Study week 3.

3.4.1 Low-Rate QAM-Type of Input Signals

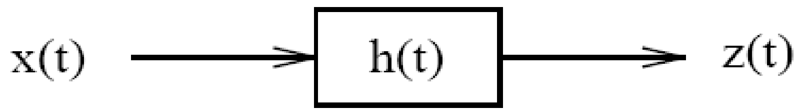


Figure 3.11: Bandpass filtering.

$$x(t) = x_I(t)\cos(\omega_c t) - x_Q(t)\sin(\omega_c t) = Re\{\tilde{x}(t)e^{j\omega_c t}\} \qquad (3.103)$$

$$\tilde{x}(t) = x_I(t) + jx_Q(t) \qquad (3.104)$$

This complex signal contains the information!

$$x(t) = x_I(t)\cos(\omega_c t) - x_Q(t)\sin(\omega_c t) = Re\{\tilde{x}(t)e^{j\omega_c t}\} \quad (3.103)$$

$$z(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau = \int_{-\infty}^{\infty} h(\tau)Re\{\tilde{x}(t-\tau)e^{j\omega_c(t-\tau)}\}d\tau =$$

$$= Re\left\{e^{j\omega_c t}\int_{-\infty}^{\infty} h(\tau)\tilde{x}(t-\tau)e^{-j\omega_c \tau}d\tau\right\} \quad (3.105)$$

3 assumptions:

1) The duration of the impulse response $h(t)$ can be considered to be equal to T_h. This means that essentially all the energy in $h(t)$ is assumed to be contained within the time interval $0 \le t \le T_h$.

2) The input signal is assumed to be a QAM-type of signal with duration $T = T_s$:

$$x(t) = \begin{cases} 0 & , \quad t < 0 \\ A\cos(\omega_c t) - B\sin(\omega_c t) = \sqrt{A^2+B^2}\cos(\omega_c t + \nu) & , \quad 0 \le t \le T_s \\ 0 & , \quad t > T_s \end{cases} \quad (3.106)$$

3) $T_s > T_h$ (“low” signaling rate).

$$\tilde{x}(t) = \begin{cases} A + jB = \sqrt{A^2+B^2}\ e^{j\nu} & , \quad 0 \le t \le T_s \\ 0 & , \quad \text{otherwise} \end{cases} \tag{3.108}$$

$\boldsymbol{T_h \le t \le T_s}$ **:**

$$\begin{aligned} z(t) &= Re\left\{ e^{j\omega_c t} \int_0^{T_h} h(\tau)\sqrt{A^2+B^2}\ e^{j\nu} e^{-j\omega_c \tau} d\tau \right\} = \\ &= Re\{\sqrt{A^2+B^2}\ e^{j\nu} \cdot H(f_c) e^{j\omega_c t}\} = \\ &= |H(f_c)|\sqrt{A^2+B^2}\cos(\omega_c t + \nu + \phi(f_c)) = A_z \cos(\omega_c t) - B_z \sin(\omega_c t) \end{aligned} \tag{3.109}$$

Hence, a QAM-signal at the output in this time interval!

However, **attenuation and rotation** compared with the input! Compare with the input x(t) in (3.106)!

$$\begin{aligned} A_z + jB_z &= (A+jB)H(f_c) = \sqrt{A^2+B^2}|H(f_c)|e^{j(\nu+\phi(f_c))} = \\ &= (A+jB)(H_{Re}(f_c) + jH_{Im}(f_c)) \end{aligned} \tag{3.110}$$

$$A_z + jB_z = (A + jB)H(f_c) = \sqrt{A^2 + B^2}|H(f_c)|e^{j(\nu+\phi(f_c))} = $$
$$= (A + jB)(H_{Re}(f_c) + jH_{Im}(f_c)) \quad (3.110)$$

A COMPACT MODEL WITH A COMPLEX CHANNEL PARAMETER!!

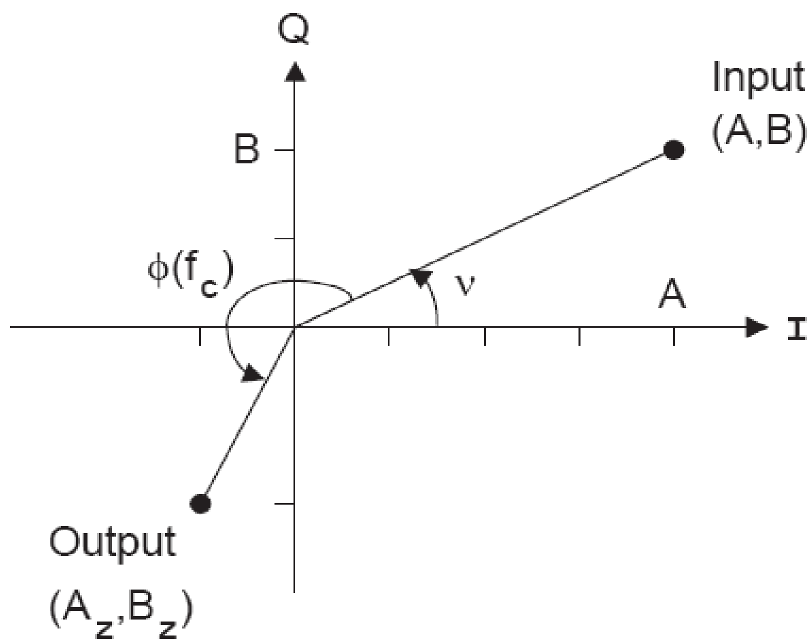


Figure 3.13: Illustrating that the input I-Q amplitudes (A,B) are scaled and rotated by the channel $H(f)$, see (3.109) and (3.110).

$$
z(t) = \begin{cases} 0 & , \quad t < 0 \\ \text{“non-stationary transient” starting interval} & , \quad 0 \le t \le T_h \\ |H(f_c)|\sqrt{A^2+B^2}\ \cos(\omega_c t + \nu + \phi(f_c)) & , \quad T_h \le t \le T_s \\ \text{“non-stationary transient” ending interval} & , \quad T_s \le t \le T_s + T_h \\ 0 & , \quad t > T_s + T_h \end{cases}
$$

and within $T_h \le t \le T_s,\ A_z + jB_z = (A + jB)H(f_c)$ (3.111)

An important result here is that the input QAM signal $x(t)$ in (3.106) is changed to a new QAM signal by $|H(f_c)|$ and $\phi(f_c)$ in the interval $T_h \le t \le T_s$, see also Figure 3.13 and (3.110) how the I-Q components are changed. Furthermore, in OFDM applications the signaling rate $1/T_s$ is low such that $T_s \gg T_h$, and many QAM signals with different carrier frequencies are sent in parallel. ***Due to linearity, the result in (3.111) can be applied to each QAM signal in the OFDM signal by replacing f_c with f_n.*** In OFDM applications the receiver uses the time interval $\Delta_h \le t \le T_s$ for detection of the output QAM signals, and the duration of this observation interval is denoted $T_{obs} = T_s - \Delta_h$ (compare with (2.110) on page 51, and $T_h \le \Delta_h$).

So, the n:th QAM signal constellation in a sent OFDM signal is attenuated and rotated by H(fn) which is the value of the channel transfer function H(f) at the carrier frequency fn.

3.4.3 N-Ray Channel Model

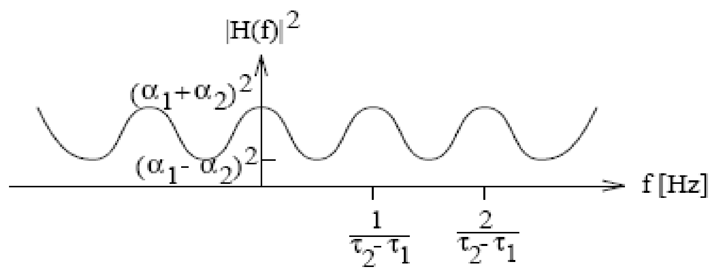
$$z(t) = x(t) * \underbrace{\left(\sum_{i=1}^{N} \alpha_i \delta(t-\tau_i)\right)}_{\text{Impulse response } h(t)} = \sum_{i=1}^{N} \alpha_i x(t-\tau_i) \qquad (3.126)$$

$$H(f) = \mathcal{F}\{h(t)\} = \sum_{i=1}^{N} \alpha_i e^{-j2\pi f \tau_i} \qquad (3.128)$$

So, **H(fc)** is easy to find!

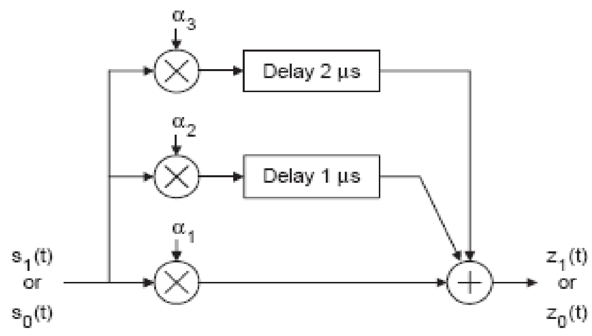
EXAMPLE 3.20

Rough sketch:



It is seen in this figure that the two signal paths add constructively or destructively (fading) depending on the frequency. Furthermore, if $\alpha_1 \approx \alpha_2$ then $|H(f)|$ is very close to zero at certain frequencies (so-called deep fades)!

EXAMPLE 3.19



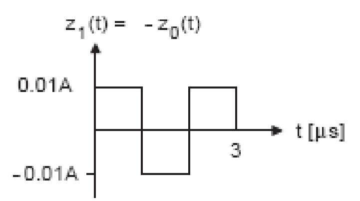
The signal $z_i(t) = s_i(t) * h(t)$ *is the output signal corresponding to the input signal* $s_i(t)$, $i = 0, 1$. *Determine and sketch* $z_0(t)$ *and* $z_1(t)$ *if* $\alpha_1 = 0.01$, $\alpha_2 = -0.01$, *and* $\alpha_3 = 0.01$.

Your conclusions concerning choice of bit rate to avoid overlapping signal alternatives after the channel?

Solution:

$$z_\ell(t) = \sum_{i=1}^{3} \alpha_i s_\ell(t - \tau_i) = 0.01 s_\ell(t) - 0.01 s_\ell(t - 10^{-6}) + 0.01 s_\ell(t - 2 \cdot 10^{-6}), \;\; \ell = 0, 1$$

yields,



Observe that the signal alternatives are changed significantly by the channel (filtering), and that the duration of both signal alternatives is increased from 1 [µs] before the channel, to 3[µs] after the channel!

If the bit rate is reduced to at most $10^6/3$ bps, *then no overlap of signal alternatives will exist after the channel.* □

See (3.109)-(3.110)!

See (3.110)!

5.34 Consider a communication system where N_t M-ary QAM signals are sent simultaneously (from N_t antennas). The n:th transmitted M-ary QAM signal is denoted $s_n(t)$,

$$s_n(t) = A(n)g(t)\cos(\omega_c t) - B(n)g(t)\sin(\omega_c t) \tag{5.133}$$

for $n = 1, 2, \ldots, N_t$. Note that the same carrier frequency is used for all N_t transmitted QAM signals!

The receiver is assumed to have N_r receiving antennas. The received signal $r_k(t)$ at the k:th receiving antenna is here modelled as

$$r_k(t) = \sum_{n=1}^{N_t} \Big(\left[H_{k,n}^{Re}A(n) - H_{k,n}^{Im}B(n)\right] g(t)\cos(\omega_c t) - \\ - \left[H_{k,n}^{Re}B(n) + H_{k,n}^{Im}A(n)\right] g(t)\sin(\omega_c t)\Big) + w_k(t) \tag{5.134}$$

for $k = 1, 2, \ldots, N_r$. The variables $H_{k,n}^{Re}$ and $H_{k,n}^{Im}$ models how the n:th transmitted QAM signal is received at the k:th receiving antenna (attenuation and rotation of the I-Q components).

After I and Q demodulation of $r_k(t)$ to baseband, the receiver obtains the noisy signal space coordinates, here collected in r_k as

$$r_k = \underbrace{\sum_{n=1}^{N_t} \left(H_{k,n}^{Re}A(n) - H_{k,n}^{Im}B(n)\right)}_{\text{received } I \text{ component}} + j\underbrace{\sum_{n=1}^{N_t} \left(H_{k,n}^{Re}B(n) + H_{k,n}^{Im}A(n)\right)}_{\text{received } Q \text{ component}} \\ + \underbrace{(w_k^{Re} + jw_k^{Im})}_{\text{due to AWGN}} \tag{5.135}$$

Note that complex notation ($j^2 = -1$) is used in (5.135)!

Let us now introduce the complex notations:

$$\begin{aligned} d_n &= A(n) + jB(n) \\ \alpha_{k,n} &= H_{k,n}^{Re} + jH_{k,n}^{Im} \\ w_k &= w_k^{Re} + jw_k^{Im} \end{aligned} \tag{5.136}$$

Then (5.135) can be formulated as,

$$r_k = \sum_{n=1}^{N_t} \alpha_{k,n} d_n + w_k \quad , k = 1, 2, \ldots, N_r \qquad (5.137)$$

A compact formulation is now obtained as

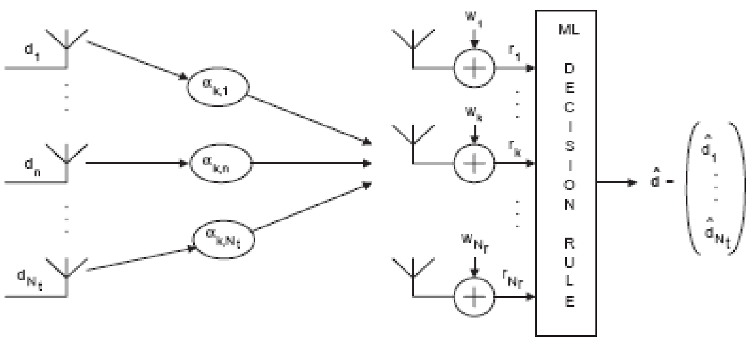
$$\boldsymbol{r} = \begin{pmatrix} r_1 \\ \vdots \\ r_{N_r} \end{pmatrix} = \boldsymbol{A} \begin{pmatrix} d_1 \\ \vdots \\ d_{N_t} \end{pmatrix} + \begin{pmatrix} w_1 \\ \vdots \\ w_{N_r} \end{pmatrix} = \boldsymbol{A}\boldsymbol{d} + \boldsymbol{w} \qquad (5.138)$$

Very important!

where the $N_r \times N_t$ matrix $\boldsymbol{A}$ contains the channel coefficients $\{\alpha_{k,n}\}$. The relationship in (5.138) is a basic model in so-called multiple-input multiple-output (MIMO) systems.

The MIMO model is illustrated in the figure below,

MIMO MODEL



SISO
SIMO
MISO
MIMO
Diversity gain
Spatial multiplexing gain

$$r_k = \sum_{n=1}^{N_t} \alpha_{k,n} d_n + w_k$$

$$\boldsymbol{r} = \begin{pmatrix} r_1 \\ \vdots \\ r_{N_r} \end{pmatrix} = \boldsymbol{A} \begin{pmatrix} d_1 \\ \vdots \\ d_{N_t} \end{pmatrix} + \begin{pmatrix} w_1 \\ \vdots \\ w_{N_r} \end{pmatrix} = \boldsymbol{A}\boldsymbol{d} + \boldsymbol{w}$$

z=Ad is the received signalpoint and w is the additive noise vector.

64-QAM+Nt=8 (48bits): ML symbol decision rule ....?

Chapter 8

Trellis-coded Signals

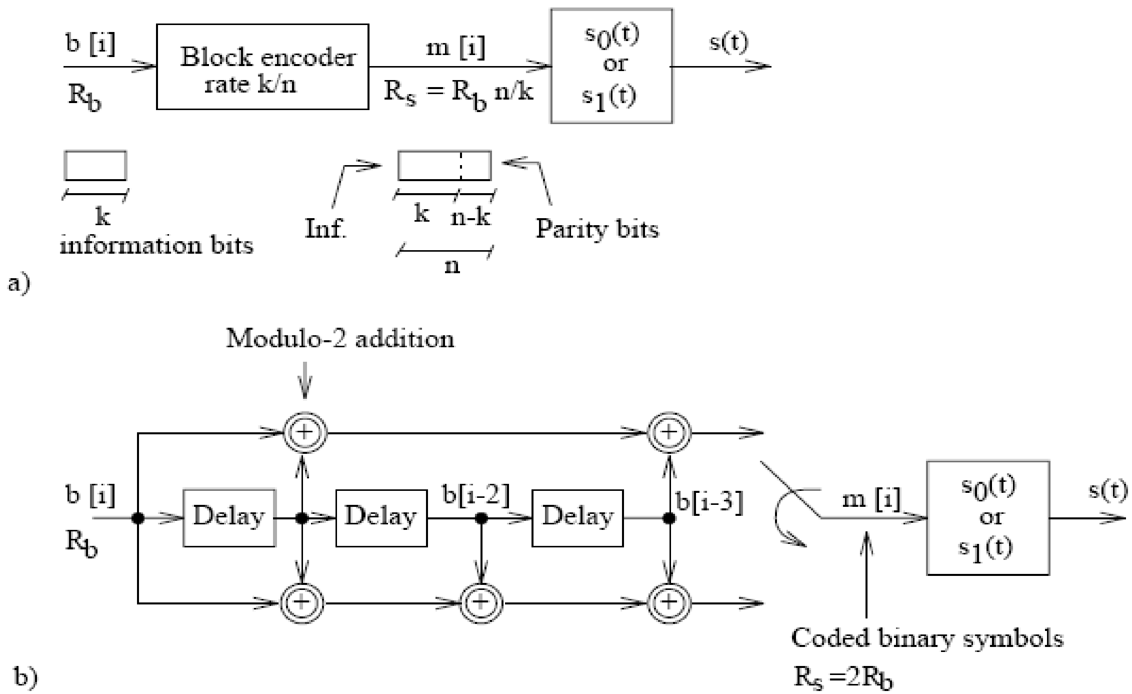


Figure 8.1: a) Block coding, $r_c = k/n$. b) Convolutional coding, $r_c = 1/2$.

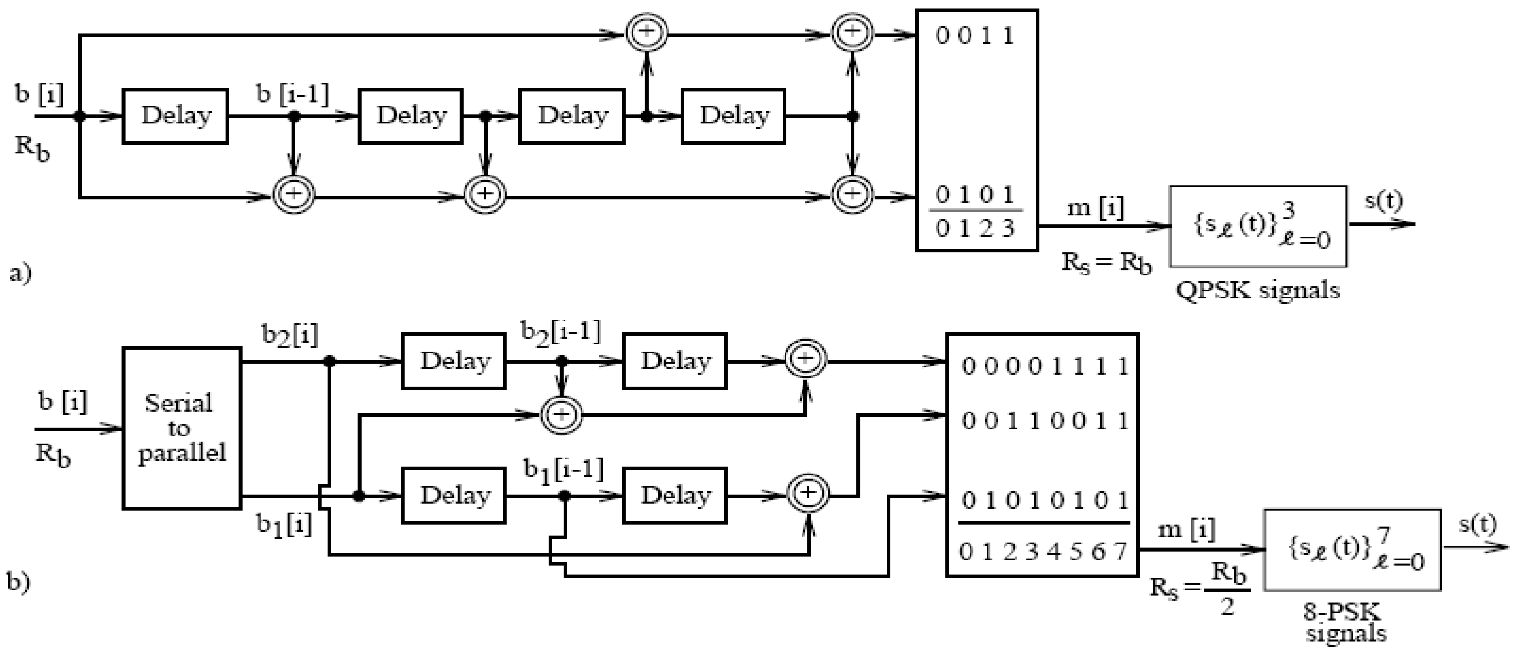
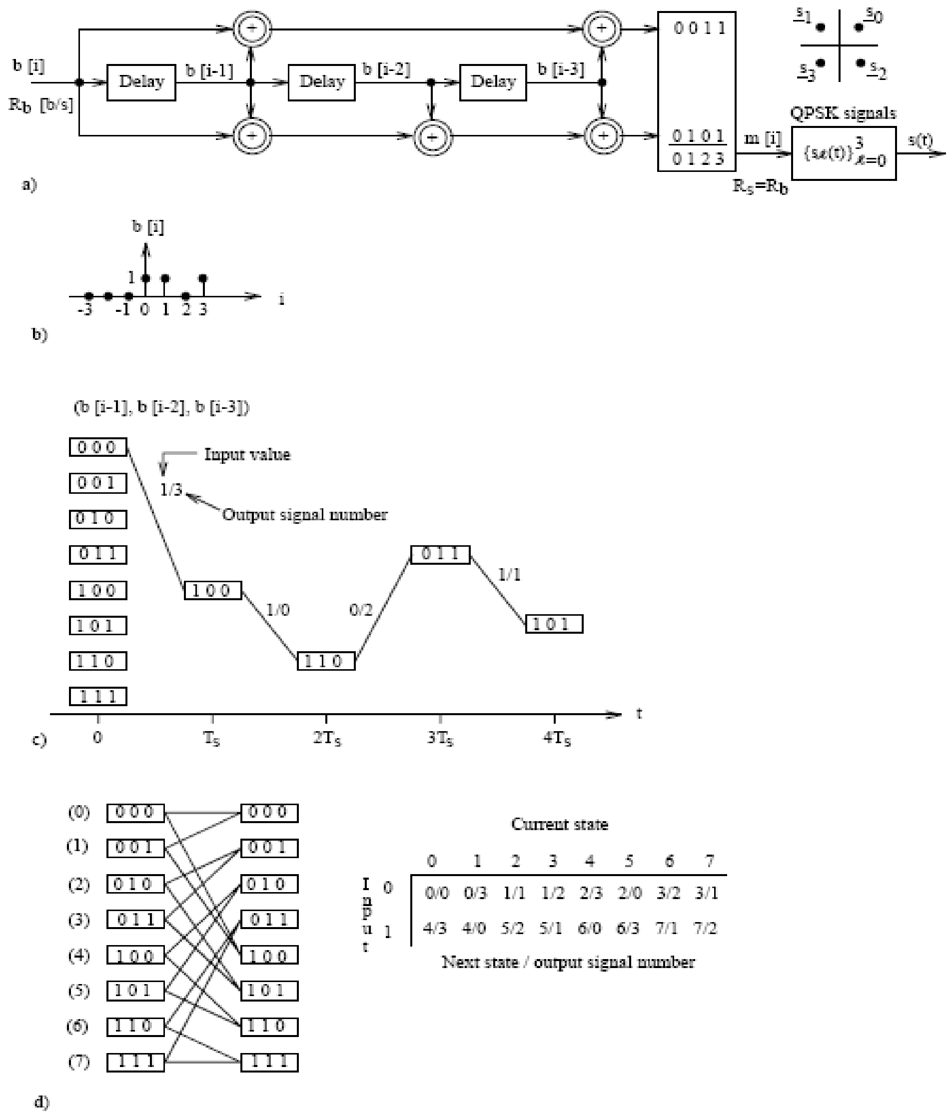


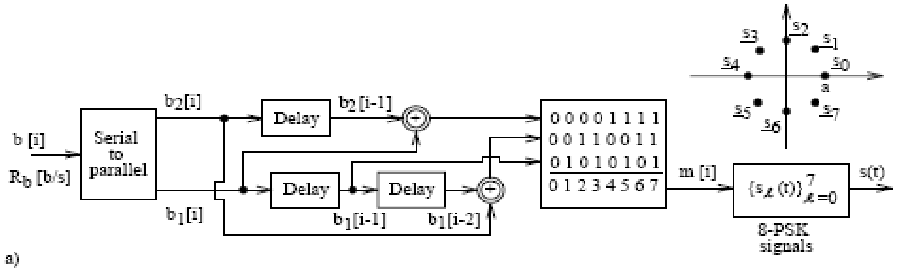
Figure 8.2: a) Rate $r_c = 1/2$ convolutional encoder combined with QPSK; b) Rate $r_c = 2/3$ convolutional encoder combined with 8-PSK, from [63], [64].



Input	Current state 0	1	2	3	4	5	6	7
0	0/0	0/3	1/1	1/2	2/3	2/0	3/2	3/1
1	4/3	4/0	5/2	5/1	6/0	6/3	7/1	7/2

Next state / output signal number

Figure 8.4: a) A rate 1/2 convolutional encoder combined with QPSK signal alternatives; b) A specific input sequence $b[i]$; c) The corresponding path in the trellis; d) A trellis section, and a table containing all relevant parameters.



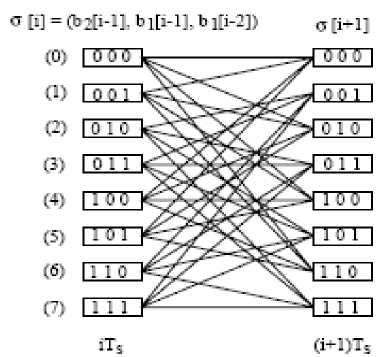
a)

Current state $\sigma[i]$

INPUT $\begin{pmatrix} b_2[i] \\ b_1[i] \end{pmatrix}$	(000) 0	(001) 1	(010) 2	(011) 3	(100) 4	(101) 5	(110) 6	(111) 7
$\binom{0}{0}$	0/0	0/2	1/1	1/3	0/4	0/6	1/5	1/7
$\binom{0}{1}$	2/4	2/6	3/5	3/7	2/0	2/2	3/1	3/3
$\binom{1}{0}$	4/2	4/0	5/3	5/1	4/6	4/4	5/7	5/5
$\binom{1}{1}$	6/6	6/4	7/7	7/5	6/2	6/0	7/3	7/1

$\sigma[i+1] / m[i]$

b)



c)

Figure 8.6: a) An example of TCM, from [63]–[64]; b) The mappings $F(\cdot,\cdot)$ and $G(\cdot,\cdot)$; c) A trellis section.

<u>Memory (redundancy, dependancy) is introduced among the sent signal alternatives!</u>

This gives us some new properties like, e.g.,:

Which of the following signal sequences are impossible?

1. $s_3(t), s_2(t-T_b), s_1(t-2T_b), s_1(t-3T_b)$
2. $s_3(t), s_2(t-T_b), s_2(t-2T_b), s_1(t-3T_b)$
3. $s_3(t), s_1(t-T_b), s_0(t-2T_b), s_2(t-3T_b)$
4. $s_3(t), s_1(t-T_b), s_3(t-2T_b), s_1(t-3T_b)$

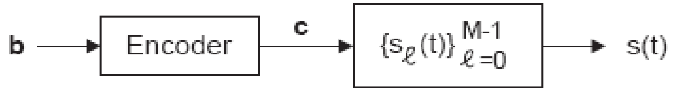
Note: In the uncoded case all signal sequences are possible.

Find the "missing" signal, in the sequence below,

$$s_1(t), s_3(t-T_b),\ ?\ , s_2(t-3T_b), s_3(t-4T_b), s_0(t-5T_b)$$

Note: This is not possible to do in the uncoded case!

2.32 Let us here study adaptive coding and modulation according to the block diagram below.



$$\bar{E}_{sent} = r_c \log_2(M) E_{b,sent} = \frac{k}{n} \log_2(M) E_{b,sent} \tag{8.4}$$

$$R_s = 1/T_s = \frac{1}{r_c} \cdot \frac{1}{\log_2(M)} \cdot R_b = \frac{1}{k/n} \cdot \frac{1}{\log_2(M)} \cdot R_b \tag{8.5}$$

$$W = c \cdot R_s \tag{8.6}$$

Typically, the bandwidth W is fixed and given but:
the rate of the encoder
the number of signal alternatives
and the bit rate can be **ADAPTIVE**, see (8.5)-(8.6)!

We have memory in the sequence of sent signal alternatives!

Some sequences are impossible, see problem!

Only ”good” sequences are sent!

OFDM - INTRO

OFDM signal = $g_{rec}(t)Re\{\sum_{n=0}^{K-1} a_n e^{j2\pi(f_0+nf_\Delta)t}\} = g_{rec}(t)Re\{\sum_{n=0}^{K-1} a_n e^{j2\pi g_n f_\Delta t} e^{j2\pi f_{rc} t}\}$ (1.12)

$f_\Delta = 1/T_{obs}$ (2.1)

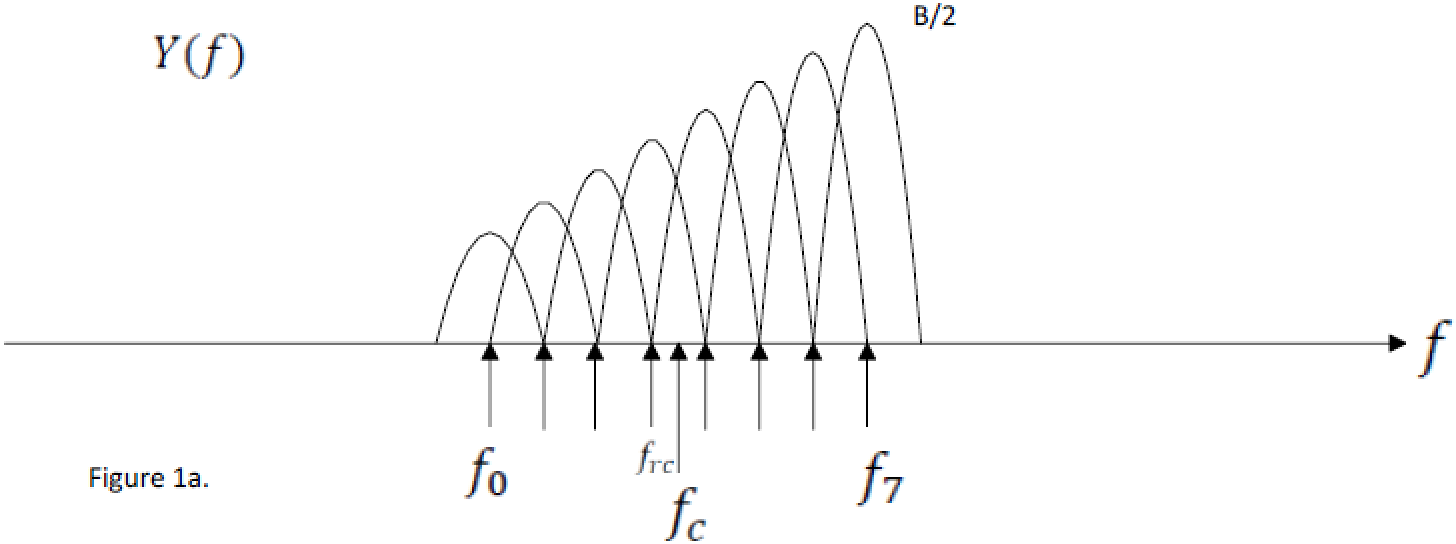


Figure 1a.

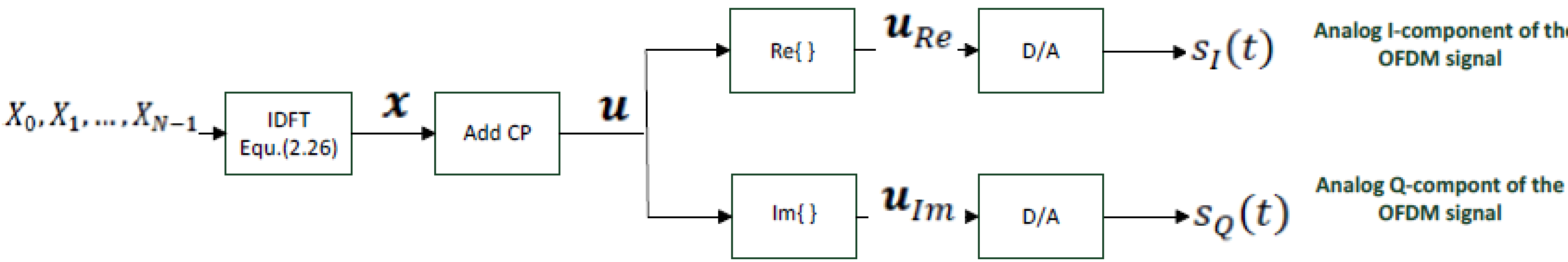


Figure 5. Block diagram illustrating the operations in the digital domain, and the transition to the analog domain.

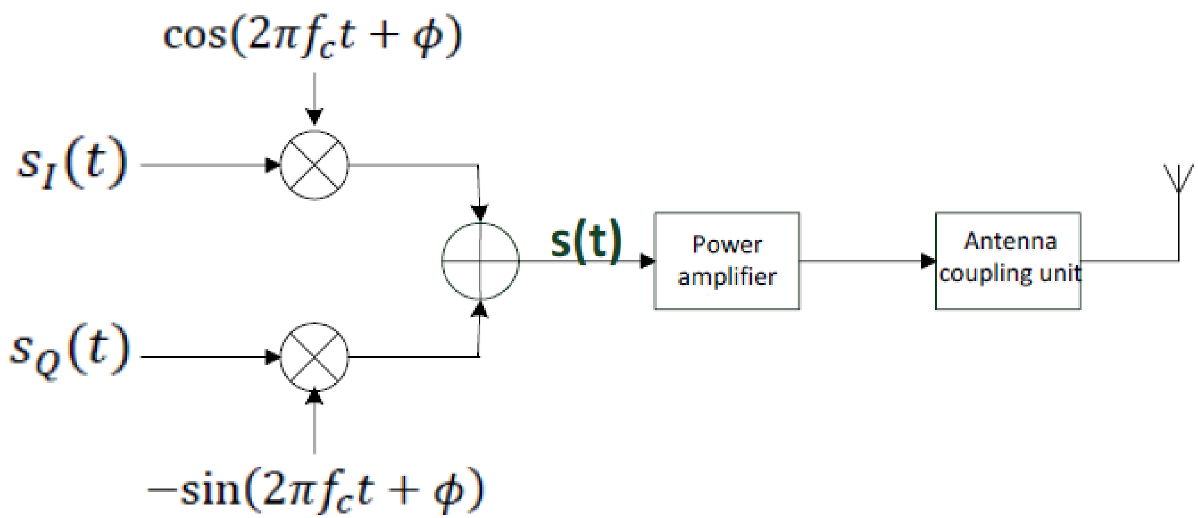


Figure 6. Block diagram illustrating frequency up-converting to the carrier frequency (K is odd), the power amplifier, and the antenna coupling unit. The OFDM signal $s(t)$ is given in equation (4.1).

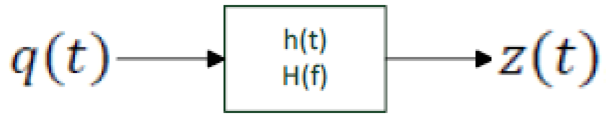


Figure 7. Illustrating the linear time-invariant filter channel.

INPUT OFDM: $As(t) = ARe\{\sum_{n=0}^{K-1} a_n \, e^{j(2\pi f_n t + \theta_n)}\},$ $\quad 0 \leq t \leq T_s$

OUTPUT OFDM: $z(t) = ARe\{\sum_{n=0}^{K-1} a_n H(f_n) \, e^{j(2\pi f_n t + \theta_n)}\},$ $\quad T_{CP} \leq t \leq T_s \quad (5.13)$

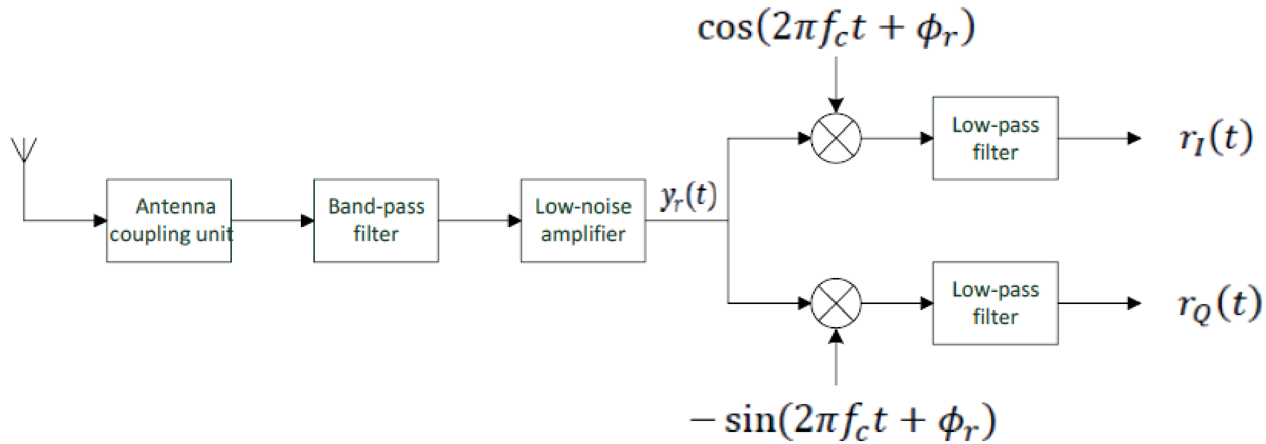


Figure 8. Illustrating the first part of the receiver: the antenna coupling unit, band-pass filter (wide), low-noise amplifier (LNA) and the homodyne unit for frequency down-converting and extracting the baseband signals $r_I(t)$ and $r_Q(t)$. It is here assumed that K is odd for which $f_{rc} = f_c$.

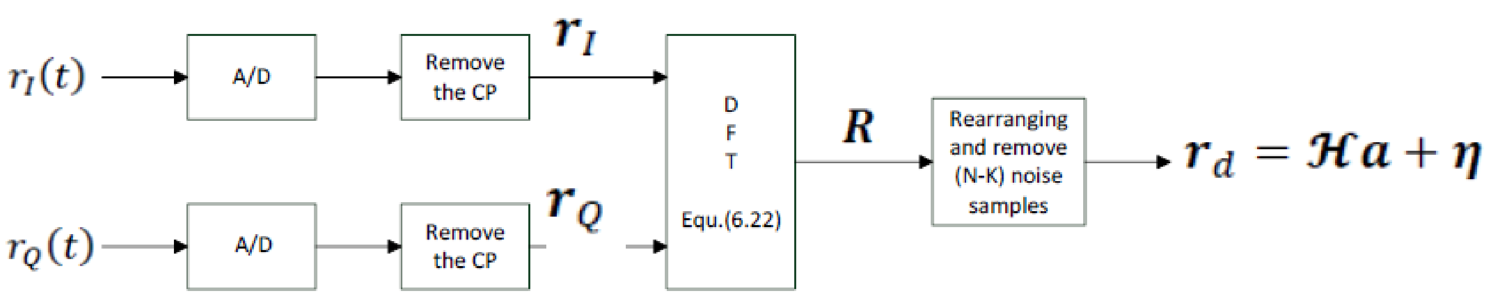


Figure 9. Illustrating sampling and the size-N DFT in the receiver to extract the K received noisy signal points collected in the size-K vector $\boldsymbol{r}_d$.

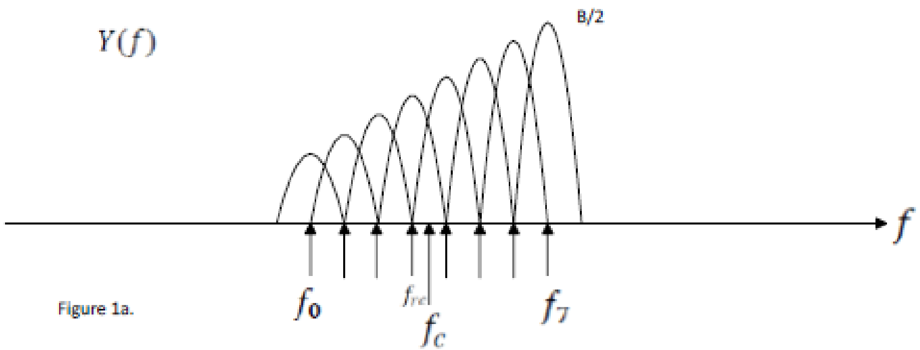


Figure 1a.

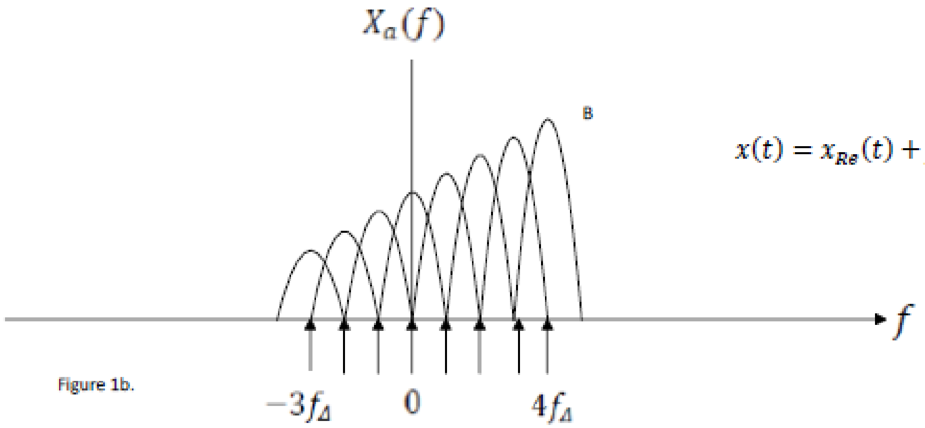


Figure 1b.

$$x(t) = x_{Re}(t) + jx_{Im}(t) = \sum_{n=0}^{K-1} a_n e^{j2\pi g_n f_\Delta t}, \qquad 0 \le t \le T_{obs} \qquad (2.3)$$

Figure 1a) A specific example where $K = 8$, illustrating the main-lobes of the eight individual QAM signals that constitute the OFDM signal $y(t)$ in equation (2.2). The side-lobes of each QAM-signal are, however, **not** shown in this figure. The Fourier transform $Y(f)$ of the OFDM signal $y(t)$ is only roughly indicated by this figure. The short arrows show the eight sub-carrier frequencies. Furthermore, $f_{rc} = f_3$ in this case. In this example it is also assumed that the specific set of K signal points to be transmitted are such that $|a_0| < |a_1| < |a_2| < \cdots < |a_6| < |a_7|$.

Figure 1b) The baseband version of Figure 1a is here considered. Illustrating the main-lobes for the eight individual *baseband* QAM signals that constitute the baseband OFDM signal $x(t)$ in equation (2.3). The Fourier transform $X_a(f)$ of the baseband OFDM signal $x(t)$ is only roughly indicated by this figure. The arrows show the 8 *baseband* sub-carrier frequencies.

$$f_{samp} = N/T_{obs} = Nf_{\Delta} > Kf_{\Delta} \qquad (2.12)$$

samples per second, and **N should be chosen larger than K**, and large enough such that the sampling theorem ([1]) is sufficiently fulfilled.

$$x_m = x(mT_{obs}/N) = \sum_{n=0}^{K-1} a_n e^{j2\pi g_n m/N} \qquad m = 0,1 \ldots, (N-1) \qquad (2.13)$$

The Fourier transform of the discrete-time signal $\boldsymbol{x}$ in equation (2.13) is defined by [1],

$$X(\nu) = \sum_{n=0}^{N-1} x_n e^{-j2\pi\nu n} \qquad (2.14)$$

Note in equation (2.14) that $X(\nu)$ is *periodic in ν with period 1*. Furthermore, the variable ν can be viewed as a **normalized** frequency variable, $\nu = f/f_{samp}$. The periodicity in ν is illustrated in Figure 2

As an example in Figure 4: since $K = 8$ and $N = 12$, the QAM symbol a_3 appears at the sampling indices $l =$ ...-24,-12,0,12,24,.. (corresponding to $\nu = \cdots, -2, -1, 0, 1, 2, \ldots$).

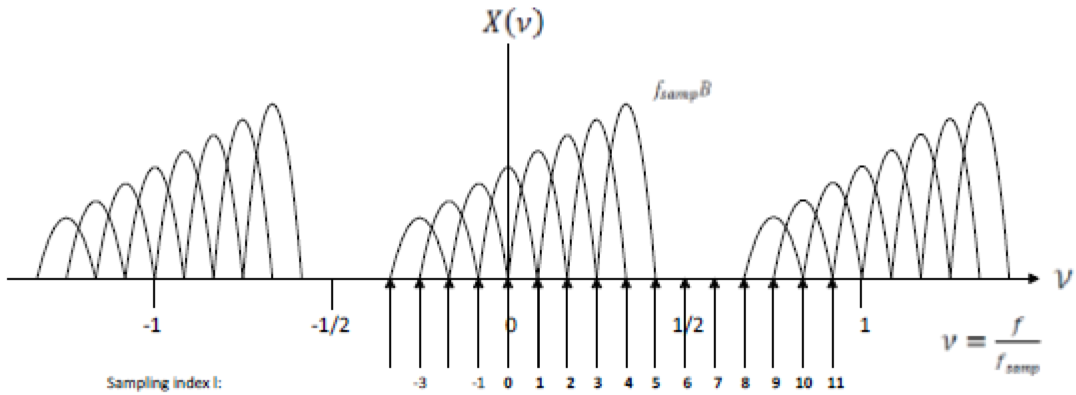


Figure 4. Roughly indicating $X(\nu)$ for the specific example given in Figure 1b where $K = 8$. It is in this figure also assumed that $N = 12$ and the arrows indicate where the frequency-samples X_l are obtained. The bold indices are those samples that are used by the size-12 IDFT. Since $K = 8$ and $N = 12$ the QAM symbol a_0 appears at the sampling indices $l =$...-15,-3,9,21,..

Furthermore, let X_k denote the k:th **frequency-domain sample** of $X(\nu)$ defined by,

$$X_k = X(\nu = k/N) = \sum_{n=0}^{N-1} x_n e^{-j2\pi kn/N}, k = 0,1,\ldots,N-1 \qquad \textbf{(DFT)} \qquad (2.15)$$

This is the definition ([1]) of the size-N **DFT** (Discrete Fourier Transform) of the sequence $\boldsymbol{x}$.

However, for the moment we are particularly interested in the size-N **IDFT** (Inverse Discrete Fourier transform) which is defined by ([1]),

$$x_n = \frac{1}{N}\sum_{k=0}^{N-1} X_k e^{j2\pi kn/N}, n = 0,1,\ldots,N-1 \qquad \textbf{(IDFT)} \qquad (2.16)$$

Hence, as soon as we have determined the samples in the frequency domain $X_0, X_1, \ldots, X_{N-1}$ we should use them in the size-N IDFT in equation (2.16) to efficiently create the desired sequence $\boldsymbol{x}$. The values X_k will be determined in step 3.

In practice, **N is a power of 2** since fast Fourier transform (FFT) algorithms then can be used to significantly speed up the calculations.