayleigh channel. The symbols 0 and 3 have been rotated 180° and all the symbols have been attenuated. Similar to previous case, we calculate Euclidean distance i.e. $D^2_{y,z0}$, $D^2_{y,z1}$, $D^2_{y,z2}$ and $D^2_{y,z3}$ and find the minimum distance from Fig.14. It can be seen that $D^2_{y,z0}$ is the minimum distance and therefore s_0 is decided to be the transmitted symbol.

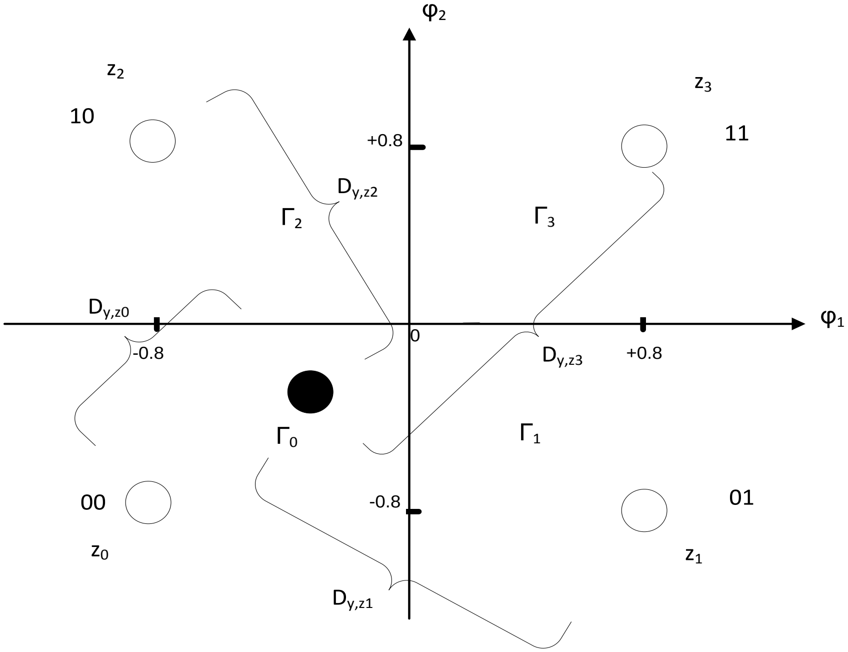

Fig.14: ML decoding with 4-QAM

Similarly, we perform same method for 16-QAM SISO to get the required result and for the SIMO, MISO and MIMO.

CHAPTER 5

COMPUTER SIMULATIONS OF SYMBOL ERROR PROBABILITY FOR DIFFERENT ANTENNA SYSTEMS

Error rate simulations are very important to get better and clear understanding of the practical behavior of symbol error rates of different multiple antenna systems. It also helps us to get better idea how the symbol error rate performances are as compared to theoretical knowledge. Section 5.1 briefly describes how the simulations were carried out. Section 5.2 derives an upper bound on the symbol error probability for some of the studied modulation schemes.

5.1 Symbol Error Probability

Simulations were carried out based on Fig.9. For all the simulations concerning SISO, SIMO, MISO and MIMO, SNR and symbol error rate were initialized. Bits were converted to symbols. Gray coding scheme was applied and these symbols were sent through AWGN, Rayleigh or Ricean channel. Then AWGN noise was added and finally decisions regarding symbols were made using optimal receiver namely ML receiver. After the decision on symbols had been made, number of symbol errors and number of symbols was updated. If the number of symbol errors was less than 700, again bits were converted to symbols and the same procedure was carried out. If the number of symbol errors was greater than 700, symbol error rate was calculated. In order to reduce the simulation time, if the symbol error rate was less than 10^{-5}, we stop the simulation otherwise we updated SNR value and symbol error rate value and take the bits, convert them to symbols and do the procedure all over again. For simulations 30 values of SNR were chosen with the starting point as 0 and the ending point to be 30 in decibels. Numbers of errors were chosen to be 700, which was a compromise between the amount of time for simulations and statistical significance.

The formula for estimating symbol error rate is

$$P_s \approx \frac{S_{err}}{S} \tag{32}$$

where P_s is the estimated symbol error rate or symbol error probability, S_{err} is the number of symbol errors, $S_{err} = 700$and S is the total number of symbols simulated.

5.1.1 Problems

First method to simulate symbol error rate was that 10^8 symbols were used for simulation in iteration. The drawback of simulations regarding aforementioned symbols was that it took large amount of time to simulate at each value of SNR. It was not a feasible method. For high level modulation schemes, the simulations were long when using 10^8 iterations for each value of SNR because $M^{M}{}_{T}$ distances had to be calculated. Statistical significance was good as the curves were sharp.

Second method which used to simulate symbol error rate was to use multiple loops i.e. more than two loops, which was again not a feasible procedure because of high time consumption. Here also the statistical significance was good.

Third method was that try was made to make further improvement on the script. Instead of simulating 10^8 iterations, an appropriate choice was to choose 10^3 symbols for each value of SNR. Two other conditions were applied to terminate the computer simulations. One condition included putting a check on number of errors. For example, if $P_s \approx 10^{-3}$, then using (26), $S \approx 700000$ if S_{err}= 700. Now if $P_s \approx 10^{-1}$ and S_{err}= 700, then $S \approx 7000$. Therefore, at low SNR and using less number of symbols, the simulation is faster. The reason why 700 was chosen because if S_{err} is very small for example S_{err}=1, then statistical significance will be bad implying that the curves would have ripples in them. Amount of time for simulation is reduced with 700. Symbol error rate limit was 10^{-5}. At low SNR, simulations are fast so symbol error rate limit is not important but at high SNR, 10^{-5} reduces time for simulation which otherwise would be very slow. For the simplest AWGN case using BPSK modulation, this value was S_{err}= 300. Another condition was to put check on number of symbols, which was as mentioned before i.e. 10^8. Finally, the aim of these limits was to terminate the simulation whenever, number of errors were greater than 300 and number of symbols were greater than 10^8. For 4-QAM and 16-QAM, the limit of number of errors used was 10000 or higher. However, the second limit is the same for all the modulation schemes and antenna systems. This was to reduce the number of loops as much as possible to two loops. For example, simulating the symbol error rate in the range of SNR i.e. when SNR = 0-1 dB, if instantaneous value of communication parameters namely *y*, *s*, *n* or A_{Ray} is used in the simulations, it requires additional loop namely that for the number of symbols apart from loop of SNR values and the two check conditions. In order to reduce the time of simulations, iteration loop of number of symbols was removed. Instead, all the values were taken simultaneously from 0 to 1000 symbol interval T_s at each value of SNR. To be specific, the value of *s* for SNR range of 0-1 dB for e.g. from 0 to T_s, actually corresponds to running the loop of number of symbols for the first time. Similarly, value of *s* from T_s to $2T_s$ corresponds to running the loop for the second time and it is repeated until 1000Ts. Same explanation is given for *y*, *n*, *A* and A_{Ray}. In mathematical terminology, matrices of size 1000 were created for each of these communication parameters. Exactly same procedure was carried out for SIMO, MISO and MIMO. For these three cases, the matrices are 3 dimensional. In this method, again for the lower modulation schemes employing up to two antennas at transmitter and receiver, the simulations were short but for higher number of antennas and higher modulation scheme like 16-QAM, it was again time consuming because as signal to noise ratio was increased, it took more time to get to that error limit or number of symbols limit. This is because for higher SNR, it is possible that the error occurs after long period of time and at very low symbol error rate like 10^{-5}.

5.1.2 Solution

Finally, the limit for the number of symbols was removed and also error limit was reduced to 700. Another limit was introduced namely for symbol error rate. If the symbol error rate is less than 10^{-5}, the program do not need to be simulated and completely exit from it. There were some ripples and edges in the simulated curves but the simulations were quiet fast.

For all the simulations, the variance, ${\sigma_h}^2$, of the Rayleigh channel was ${\sigma_h}^2 = \frac{1}{20}$ in both the real and imaginary part and mean and variance of AWGN should be as mentioned before in (8). In order to compare symbol error rate performances of different modulation schemes, average transmitted bit energy was also set to one. Flowchart below shows briefly the procedure of the fourth, and final, method discussed in this subsection.

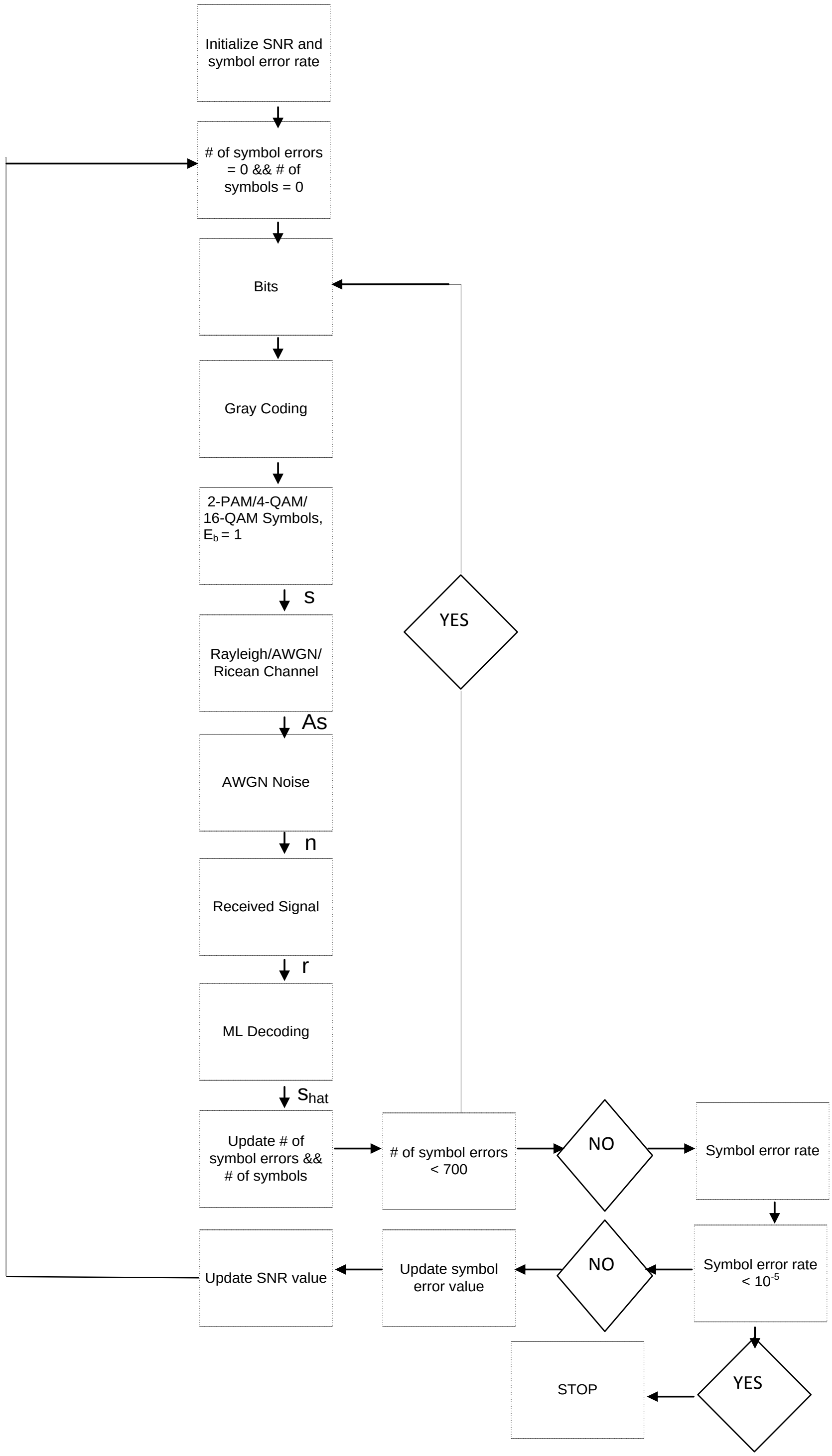

Fig. 15: Flowchart of Symbol Error Rate Simulations

5.2 Derivation of Upper Bound on the Symbol Error Probability

5.2.1 4-QAM

An upper bound is a theoretical result that often is used to complement the simulated results. It is also of interest how much the simulated results deviate from the upper bound. It is basically an upper limit on the symbol error probability. The upper bound for the symbol error probability of AWGN channel is different from that of Rayleigh and Ricean channel.

Let us now derive an upper bound for the symbol error probability of Rayleigh channel for all four antenna system configurations since Rayleigh case is one of the most important special cases to be considered for all antenna configurations.

Assume there are M^{M_T} signal points in the vector space diagram illustrated below

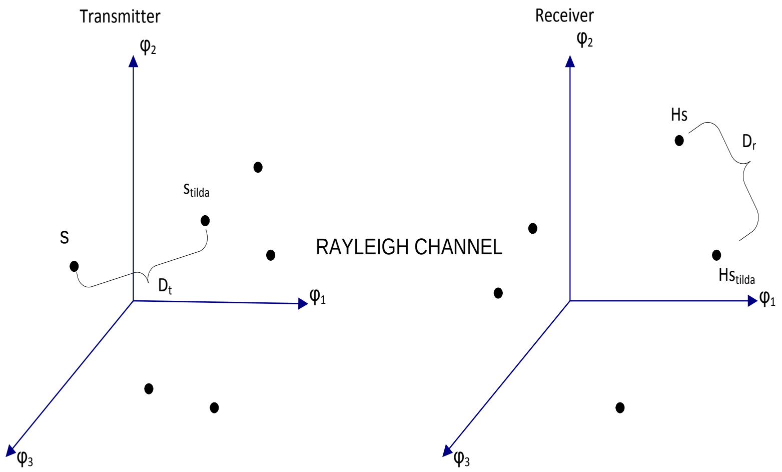

Fig.16

where *s* is one possible transmitted symbol vectors and $\tilde{s}$, denoted by s_{tilda}, is another possible transmitted symbol vectors.

For the binary AWGN case, it is known that the bit error probability for the minimum Euclidean distance ML receiver [9] is

$$P_s = P_b = Q\left(\sqrt{\frac{D_{0,1}{}^2}{2N_0}}\right) \tag{33}$$

We know that an upper bound on $Q(x)$ is

$$Q(x) \le \frac{1}{2} e^{\frac{-x^2}{2}}, x \ge 0 \tag{34}$$

[8] where $Q(x)$ is the tail of the Gaussian distribution given by

$$Q(x) = \int_x^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} dy$$

It is also known that the so called pair-wise error probability for the Rayleigh fading case is denoted by $P_2(i,j)$ is [12]

$$P_2(i,j) = E\left\{Q\left(\sqrt{\frac{D_{i,j}{}^2}{2N_0}}\right)\right\} \le E\left\{\frac{e^{\frac{-D^2}{4N_0}}}{2}\right\} = \frac{1}{2} E\left\{e^{\frac{-D^2}{4N_0}}\right\} \tag{35}$$

$$D_{i,j}{}^2 = \|As - A\tilde{s}\|^2 = \sum_{i=1}^{M_R}\left(X_i{}^2 + Y_i{}^2\right) \tag{36}$$

where X_i and Y_j are zero mean Gaussian independent and identically distributed for all i and j. Also

$$\sigma_x{}^2 = \sigma_y{}^2 = \sigma_h{}^2 D_t^2 \tag{37}$$

where $\sigma_h{}^2$ is variance in the real and imaginary part of Rayleigh channel and it is taken to be 1.

We need expected value because the realizations are Rayleigh and they are varying over time so in order to get the average, we need expected operator.

The so called union bound is a well known upper bound given by right hand side below,

$$P_s \le \frac{1}{M^{M_T}} \sum_{j=1}^{M^{M_T}} \sum_{\substack{i=1 \\ j \ne i}}^{M^{M_T}} E\left\{Q\left(\sqrt{\frac{D_{i,j}{}^2}{2N_0}}\right)\right\} \tag{38}$$

After substituting (36) in (35), we get

$$\frac{1}{2}\mathrm{E}\left\{e^{\frac{-\mathrm{D}^2}{4\mathrm{N}_0}}\right\} = \frac{1}{2}\mathrm{E}\left\{\mathrm{e}^{\frac{-\sum_{i=1}^{M_R}\left({X_i}^2+{Y_i}^2\right)}{4\mathrm{N}_0}}\right\} =$$

$$= \frac{1}{2}\mathrm{E}\left\{\mathrm{e}^{\frac{-\left({X_1}^2+{Y_1}^2\right)}{4\mathrm{N}_0}} \cdot \mathrm{e}^{\frac{-\left({X_2}^2+{Y_2}^2\right)}{4\mathrm{N}_0}} \cdots \mathrm{e}^{\frac{-\left({X_{M_R}}^2+{Y_{M_R}}^2\right)}{4\mathrm{N}_0}}\right\}$$

$$= \frac{1}{2}\mathrm{E}\left\{\mathrm{e}^{\frac{-\left({X_1}^2+{Y_1}^2\right)}{4\mathrm{N}_0}}\right\} . \mathrm{E}\left\{\mathrm{e}^{\frac{-\left({X_2}^2+{Y_2}^2\right)}{4\mathrm{N}_0}}\right\} \ldots \mathrm{E}\left\{\mathrm{e}^{\frac{-\left({X_{M_R}}^2+{Y_{M_R}}^2\right)}{4\mathrm{N}_0}}\right\}$$

$$= \frac{1}{2}\mathrm{E}\left\{\mathrm{e}^{\frac{-{X_1}^2}{4\mathrm{N}_0}} . \mathrm{e}^{\frac{-{Y_1}^2}{4\mathrm{N}_0}}\right\} . \mathrm{E}\left\{\mathrm{e}^{\frac{-{X_2}^2}{4\mathrm{N}_0}} . \mathrm{e}^{\frac{-{Y_2}^2}{4\mathrm{N}_0}}\right\} \ldots \mathrm{E}\left\{\mathrm{e}^{\frac{-{X_{M_R}}^2}{4\mathrm{N}_0}} . \mathrm{e}^{\frac{-{Y_{M_R}}^2}{4\mathrm{N}_0}}\right\} \tag{39}$$

From above expression, the expression $\mathrm{E}\left\{e^{\frac{-{X_1}^2}{4\mathrm{N}_0}}\right\}$ is same as $\mathrm{E}\left\{e^{-a{X_1}^2}\right\}$, where a is a constant and $a = \frac{1}{4\mathrm{N}_0}$.

General expression for evaluation of $\mathrm{E}\{X\}$, which is expected value of Gaussian random variable X and is given by

$$\mathrm{E}\{X\} = \int_{-\infty}^{\infty} X pdf(X) dX \tag{40}$$

where pdf(X) in this case is probability density function of Gaussian random variable X.

Probability density function of Gaussian distribution is given by

$$\frac{e^{\frac{-(X_1-\mu)^2}{2\sigma^2}}}{\sigma\sqrt{2\pi}} \tag{41}$$

where μ is the mean and σ^2 is the variance.

Therefore [9],

$$\mathrm{E}\left\{e^{-a{X_1}^2}\right\} = \int_{-\infty}^{\infty} \mathrm{e}^{-a{X_1}^2} \frac{e^{\frac{-(X_1-\mu)^2}{2\sigma^2}}}{\sigma\sqrt{2\pi}} \, dX_1$$

$$= \int_{-\infty}^{\infty} e^{-aX_1{}^2} \frac{\mathbf{e}^{\frac{-(X_1 - \mathbf{2}\mu X_1 + \mu^2)}{2\sigma^2}}}{\sigma\sqrt{2\pi}} dX_1$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-aX_1{}^2} e^{\frac{-(X_1 - \mathbf{2}\mu X_1 + \mu^2)}{2\sigma^2}} dX_1$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\left(a + \frac{1}{2\sigma^2}\right)X_1{}^2 + \frac{2\mu X_1}{2\sigma^2} - \frac{\mu^2}{2\sigma^2}} dX_1$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{\frac{\left(-(2\sigma^2 a + 1)X_1{}^2 + 2\mu X_1 - \mu^2\right)}{2\sigma^2}} dX_1$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{\left(-X_1{}^2 + \frac{2\mu X_1}{(2\sigma^2 a + 1)} - \frac{\mu^2}{(2\sigma^2 a + 1)}\right)\frac{(2\sigma^2 a + 1)}{2\sigma^2}} dX_1$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\left(X_1{}^2 - \frac{2\mu X_1}{(2\sigma^2 a + 1)} + \frac{\mu^2}{(2\sigma^2 a + 1)}\right)\frac{(2\sigma^2 a + 1)}{2\sigma^2}} dX_1$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\left(\left(X_1{}^2 - \frac{\mu}{2\sigma^2 a + 1}\right)^2 + \frac{\mu^2}{2\sigma^2 a + 1} - \frac{\mu^2}{(2\sigma^2 a + 1)^2}\right)\frac{(2\sigma^2 a + 1)}{2\sigma^2}} dX_1$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} \left(e^{-\left(X_1 - \frac{\mu}{2\sigma^2 a + 1}\right)^2} e^{-\left(\frac{\mu^2}{2\sigma^2 a + 1} - \frac{\mu^2}{(2\sigma^2 a + 1)^2}\right)\frac{(2\sigma^2 a + 1)}{2\sigma^2}} \right) dX_1$$

$$= \frac{e^{-\left(\left(\frac{\mu^2}{2\sigma^2 a + 1} - \frac{\mu^2}{(2\sigma^2 a + 1)^2}\right)\frac{(2\sigma^2 a + 1)}{2\sigma^2}\right)}}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\left(\left(X_1 - \frac{\mu}{2\sigma^2 a + 1}\right)^2\right)\frac{(2\sigma^2 a + 1)}{2\sigma^2}} dX_1$$

$$= \frac{e^{\frac{-a\mu^2}{(2\sigma^2 a+1)}}}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\left(\left(X_1 - \frac{\mu}{2\sigma^2 a+1}\right)^2\right)\frac{(2\sigma^2 a+1)}{2\sigma^2}}\, dX_1$$

Let $\widetilde{\sigma^2} = \frac{\sigma^2}{(2\sigma^2 a+1)}$

$$\mathrm{E}\left\{e^{-aX_1{}^2}\right\} = \frac{e^{\frac{-a\mu^2}{(2\sigma^2 a+1)}}}{\sigma\sqrt{2\pi}}\tilde{\sigma}\sqrt{2\pi}\underbrace{\int_{-\infty}^{\infty}\frac{e^{\frac{-\left(X_1-\frac{\mu}{2\sigma^2 a+1}\right)^2}{2\widetilde{\sigma^2}}}}{\tilde{\sigma}\sqrt{2\pi}}}_{=1}\, dX_1$$

The total area under a probability density function equals 1.

$$\therefore \mathrm{E}\left\{e^{-aX_1{}^2}\right\} = \frac{e^{\frac{-a\mu^2}{(2\sigma^2 a+1)}}}{\sqrt{2\sigma^2 a+1}} \tag{42}$$

Since the random variable is X, the variance as given in (31) is $\sigma_X{}^2 = \sigma_h{}^2 \mathrm{D_t}{}^2$.

$$\mathrm{E}\left\{e^{-aX_1{}^2}\right\} = \frac{e^{\frac{-a\mu^2}{(2\sigma_X{}^2 a+1)}}}{\sqrt{2\sigma_X{}^2 a+1}}$$

Since Y is identical to X, we have

$$\mathrm{E}\left\{e^{-aY_1{}^2}\right\} = \frac{e^{\frac{-a\mu^2}{(2\sigma_Y{}^2 a+1)}}}{\sqrt{2\sigma_Y{}^2 a+1}}$$

Considering the first expression from (30) namely $\mathrm{E}\left\{e^{\frac{-X_1{}^2}{4\mathrm{N_0}}} . e^{\frac{-Y_1{}^2}{4\mathrm{N_0}}}\right\}$

$$\mathrm{E}\left\{e^{\frac{-X_1{}^2}{4\mathrm{N_0}}} . e^{\frac{-Y_1{}^2}{4\mathrm{N_0}}}\right\} = \int_{-\infty}^{\infty} e^{\frac{-X_1{}^2}{4\mathrm{N_0}}}\, \mathrm{pdf}(X_1)\, dX_1 e^{\frac{-Y_1{}^2}{4\mathrm{N_0}}}\, \mathrm{pdf}(Y_1) dY_1$$

$$= \int_{-\infty}^{\infty} e^{\frac{-X_1{}^2}{4\mathrm{N_0}}}\, \mathrm{pdf}(X_1) dX_1 \int_{-\infty}^{\infty} e^{\frac{-Y_1{}^2}{4\mathrm{N_0}}}\, \mathrm{pdf}(Y_1) dY_1$$

$$= \frac{e^{\frac{-a\mu^2}{(2\sigma_X{}^2 a+1)}}}{\sqrt{2\sigma_X{}^2 a+1}} \frac{e^{\frac{-a\mu^2}{(2\sigma_Y{}^2 a+1)}}}{\sqrt{2\sigma_Y{}^2 a+1}} \tag{43}$$

Assume E_t is the total energy transmitted from all M_T antennas and ϵ is the total received energy.

$$\therefore \epsilon = E(As) = \underbrace{M_R\,.2\sigma_h{}^2}_{<1}\,.\,E_t$$

In order to make total received energy smaller than the total transmitted energy, $\sigma_h{}^2 = \frac{1}{20}$ is chosen instead of $\sigma_h{}^2 = 1$ with M_R less than ten.

By using $\mu = 0$ and $\sigma^2 = \frac{1}{20}$ and substituting them into (37) and (43), we get

$$E\left\{e^{\frac{-X_1{}^2}{4N_0}}\,.\,e^{\frac{-Y_1{}^2}{4N_0}}\right\} = \frac{1}{1+2.\frac{1}{20}.D_t{}^2.\frac{1}{4N_0}} \tag{44}$$

We need to find the minimum D_t , namely $D_{t,min}$. Alternatively, we have to find all the possible Euclidean distances for a signal constellation.

By referring to Fig.3 and Section 2.2.1, $D_{0,1}$, $D_{0,2}$, $D_{0,3}$, $D_{1,2}$, $D_{1,3}$, $D_{2,3}$ have to be calculated.

$$D_{0.1}{}^2 = \int_0^{T_s} \big(s_0(t) - s_1(t)\big)^2 dt =$$

$$= \int_0^{T_s} \big(s_0{}^2(t) + s_1{}^2(t) - 2s_0(t)s_1(t)\big)\, dt$$

$$= E_0 + E_1 - 2\int_0^{T_s} s_0(t)s_1(t)\, dt \tag{45}$$

The term $\int_0^{T_s} s_0(t)s_1(t)\, dt = 0$ when two symbols are orthogonal and it is the case for QAM as it is an orthogonal modulation scheme.

$\therefore D_{0.1}{}^2 = E_0 + E_1 = 4.$

Similarly, $D_{1,3}{}^2 = D_{2,3}{}^2 = D_{0,2}{}^2 = D_{0,1}{}^2$.

To calculate $D_{0,3}$

$$D_{0,3}{}^2 = D_{0,2}{}^2 + D_{2,3}{}^2$$

since $D_{0,3}$ is the hypotenuse of the triangle with $D_{0,2}$ as the adjacent side and $D_{2,3}$ as the opposite side.

Now using (36),

$$D_{0,3}{}^2 = E_0 + E_2 + E_0 + E_3$$

$$= 2+2+2+2 = 8$$

Similarly, $D_{1,2}{}^2 = D_{0,3}{}^2$.

From the above calculations, it is clear that $D_{t,min}{}^2 = 4$. Substituting it in (42), we obtain

$$E\left\{e^{\frac{-X_1{}^2}{4N_0}} . e^{\frac{-Y_1{}^2}{4N_0}}\right\} = \frac{1}{1+2.\frac{1}{20}.\frac{1}{N_0}} \tag{46}$$

Since snr = $\frac{E_b}{N_0}$ and E_b=1 as seen from Section 2.2.1, (46) can be rewritten as

$$E\left\{e^{\frac{-X_1{}^2}{4N_0}} . e^{\frac{-Y_1{}^2}{4N_0}}\right\} = \frac{1}{1+2.\frac{1}{20}.\frac{E_b}{N_0}}$$

$$= \frac{1}{1+2.\frac{1}{20}.snr} \tag{47}$$

∴ Expression in (39) results in

$$\frac{1}{2}\mathrm{E}\left\{\mathrm{e}^{\frac{-X_1{}^2}{4\mathrm{N}_0}}\cdot\mathrm{e}^{\frac{-Y_1{}^2}{4\mathrm{N}_0}}\right\}.\mathrm{E}\left\{\mathrm{e}^{\frac{-X_2{}^2}{4\mathrm{N}_0}}\cdot\mathrm{e}^{\frac{-Y_2{}^2}{4\mathrm{N}_0}}\right\}\ldots\mathrm{E}\left\{\mathrm{e}^{\frac{-X_{M_R}{}^2}{4\mathrm{N}_0}}\cdot\mathrm{e}^{\frac{-Y_{M_R}{}^2}{4\mathrm{N}_0}}\right\}=$$

$$\frac{1}{2}\left(\frac{1}{1+2.\frac{1}{20}.snr}\right)^{\mathrm{M_R}}$$

Finally, using (28), we obtain the upper bound for the symbol error probability which is

$$\mathrm{P_s}\leq\frac{1}{\mathrm{M}^{\mathrm{M_T}}}\sum_{j=1}^{M^{M_T}}\sum_{\substack{i=1\\ j\neq i}}^{M^{M_T}}\frac{1}{2}\left(\frac{1}{1+2.\frac{1}{20}.snr}\right)^{\mathrm{M_R}} \tag{48}$$

(48) can be simplified to

$$\mathrm{P_s}\leq\frac{1}{\mathrm{M}^{\mathrm{M_T}}}\mathrm{M}^{\mathrm{M_T}}(\mathrm{M}^{\mathrm{M_T}}-1)\left(\frac{1}{2}\left(\frac{1}{1+2.\frac{1}{20}.snr}\right)^{\mathrm{M_R}}\right)$$

$$\mathrm{P_s}\leq\frac{1}{2}(\mathrm{M}^{\mathrm{M_T}}-1)\left(\frac{1}{1+2.\frac{1}{20}.snr}\right)^{\mathrm{M_R}} \tag{49}$$

This upper bound expression is valid for 2-PAM and 4-QAM case. The above expression was shown when $\mathrm{M_T}=1$. When $\mathrm{M_T}>1$, the expression for $\mathrm{P_s}$ is still given by (43). $\mathrm{D_{t,min}}{}^2$ will still be 4. Consider $\mathrm{M_T}=3$. The vector s is given by s$=\begin{pmatrix}s_1\\ s_2\\ s_3\end{pmatrix}$ and $\hat{s}=\begin{pmatrix}\widehat{s_1}\\ \widehat{s_2}\\ \widehat{s_3}\end{pmatrix}$. Assume $s_2=\widehat{s_2}$ and $s_3=\widehat{s_3}$.Therefore, we have to only find distance from $s_1$to $\widehat{s_1}$, which will be $\mathrm{D_{t,min}}{}^2$. For 4-QAM, $\mathrm{D_{t,min}}{}^2=4$. From (49), the diversity gain is given by $\mathrm{M_R}$.

5.2.2 16-QAM

Similarly, for 16-QAM modulation schemes, we find all possible Euclidean distances and then calculate the minimum Euclidean distance which is the same as for 4-QAM case i.e. 2.

From Fig.4, s_0, s_2, s_8,and s_{10} are equidistant from each other.

$$\therefore\ \mathrm{D_{0.2}}{}^2=\mathrm{D_{0.8}}{}^2=\mathrm{D_{8.10}}{}^2=\mathrm{D_{2.10}}{}^2$$

From Section 2.3.1 and using (31), ${D_{0.2}}^2 = E_0 + E_2$= 18 + 18 = 36

s_1, s_3, s_9, s_{11}, s_4, s_{12}, s_9, and s_{14} have same distances from origin.
Therefore using Section 2.3.1, ${D_{1.3}}^2 = {D_{4.12}}^2 = {D_{9.11}}^2 = {D_{6.14}}^2 = 20$

Now s_5, s_7, s_{13},and s_{15} are equidistant from each other and origin similar to s_0, s_2, s_8,and s_{10}.

$\therefore {D_{5.7}}^2 = {D_{5.13}}^2 = {D_{13.15}}^2 = {D_{7.15}}^2 = 4.$

There are many other possibilities of squared Euclidean distances from Fig.4 but they have values greater than ${D_{t,min}}^2$. Hence, ${D_{t,min}}^2$=4.

Now since E_b=2.5 and using (38)

$$E\left\{e^{\frac{-{X_1}^2}{4N_0}} . e^{\frac{-{Y_1}^2}{4N_0}}\right\} = \frac{1}{1+2.\frac{1}{20}.{D_t}^2.\frac{2.5}{4.2.5.N_0}}$$

$$= \frac{1}{1+0.8.\frac{1}{20}.snr}$$

Hence upper bound for the symbol error probability is given by

$$P_s \leq \frac{1}{2}\left(M^{M_T} - 1\right)\left(\frac{1}{1+0.8.\frac{1}{20}.snr}\right)^{M_R} \quad (50)$$

CHAPTER 6

COMPUTER SIMULATION RESULTS

6.1 Rayleigh fading: Introduction

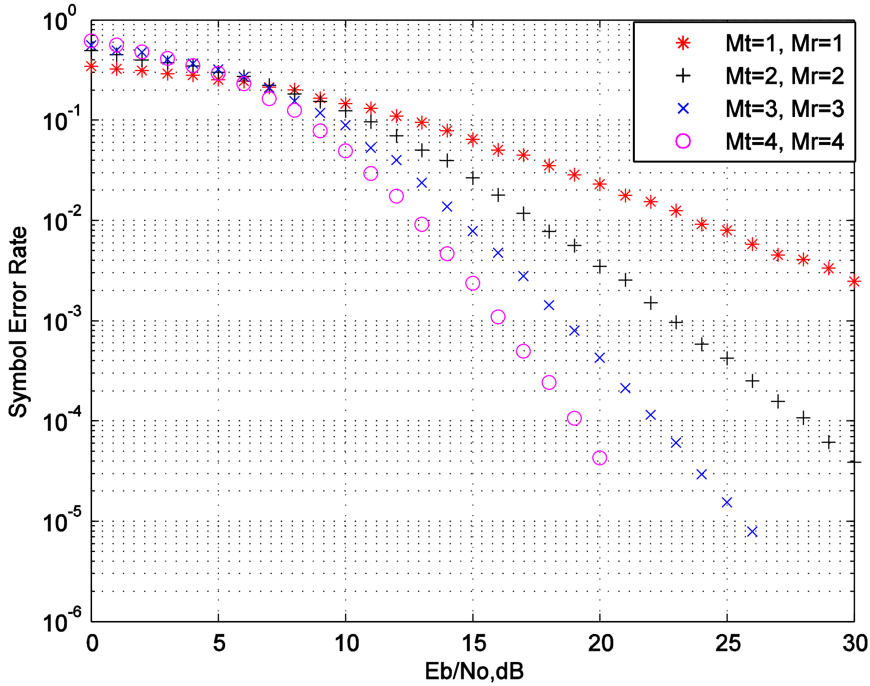

Fig.17: 2-PAM scheme using Rayleigh channel

In Fig.17, we have one SISO and three MIMO schemes using BPSK and Rayleigh channel. The MIMO schemes are employing equal number of antennas both at transmitter and at receiver. One scheme is using two antennas at transmitter and two at receiver, second one is using three at transmitter and three at receiver and the third one is using four at transmitter and four at receiver. Since the symbol time T_s is constant, bandwidth is also constant, and the bit rate is $R_b = \frac{M_T}{T_s}$.

According to (7), since four transmit and four receive antennas have highest number of bits of all other schemes, it has the highest bit rate followed by three transmit and three receive with SISO scheme having the least bit rate of these four.

Also according to (43), diversity gain of MIMO system is M_R. Therefore, MIMO schemes have high diversity as compared to SISO, which has no diversity gain.

Finally from Fig.17, it can be seen that at low SNR, MIMO scheme with four transmit and four receive antennas have highest symbol error rate followed respectively by three transmit and three receive, two transmit and two receive and finally one transmit and one receive antenna. This is because at low SNR, AWGN noise is high; therefore receiver is not able to make correct symbol decisions among many possible signal alternatives. At high SNR, AWGN is considerably reduced and in combination with higher diversity gain it is easier for the receiver to correctly determine the symbols, therefore, four transmit and four receive antennas has least symbol error probability of other three schemes.

6.2 Rayleigh SIMO

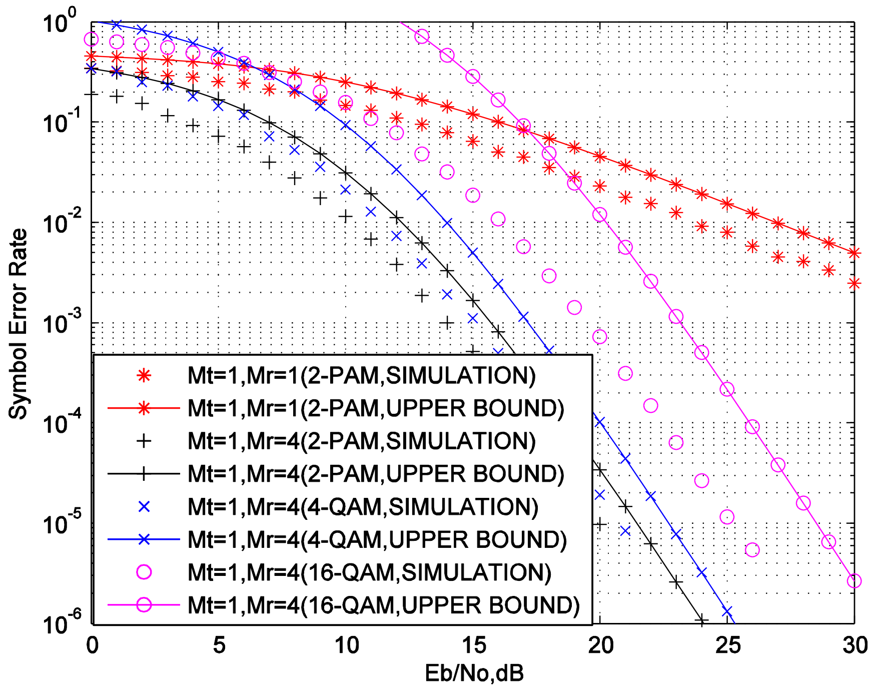

Fig.18: 2-PAM, 4-QAM and 16-QAM scheme using Rayleigh channel

In Fig.18, we have one SISO and three SIMO schemes using Rayleigh channel. All the four schemes also have their respective upper bounds as shown in Fig.18. SISO scheme is using 2-PAM. Each SIMO scheme is using 2-PAM, 4-QAM and 16-QAM schemes at transmitter and receiver. SIMO schemes are employing one antenna at transmitter and four antennas at receiver. According to (7), SIMO scheme using 16-QAM has the highest bit rate followed by SIMO scheme using 4-QAM and then SIMO scheme using 2-PAM. SISO scheme will have the same bit rate as the SIMO scheme using 2-PAM.

Also according to (43), there is diversity in SIMO schemes whereas SISO has no diversity. The SIMO schemes have equal diversity orders of M_R.

SIMO scheme having 16-QAM has the highest symbol error rate among other two SIMO schemes because of high modulation scheme used. At lower SNR, SIMO scheme with 16-QAM scheme has higher symbol error rate than the SISO scheme because receiver is not able to make correct symbol decisions due to presence of AWGN noise. But at high SNR, SIMO scheme employing 16-QAM has less symbol error rate than SISO due to lowering of AWGN noise and addition of diversity gain.

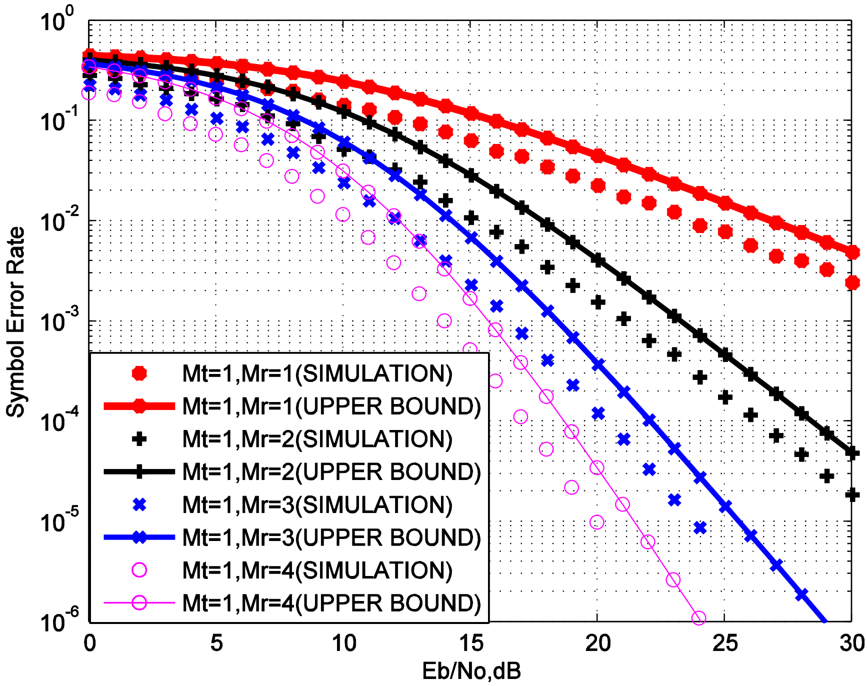

Fig.19: 2-PAM modulation scheme

In Fig.19, we have one SISO and three SIMO schemes. All the four schemes also have their respective upper bounds to illustrate that the simulated curves are within their limits. All schemes are using 2-PAM modulation schemes. SISO scheme has the worst symbol error rate among the four schemes. Among SIMO schemes, the scheme employing four receive antennas has the least symbol error rate followed by scheme employing three receive antennas, then two. This is because the receiver is easily able to distinguish the symbols when there are four antennas than two or three due to diversity since according to (43), there is diversity in SIMO schemes whereas SISO has no diversity. The SIMO schemes have equal diversity orders of M_R.

According to (7), all the schemes have equal bit rates. $R_b = \frac{M_T}{T_s}$

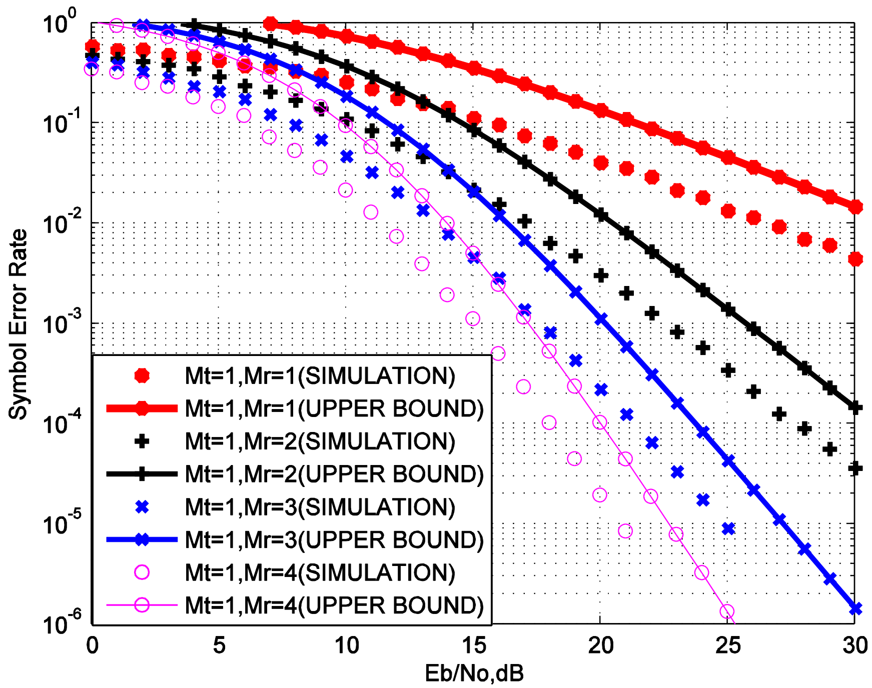

Fig.20: 4-QAM modulation scheme

In Fig.20, we have one SISO and three SIMO schemes. The explanation for this figure is similar to Fig.19 with only exception is that all schemes are using 4-QAM modulation schemes.

In [11], SNR performance of the ML receiver for M_T =1 and M_R=2 using 4-QAM modulation has been discussed. This system has M_R diversity, which is the same in Fig.20. In [11], the SER at SNR = 25dB for M_T =1 and M_R=2 is approximately 10^{-5} whereas in Fig.20, it is approximately 6×10^{-3}. The simulation is stopped for SER in [11] beyond 25dB but in Fig.20, SER exists for SNR = 30 dB. The reason for these differences are that the value of σ_h^2 is different, and also the SNR definition.

6.3 Rayleigh MIMO

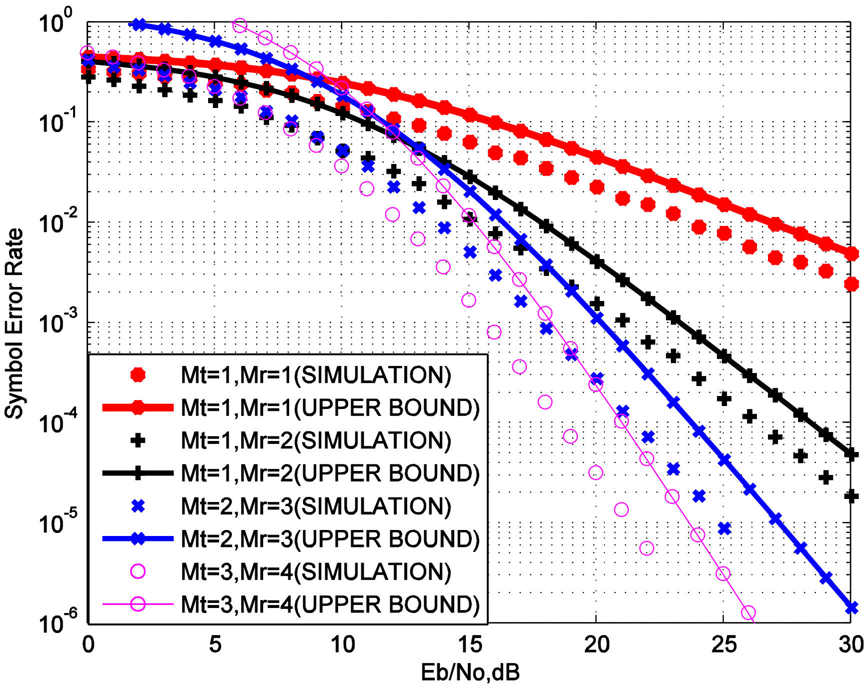

Fig.21: 2-PAM modulation scheme

In Fig.21, there are two MIMO schemes, one SIMO and one SISO. All the four schemes also have their respective upper bounds. SIMO scheme is employing two antennas at receiver. One MIMO scheme is employing two antennas at transmitter and three at receiver whereas other is using three antennas at transmitter and four at receiver. At low SNR, MIMO scheme using three transmit and four receive antennas has the highest symbol error rate among the four schemes followed by the other MIMO scheme, SISO and then SIMO. The explanation for low SNR and high SNR is the same as provided in Fig.17.

According to (43), SIMO and MIMO schemes experience diversity whereas SISO does not. They both have diversity gain of M_R.

According to (7), MIMO scheme with three transmit antennas has the highest bit rate followed by MIMO scheme having two transmit antennas. SIMO and SISO schemes have equal bit rates.

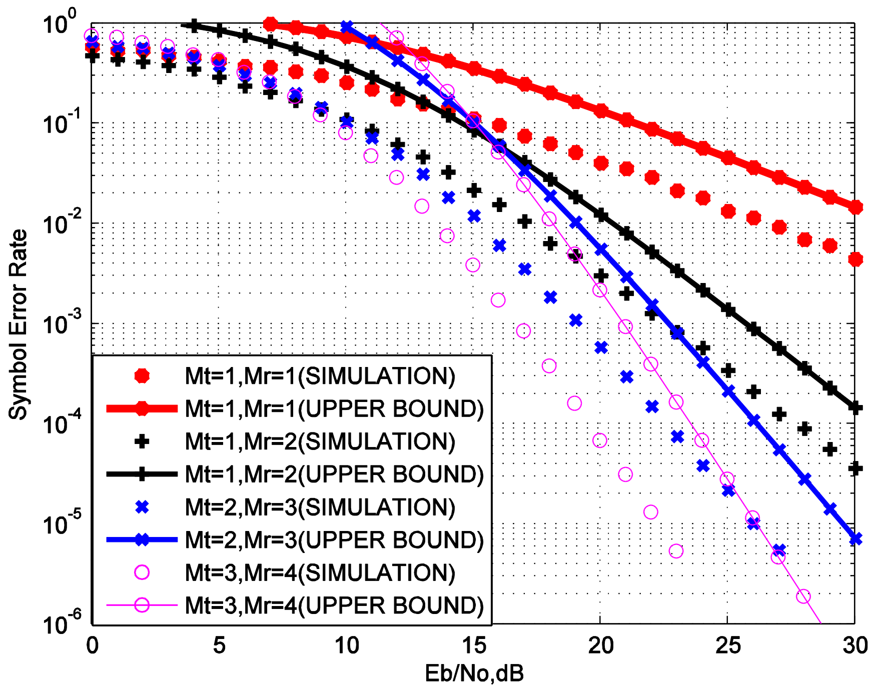

Fig.22: 4-QAM modulation scheme

Fig.22 is similar to Fig.21 with the only difference is that 4-QAM is used instead of 2-PAM. The rest of the explanation is same as in previous figure.

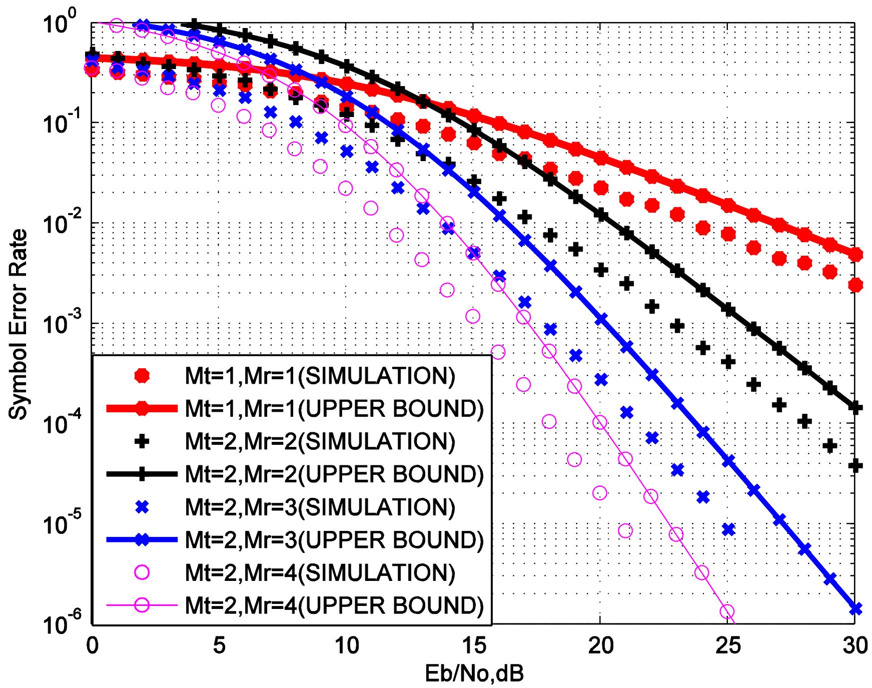

Fig.23: 2-PAM modulation scheme

In Fig.23, we have one SISO and three MIMO schemes using 2-PAM modulation scheme. All the four schemes also have their respective upper bounds. At low SNR, MIMO schemes have higher symbol error rate than the SISO scheme. At high SNR, scheme employing two transmit and four receive antennas has the lowest symbol error rate followed by scheme using two transmit and three receive and then two transmit and two receive with SISO having highest symbol error rate among the four schemes. The same explanation is given as in Fig.17 and Fig.19.

According to (7), MIMO schemes will have equal bit rates whereas SISO scheme will have the lowest bit rate. $R_b = \frac{M_T}{T_s}$

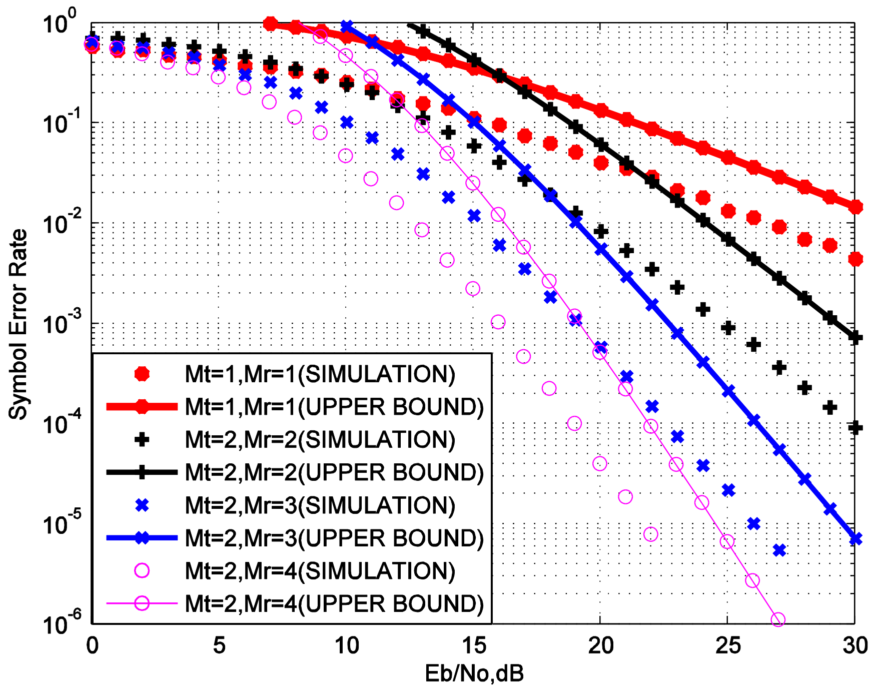

Fig.24: 4-QAM modulation scheme

Fig.24 is similar to Fig.23 with the only difference is that 4-QAM scheme is used instead of 2-PAM. At low SNR, MIMO schemes will have high symbol error rate as compared to SISO scheme but at high SNR, it is the opposite. See previous case.

In [11], SNR performance of the ML receiver for M_T =2 and M_R=2 using 4-QAM modulation has been discussed. This system has M_R diversity, which is the same in Fig.24. In [11], the SER at SNR = 25dB for M_T =2 and M_R=2 is approximately 10^{-4} whereas in Fig.24, it is approximately 10^{-3}. The simulation is stopped for SER in [11] beyond 25dB but in Fig.24, SER exists for SNR = 30 dB. The reason for these differences are that the value of σ_h^2 is different, and also the SNR definition.

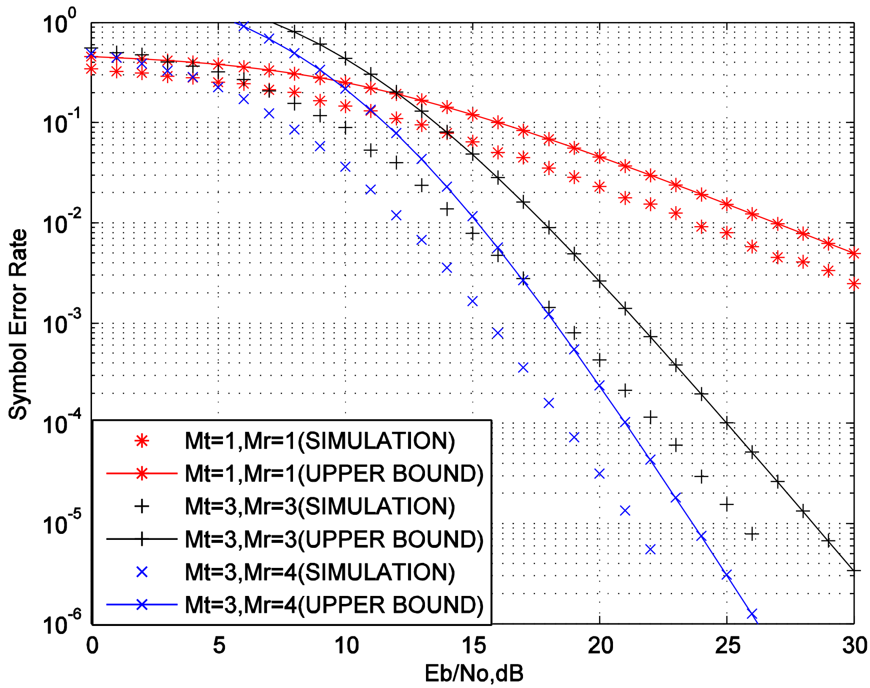

Fig.25: 2-PAM modulation scheme

In Fig.25, we have one SISO scheme and two MIMO schemes using 2-PAM modulation scheme. One MIMO scheme is using three antennas at transmitter and three at receiver. Other MIMO scheme is using three antennas at transmitter and four at receiver. All the three schemes also have their respective upper bounds. At low SNR, both MIMO schemes have higher symbol error rate than the SISO scheme. Refer to explanation given for Fig.17. At high SNR, MIMO schemes have low symbol error rate as compared to SISO. Again refer to Fig.17 for explanation. MIMO scheme using same number of transmitter and receiver have lower symbol error rate as compared to other MIMO scheme due to diversity at receiver.

According to (43), MIMO schemes experience diversity whereas SISO does not. MIMO has M_R diversity order.

According to (7), MIMO schemes will have equal bit rates whereas SISO scheme will have the lowest bit rate. $R_b = \frac{M_T}{T_s}$.

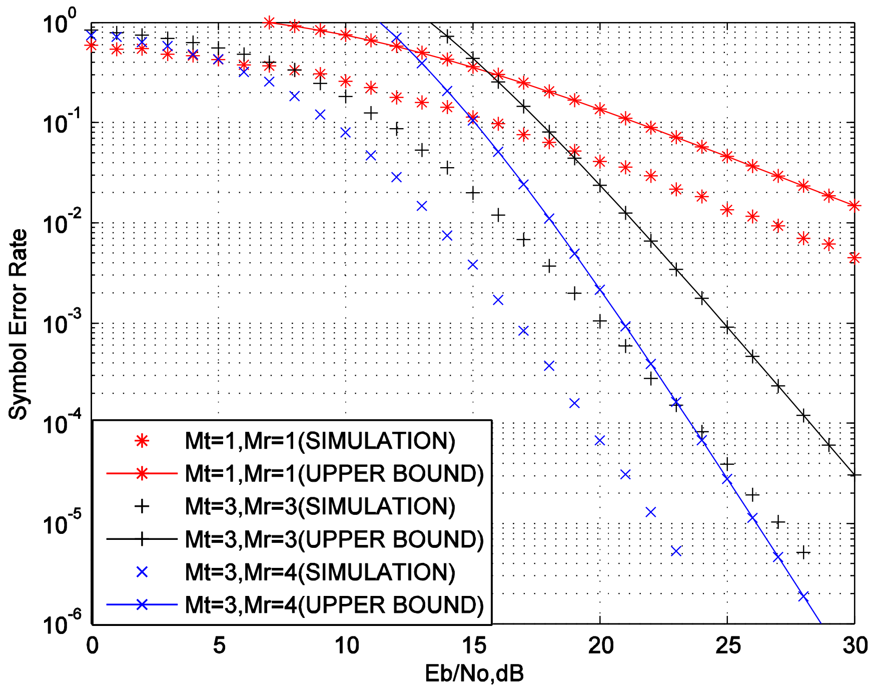

Fig.26: 4-QAM modulation scheme

Fig.26 is similar to Fig.25 with the only difference is that 4-QAM scheme is used instead of 2-PAM. At low SNR, MIMO schemes will have high symbol error rate as compared to SISO scheme but at high SNR, it is the opposite. See previous case.

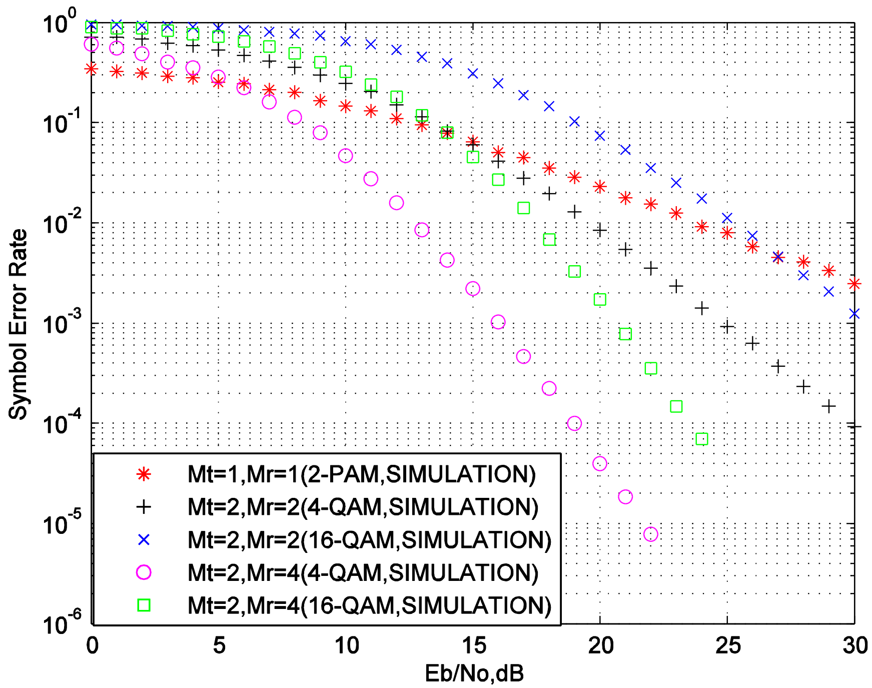

Fig.27: 2-PAM, 4-QAM and 16-QAM scheme using Rayleigh channel

In Fig.27, we have one SISO and four MIMO schemes. SISO scheme is using 2-PAM. Two MIMO schemes are using two transmit and two receive antennas with 4-QAM and 16-QAM. Other two MIMO schemes are using two transmit and four receive antennas with 4-QAM and 16-QAM as the modulation schemes. At low SNR, all the MIMO schemes have higher symbol error rate as compared to SISO. At high SNR, MIMO schemes have lower symbol error rate as compared to SISO with 4-QAM scheme using two antennas at transmitter and four at receiver having the least symbol error rate followed by 16-QAM using two antennas at transmitter and four at receiver, then 4-QAM scheme employing two antennas at transmitter and two at receiver and finally 16-QAM scheme using two antennas at transmitter and two at receiver with SISO scheme having the highest symbol error rate. Refer to the explanation given for Fig.17.

According to (7), MIMO schemes employing 16-QAM have the highest bit rates followed by MIMO schemes employing 4-QAM. Another important thing to note is that MIMO scheme with two transmit and two receive antenna having 4-QAM and MIMO scheme with two transmit and four receive antenna also having 4-QAM have equal bit rates. It is given by $R_b = \frac{2M_T}{T_s}$. Same goes for 16-QAM case as well whereas SISO scheme has the least bit rate.It is given by $R_b = \frac{4M_T}{T_s}$.

According to (43), MIMO schemes with two transmit and four receive antennas have highest diversity order. Other two MIMO schemes follow next.

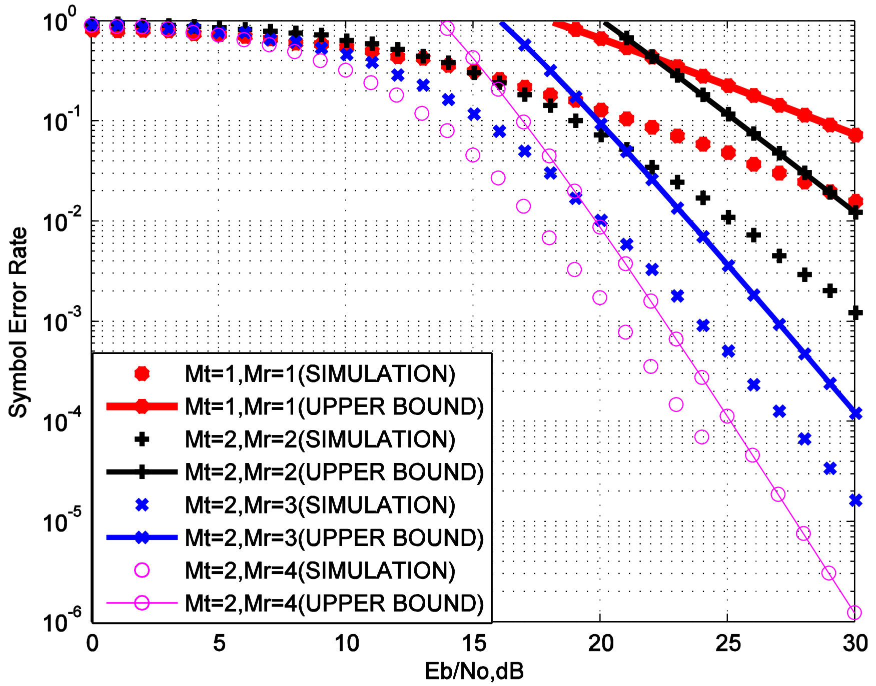

Fig.28: 16-QAM modulation scheme

In Fig.28, we have one SISO and three MIMO schemes using 16-QAM.One MIMO scheme is using two transmit and two receive antennas, second one is using two transmit and three receive antennas and the third one is using two transmit and four receive antennas. At low SNR, MIMO schemes have higher symbol error rate than SISO scheme, whereas at high SNR, MIMO schemes have lower symbol error rate as compared to SISO scheme with MIMO scheme using two antennas at transmitter and four at receiver having the least symbol error rate followed by MIMO scheme using two transmit and three receive antennas and finally MIMO scheme using two transmit and two receive antennas. SISO scheme has the largest symbol error rate. Refer to explanation for Fig.17.

According to (44), MIMO schemes experience diversity whereas SISO does not. MIMO has M_R diversity order.

According to (7), all MIMO schemes have equal bit rates, whereas SISO scheme will have the lowest bit rate among the four schemes. $R_b = \frac{4M_T}{T_s}$.

6.4 Ricean fading: Introduction

By using (19) and certain antenna configurations, we determine value certain cases of $|l|^2$. We take some cases of SISO, SIMO and MIMO. The Ricean fading behavior for SISO was determined at $|l|^2 = 0.1, |l|^2 = 0.8$ and $|l|^2 = 0$. By using (19), the Ricean factor will become 1, 8 and 0 respectively. For SIMO $(M_T = 1, M_R = 4)$ and MIMO $(M_T = 2, M_R = 4)$ $|l|^2 = 0.05, |l|^2 = 0.1$ and $|l|^2 = 0$ were used. The Ricean factor will be 0.5, 1 and 0 respectively.

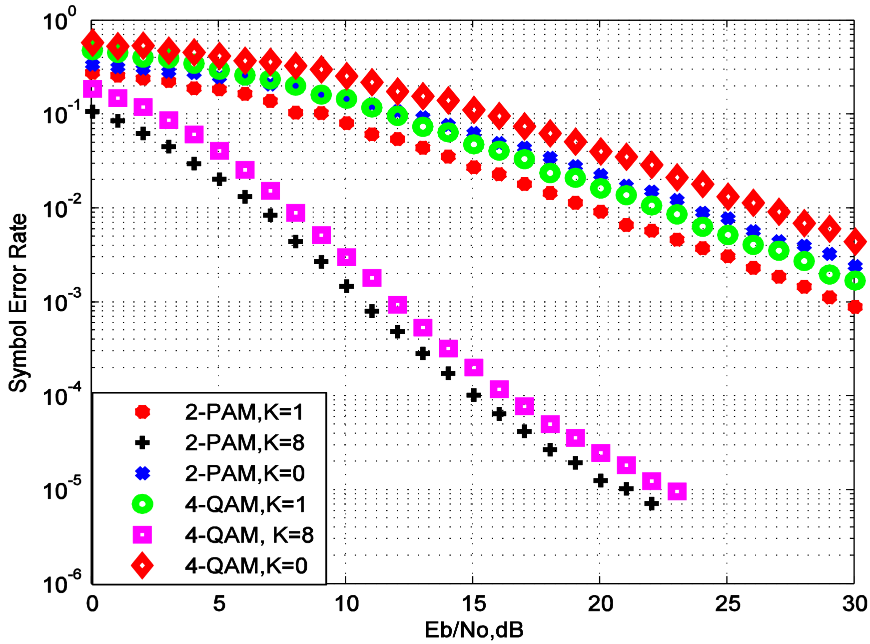

Fig.29: 2-PAM and 4-QAM modulation schemes using SISO and Ricean channel

In Fig.29, K = 0 is namely Rayleigh fading whereas K = 8 is a strong LOS factor as compared to K = 0, 1. It is clear from the figure that symbol error rate decreases with increase in LOS component. Also, 4-QAM scheme has larger symbol error rate as compared to 2-PAM scheme.

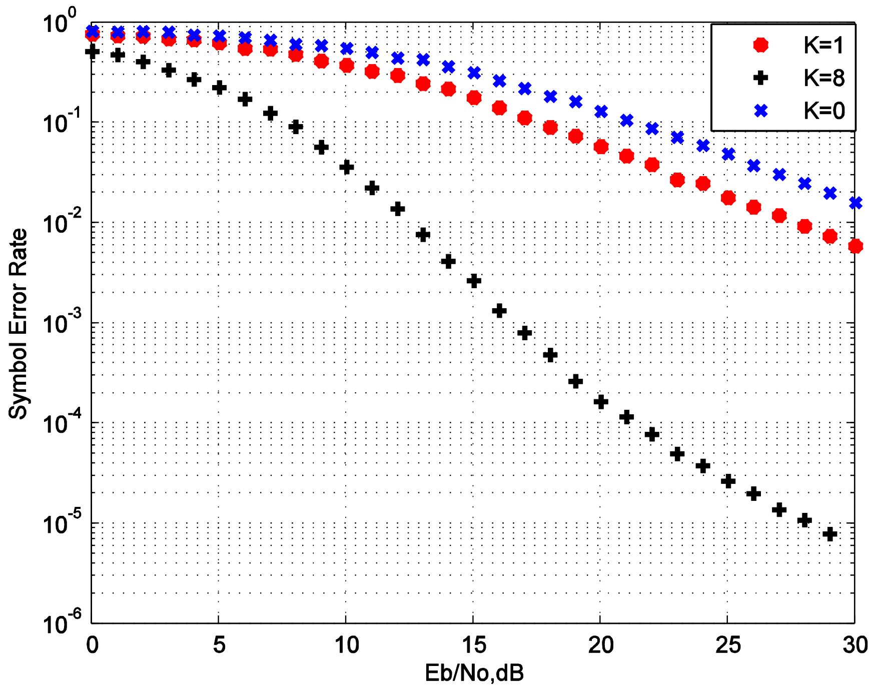

Fig.30: 16-QAM modulation scheme, SISO scheme

In Fig.30, the modulation scheme used is 16-QAM. As we increase the Ricean factor, the symbol error rate decreases because of presence of LOS component.

6.5 Ricean SIMO

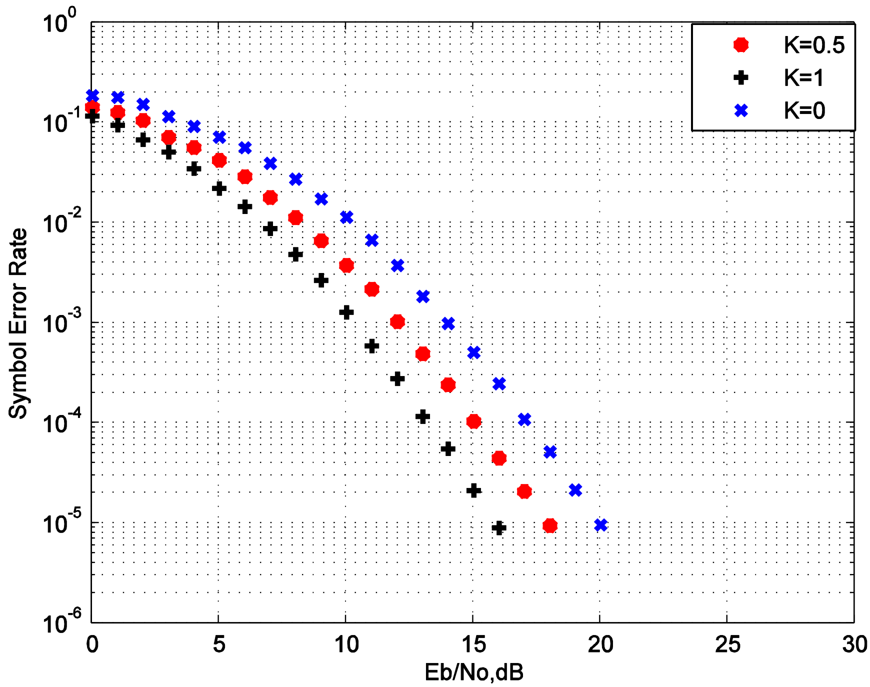

Fig.31: 2-PAM modulation scheme, (M_T = 1, M_R=4)

In Fig.31, the modulation scheme used is 2-PAM with one transmit and four receive antennas. The Ricean factors used are 0, 0.5 and 1. As we increase the Ricean factor, the symbol error rate decreases because of presence of LOS component.

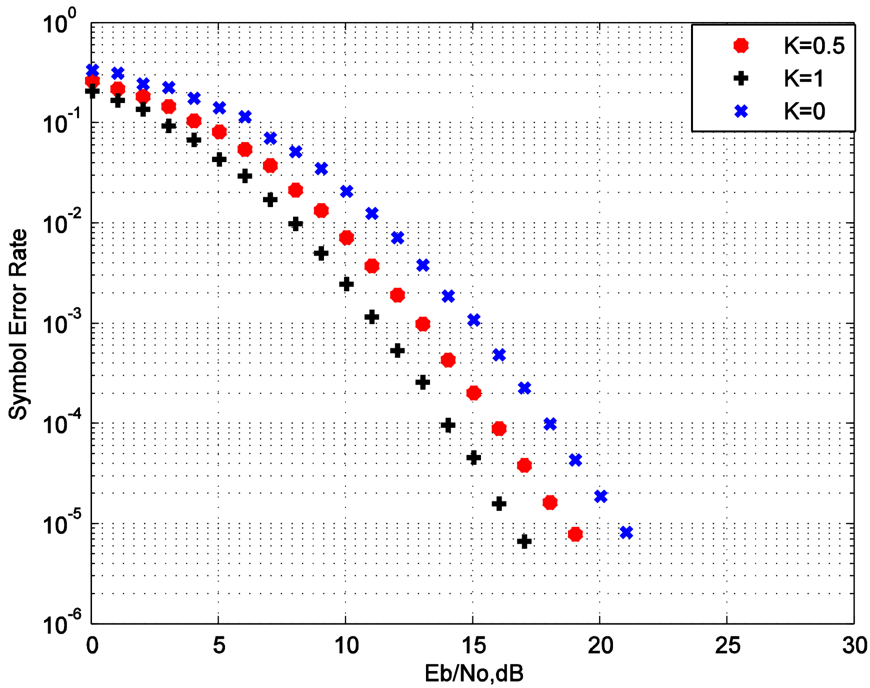

Fig.32: 4-QAM modulation scheme, (M_T = 1, M_R=4)

In Fig.32, the modulation scheme used is 4-QAM with one transmit and four receive antennas. The Ricean factors used are 0, 0.5 and 1. As we increase the Ricean factor, the symbol error rate decreases because of presence of LOS component.

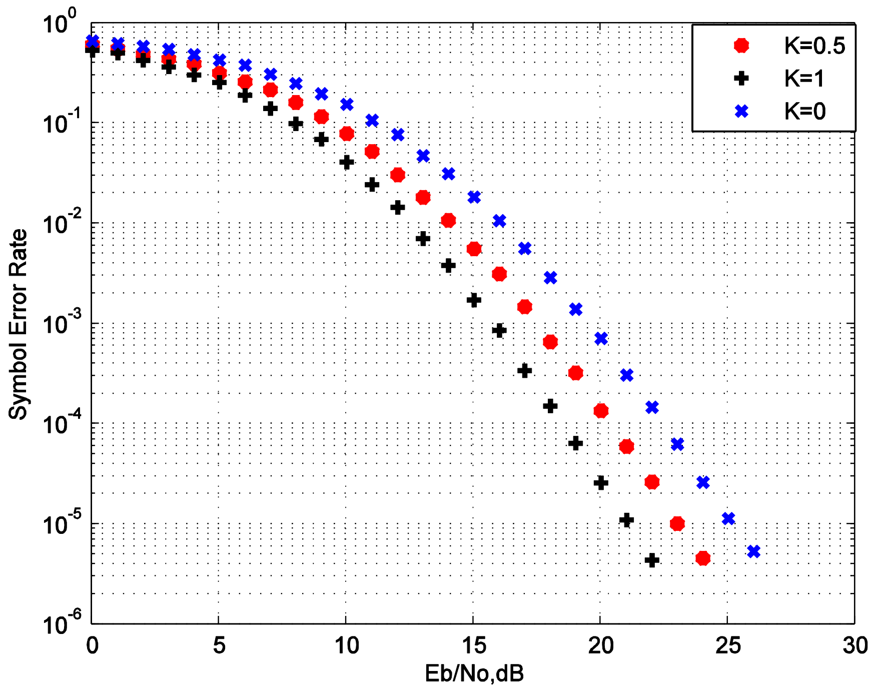

Fig.33: 16-QAM modulation scheme, (M_T = 1, M_R=4)

In Fig.33, the modulation scheme used is 16-QAM with one transmit and four receive antennas. The Ricean factors used are 0, 0.5 and 1. As we increase the Ricean factor, the symbol error rate decreases because of presence of LOS component.

6.6 Ricean MIMO

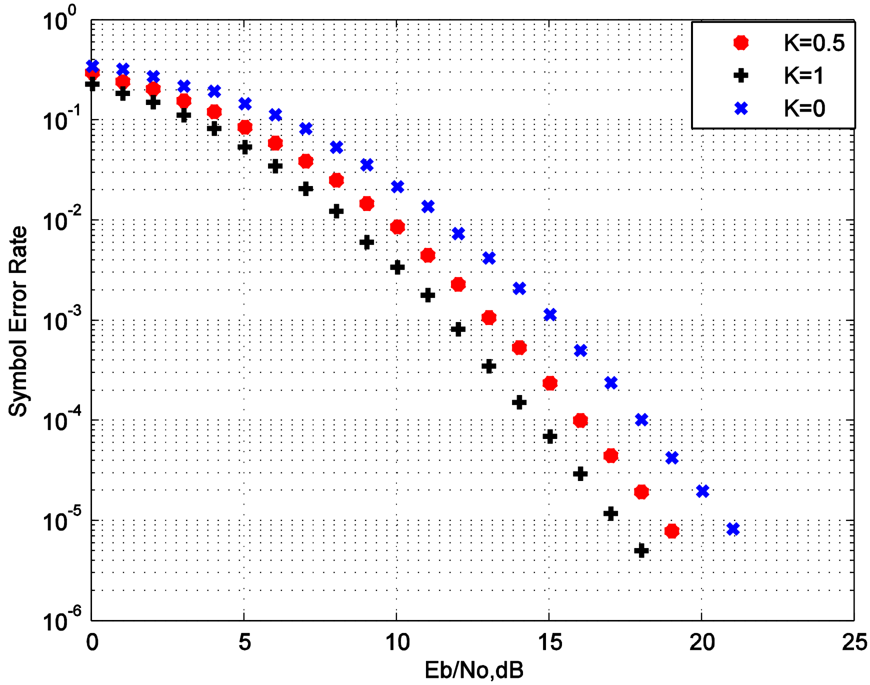

Fig.34: 2-PAM modulation scheme, (M_T = 2, M_R=4)

In Fig.34, the modulation scheme used is 2-PAM with two transmit and four receive antennas. The Ricean factors used are 0, 0.5 and 1. As we increase the Ricean factor, the symbol error rate decreases because of presence of LOS component.

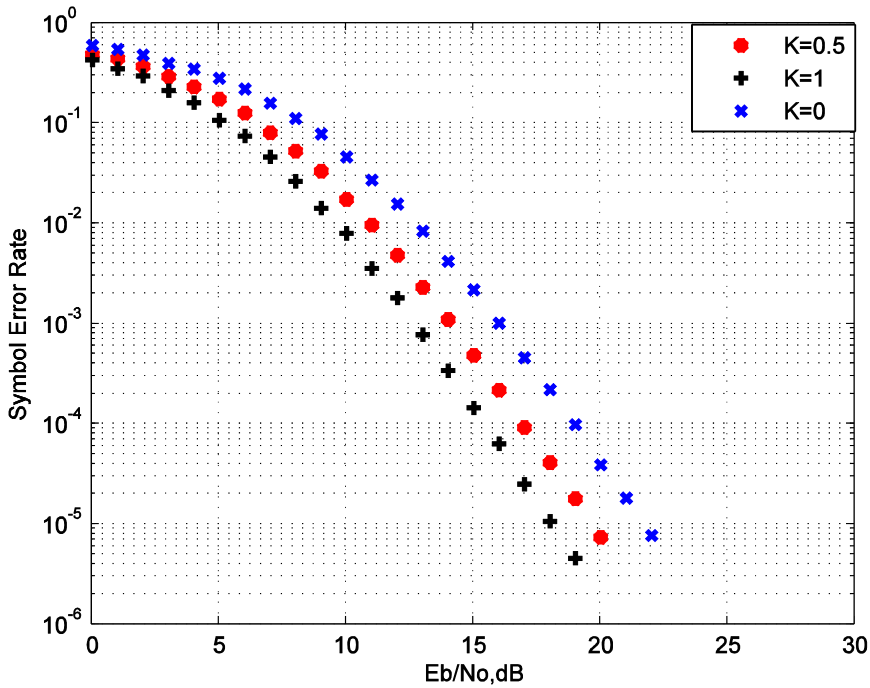

Fig.35: 4-QAM modulation scheme, (M_T = 2, M_R=4)

In Fig.35, the modulation scheme used is 4-QAM with two transmit and four receive antennas. The Ricean factors used are 0, 0.5 and 1. As we increase the Ricean factor, the symbol error rate decreases because of presence of LOS component.

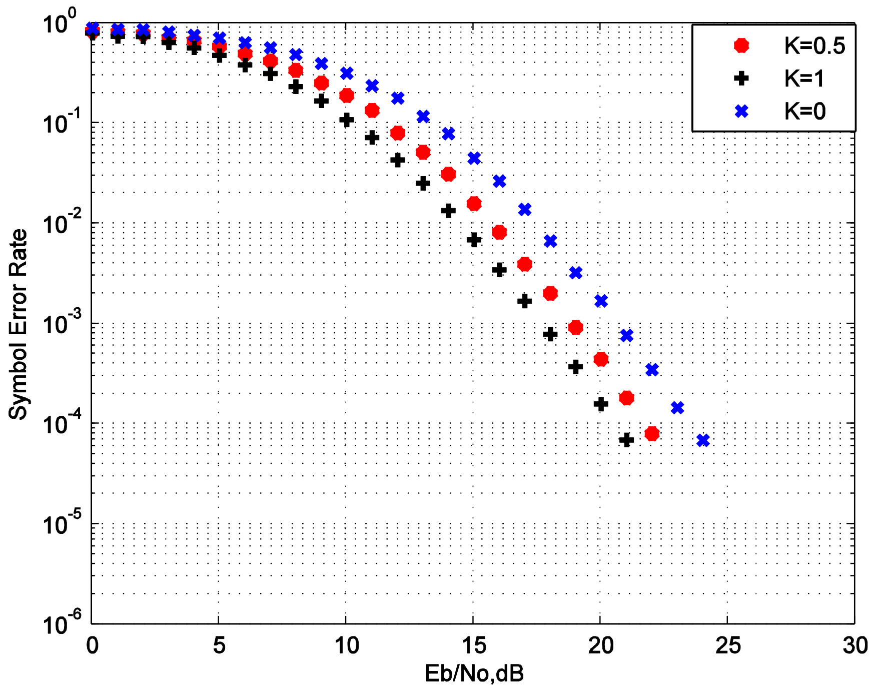

Fig.36: 16-QAM modulation scheme, (M_T = 2, M_R=4)

In Fig.36, the modulation scheme used is 16-QAM with two transmit and four receive antennas. The Ricean factors used are 0, 0.5 and 1. As we increase the Ricean factor, the symbol error rate decreases because of presence of LOS component.

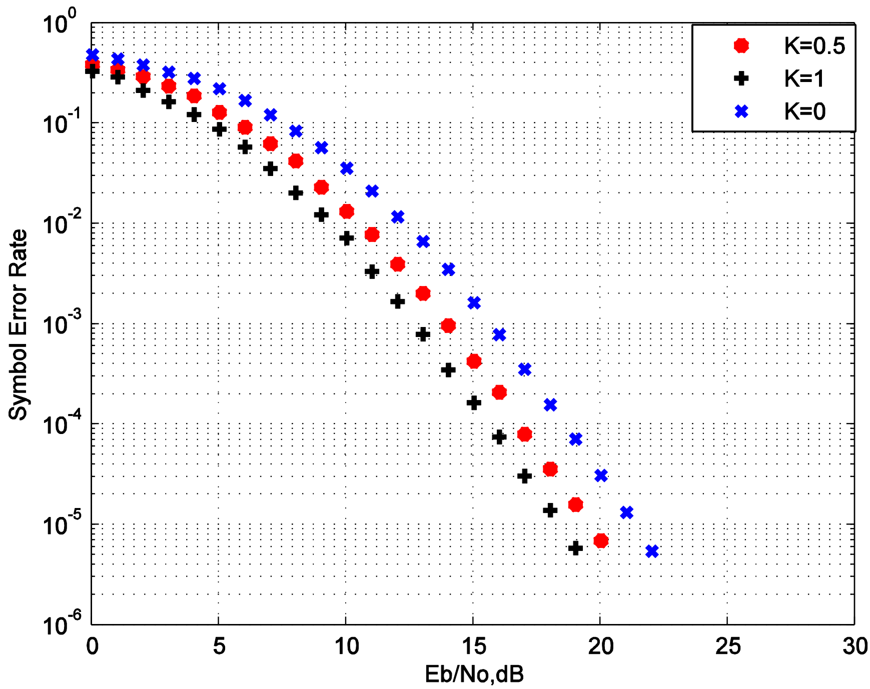

Fig.37: 2-PAM modulation scheme, (M_T = 3, M_R=4)

In Fig.37, the modulation scheme used is 2-PAM with three transmit and four receive antennas. The Ricean factors used are 0, 0.5 and 1. As we increase the Ricean factor, the symbol error rate decreases because of presence of LOS component.

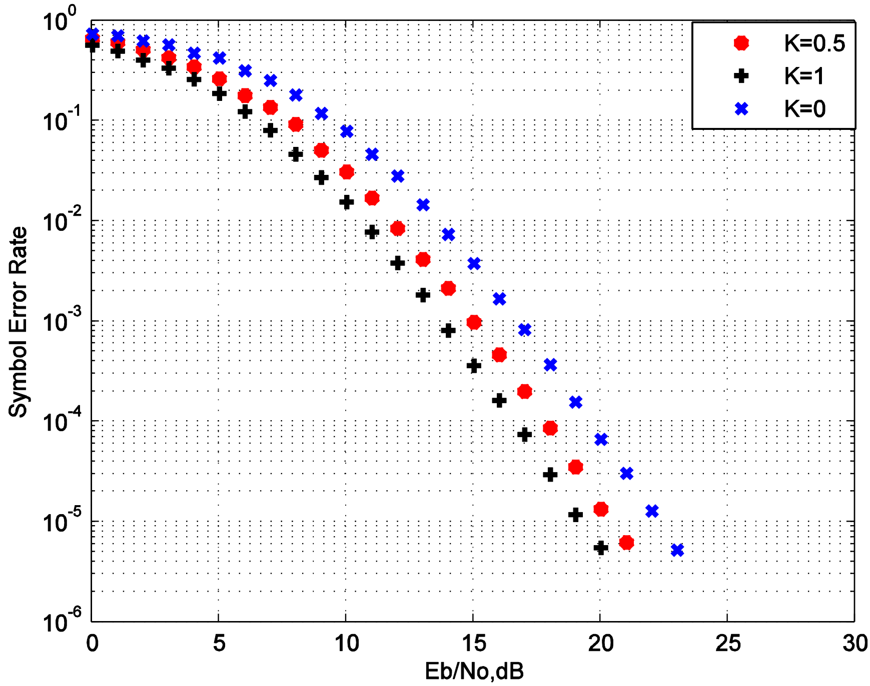

Fig.38:4-QAM modulation scheme, (M_T = 3, M_R=4)

In Fig.38, the modulation scheme used is 4-QAM with three transmit and four receive antennas. The Ricean factors used are 0, 0.5 and 1. As we increase the Ricean factor, the symbol error rate decreases because of presence of LOS component.

CHAPTER 7

CONCLUSIONS

Through this thesis, symbol error rate, bit rate performances and diversity concept of MIMO systems for Rayleigh channel regarding 2-PAM, 4-QAM and 16-QAM were discussed. Also symbol error rate performance for Ricean channel was studied for some specific cases.

It was found out that MIMO systems have the best symbol error rate, bit rate and diversity performance as compared to SISO, SIMO and MISO systems at high SNRs when both M_T and M_R are increased. SIMO channels were only slightly better than MIMO channels when only M_T is increased. It was also demonstrated that the error probability performance is significantly improved when only M_R is increased.

In this thesis, the discussion was basically focused on Rayleigh characterization used for all the forms of MIMO. Future work may include many important topics like introducing frequency selective fading in the symbol error rate simulations. Coding combined with interleaving is another very interesting topic which is becoming very important since it improves symbol error rate considerably and thus the communication system performance. Adaptive coding can be considered for different signal to noise ratios. Performance of MIMO systems can also be evaluated by using different receivers apart from ML receiver. Good examples are zero forcing and minimum mean square error receivers. One can evaluate the symbol error rate performance of MIMO systems using these two receivers. Correlated channel coefficients for Rayleigh channel is another major topic to be considered.

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List of Acronyms

AWGN	Additive White Gaussian Noise
BLAST	Bell Labs Space-Time Architecture
4G	4^{th} generation mobile telecommunications
HDTV	High-Definition Television
HSDPA	High-Speed Downlink Packet Access
LTE	Long Term Evolution
MAP	Maximum a posteriori probability
MIMO	Multiple Input Multiple Output
MISO	Multiple Input Single Output
ML	Maximum Likelihood
OFDM	Orthogonal Frequency Division Multiplex
OFDMA	Orthogonal Frequency Division Multiple Access
PAM	Pulse Amplitude Modulation
QAM	Quadrature Amplitude Modulation
SIMO	Single Input Multiple Output
SISO	Single Input Single Output
SNR	Signal-to-Noise Ratio
STC	Space Time Coding
UMTS	Universal Mobile Telecommunications System
Wi-Fi	Wireless Fidelity
WiMAX	Worldwide Interoperability for Microwave Access
WLAN	Wireless Local Area Network