Detection Theory

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Quick facts

- **Textbook**
  - Steven M. Kay, ***Fundamentals of Statistical Signal Processing – Detection Theory***, Volume II, Prentice Hall Signal Processing Series, Prentice Hall, 1998.
- **Lectures**
  - Given by students
  - Preparation assisted by seniors
  - **Tuedsdays at 13.15 in E:2349 (Köket)**
- **Exercise classes**
  - Students present solutions
  - **Mondays at 15.15 in E:2349 (Köket)**
- **Examination requirements**
  - Giving one lecture
  - 80% of lecture attendance
  - 80% of exercise class attandance
  - Solving a small set of examination problems
- **Course web pages**
  - http://www.eit.lth.se/course/PHD009

Course start: **October 19**
Course end: **January 24**

9 ECTS

Lecture schedule

Chapter	Topic	Student	Senior[1]	Date
3	Statistical Decision Theory I	Isael	FR	Oct 26
4	Deterministic Signals	Taimoor	BM	Nov 2
5	Random Signals	Meifang	FR	Nov 9
6	Statistical Decision Theory II	Pablo	BM	Nov 16
7	Deterministic Signals with Unknown Parameters	Nafiseh	OE	Nov 23
8	Random Signals with Unknown Parameters	Farzad	FR	Nov 30
9	Unknown Noise Parameters	Marco	BM	Dec 7
10	NonGaussian Noise	Reza	OE	Dec 14
12	Model Change Detection	Xiang	FR	Jan 11
13	Complex/Vector Extensions, and Array Processing	Peter	OE	Jan 18
Extra	Transition from continuous time to discrete time	Senior		Jan 25

[1] FR – Fredrik Rusek, BM – Bengt Mandersson, OE – Ove Edfors

Preparing lectures

- A PowerPoint template is available (can be downloaded from course web page)
- A few tips:
  - Concentrate on ideas, methodology and principles
  - Avoid excessive mathematical detail
  - Use graphical illustrations, whenever possible (be careful!)
  - Highlight main points
  - Use application examples to demonstrate main concepts, whenever possible
  - Provide external references, if appropriate
- Don't forget to select exercises for the next exercise class
- Any slides should be available the day before the lecture (seniors put them on the course web page)

Textbook/course overview

1. Introduction Self study!

Detection Theory in Signal Processing. The Detection Problem. The Mathematical Detection Problem. Hierarchy of Detection Problems. Role of Asymptotics. Some Notes to the Reader.

2. Summary of Important PDFs Self study!

Fundamental Probability Density Functions and Properties. Quadratic Forms of Gaussian Random Variables. Asymptotic Gaussian PDF. Monte Carlo Performance Evaluation. Number of Required Monte Carlo Trials. Normal Probability Paper. MATLAB Program to Compute Gaussian Right-Tail Probability and its Inverse. MATLAB Program to Compute Central and Noncentral Right-Tail Probability. MATLAB Program for Monte Carlo Computer Simulation.

Textbook/course overview

3. Statistical Decision Theory I

Neyman-Pearson Theorem. Receiver Operating Characteristics. Irrelevant Data. Minimum Probability of Error. Bayes Risk. Multiple Hypothesis Testing. Minimum Bayes Risk Detector - Binary Hypothesis. Minimum Bayes Risk Detector - Multiple Hypotheses.

4. Deterministic Signals

Matched Filters. Generalized Matched Filters. Multiple Signals. Linear Model. Signal Processing Examples. Reduced Form of the Linear Model.

5. Random Signals

Estimator-Correlator. Linear Model. Estimator-Correlator for Large Data Records. General Gaussian Detection. Signal Processing Example. Detection Performance of the Estimator-Correlator.

Textbook/course overview

6. Statistical Decision Theory II

Composite Hypothesis Testing. Composite Hypothesis Testing Approaches. Performance of GLRT for Large Data Records. Equivalent Large Data Records Tests. Locally Most Powerful Detectors. Multiple Hypothesis Testing. Asymptotically Equivalent Tests - No Nuisance Parameters. Asymptotically Equivalent Tests - Nuisance Parameters. Asymptotic PDF of GLRT. Asymptotic Detection Performance of LMP Test. Alternate Derivation of Locally Most Powerful Test. Derivation of Generalized ML Rule.

7. Deterministic Signals with Unknown Parameters

Signal Modeling and Detection Performance. Unknown Amplitude. Unknown Arrival Time. Sinusoidal Detection. Classical Linear Model. Signal Processing Examples. Asymptotic Performance of the Energy Detector. Derivation of GLRT for Classical Linear Model.

Textbook/course overview

8. Random Signals with Unknown Parameters

Incompletely Known Signal Covariance. Large Data Record Approximations. Weak Signal Detection. Signal Processing Example. Derivation of PDF for Periodic Gaussian Random Process.

9. Unknown Noise Parameters

General Considerations. White Gaussian Noise. Colored WSS Gaussian Noise. Signal Processing Example. Derivation of GLRT for Classical Linear Model for Unknown noise power. Rao Test for General Linear Model with Unknown Noise Parameters. Asymptotically Equivalent Rao Test for Signal Processing Example.

10. NonGaussian Noise

NonGaussian Noise Characteristics. Known Deterministic Signals. Deterministic Signals with Unknown Parameters. Signal Processing Example. Asymptotic Performance of NP Detector for Weak Signals. BRao Test for Linear Model Signal with IID NonGaussian Noise.

Textbook/course overview

Self study!

11. Summary of Detectors
Detection Approaches. Linear Model. Choosing a Detector. Other Approaches and Other Texts.

12. Model Change Detection
Description of Problem. Extensions to the Basic Problem. Multiple Change Times. Signal Processing Examples. General Dynamic Programming Approach to Segmentation. MATLAB Program for Dynamic Programming.

13. Complex/Vector Extensions, and Array Processing
Known PDFs. PDFs with Unknown Parameters. Detectors for Vector Observations. Estimator-Correlator for Large Data Records. Signal Processing Examples. PDF of GLRT for Complex Linear Model. Review of Important Concepts. Random Processes and Time Series Modeling.

Additional Material

Throughout the textbook a discrete time representation is assumed.

At the end we will give a lecture that treats the transition from continuous time to discrete time.

Material will be provided later.

What is the difference between Detection and Estimation?

Detection:

Discrete set of hypotheses.

One cares whether the decision is **right or wrong**

Estimation:

Infinite, or at least large, set of hypotheses.

The decision is almost always wrong – make **error as small as possible**.

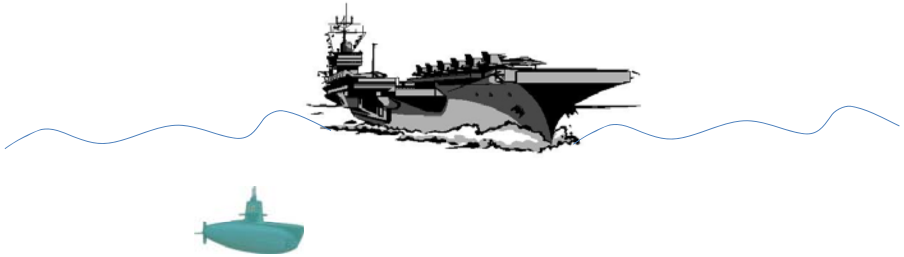
Example

Detection problem for the warship:

To figure out whether there is an enemy submarine present or not (binary decision)

Estimation problem for the warship:

To find the location of the submarine (continuous decision)



Detection or estimation?

Finding out ...

- ... if an intruder is present (burglar alarm)
- ... if a car is speeding on a 90 km/h road (speed camera)
- ... the expected number of tanks in the enemy's army, by observing their "serial" numbers (numbered from 1 to #of tanks)
- ... if the enemy has 0-9, 10-99, 100-999, or more than 1000 tanks (under the same conditions as above)

Different approaches

- **Neyman-Pearson**
  For a *fixed probability of false alarm*, find the decision rule that gives the *maximal probability of detection*.
- **Bayesian**
  Given a *Bayes Risk* (an expected "cost")

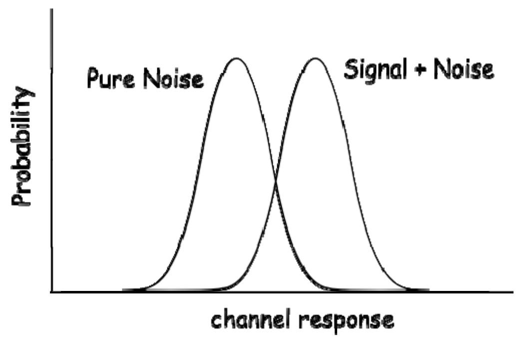
$$R = E(C) = \sum_{i,j} C_{i,j} P(H_i \mid H_j) P(H_j)$$

  find the decision rule that gives the *minimal R*.

Signal Detection – the most basic example

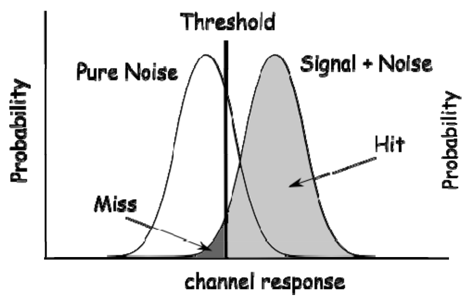
Detection of binary signal 0/1 in additive Gaussian Noise.

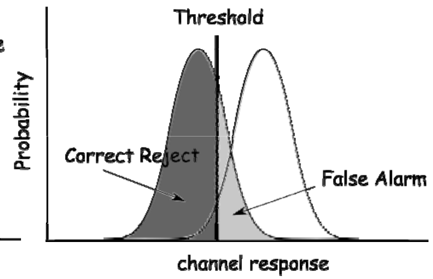
It is inevitable that some mistakes will be made



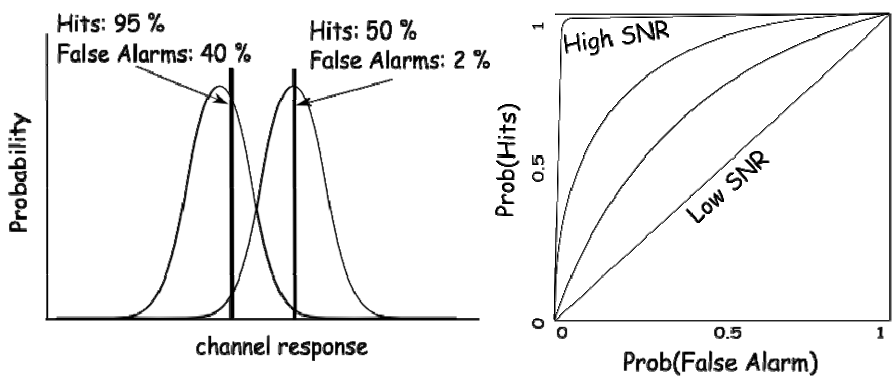
Signal detection – the most basic example

It is natural to define the following concepts.



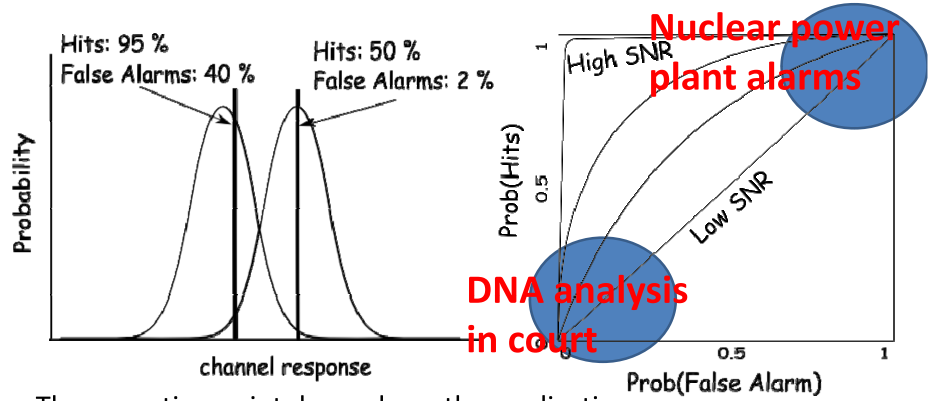


Receiver Operating Characteristics (ROC)



The operating point depends on the application.

Operating Region of the Detection

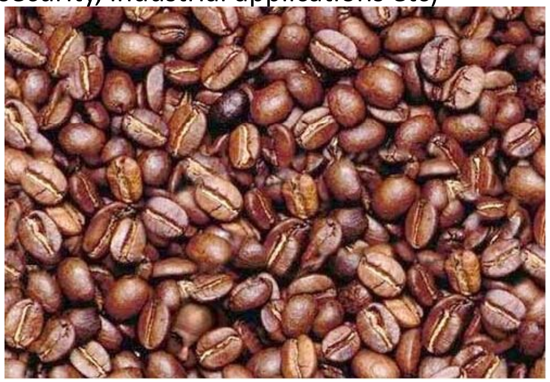


The operating point depends on the application.

Neyman-Pearson: Maximize Prob(Hit) for fixed Prob(False alarm)

Computer detection of objects, not covered in this course

Increasing popularity in computer science etc. Falls within detection theory. Machines should identify certain objects from pictures (airport security, industrial applications etc)



Find the face!

Communication theory

And of course.......a digital communication example.

Transmitted signal is $S(t)$, received is $Y(t)$, noise is $N(t)$

Hypothesis testing

$$Y(t) = N(t) \qquad H_0$$

$$Y(t) = S(t) + N(t) \qquad H_1$$

If $S(t)$ is a known signal

-> Matched filter receiver

If $S(t)$ is a random signal

-> Estimator-Correlator receiver

What if $N(t)$ is not Gaussian?

Detection of rare events is hard!

Suppose that a medical company devices a test for a rare disease. The test idenfifies the disease with 99% probability and gives false alarm with 1% probability, i.e.

Prob (positive test | sick person) = .99
Prob (negative test| healthy person) = .99

It appears that satisfactory detection of the disease is provided by the new test.

But is it really so….?

Detection of rare events is hard!

Suppose that 0.5 % of the population is infected (sick).
Apply Bayes' rule and compute the posterior probability that a person is healthy, even though the test is positive ('+')

$$\text{Prob (healthy} \mid +) = \frac{\text{Prob} (+ \mid \text{healthy}) \times \text{Pro (healthy)}}{\text{Prob} (+)}$$

$$= \frac{.01 \times .995}{\text{Prob}(+|\text{sick}) \times \text{Prob (sick)} + \text{Prob}(+|\text{healthy}) \times \text{Prob (healthy)}}$$

$$= \frac{.99 \times .005}{.99 \times .005 + .01 \times .995}$$

= **0.6678**, Two thirds of all that test as sick are healthy!

Detection of rare events is hard!

This has implications on e.g. synchronization.
In noisy environments there may be far too many locks due to noise.

This will make your cell-phone ringing, but there will be nobody to talk with when you answer.

For sparse signals, extreme precision for detection is needed.

Exercises

- The exercises for this lecture are only recommended as a "warm up" (no exercise class on Monday)
- Chapter 1
  - 1.2 Detector performance, simple example
  - 1.3 Heuristic design of a test
  - 1.6 & 1.7 Calculation of *deflection coefficient*
- Chapter 2
  - 2.2 Approximation of Q(x)
  - 2.4-2.6 Quadratic forms of random variables
  - 2.7 Bounds on eigenvalues of autocorrelation matrices
  - 2.9 Simple example regarding WSS processes
  - 2.10-2.11 DFTs and asymptotic eigenvectors of autocorrelation matrices
  - 2.12-2.13 Eigendeompositions of autocorrelation matrices

If you feel that your statistics and/or linear algebra skills are a bit rusty … give these a go!