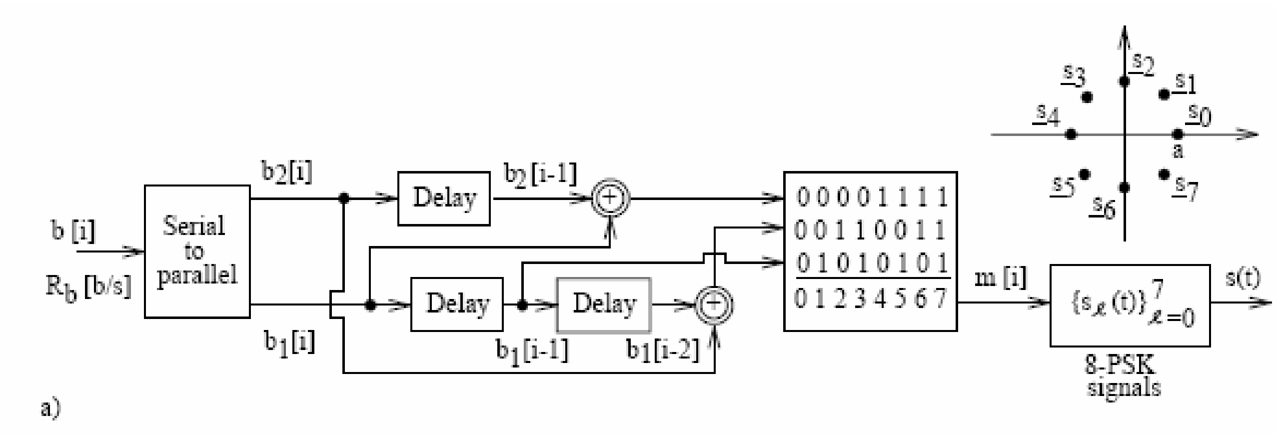


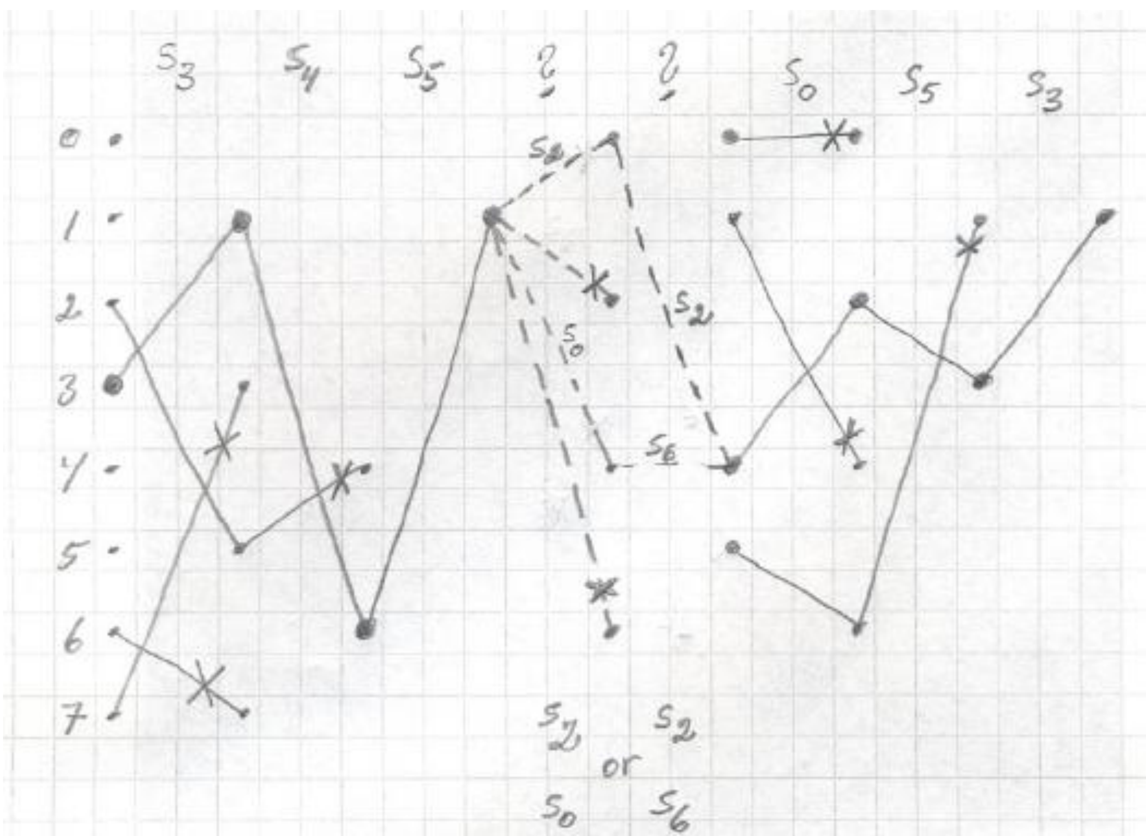
Given: ……..s3, s4, s5, s?, s?, s0, s5, s3

Find: s?, s?

Current state σ [i]

INPUT $\begin{pmatrix} b_2[i] \\ b_1[i] \end{pmatrix}$	(000) 0	(001) 1	(010) 2	(011) 3	(100) 4	(101) 5	(110) 6	(111) 7
$\binom{0}{0}$	0/0	0/2	1/1	1/3	0/4	0/6	1/5	1/7
$\binom{0}{1}$	2/4	2/6	3/5	3/7	2/0	2/2	3/1	3/3
$\binom{1}{0}$	4/2	4/0	5/3	5/1	4/6	4/4	5/7	5/5
$\binom{1}{1}$	6/6	6/4	7/7	7/5	6/2	6/0	7/3	7/1

σ [i+1] / m [i]



Answer to problem 8.10: additional information.

To be able to solve this problem you need to first obtain the table that gives the next state/output signal number, for a given current state and input value.

The table you obtain should be as given below:

	0	1	2	3	4	5	6	7
0	0/0	0/2	1/1	1/3	2/2	2/0	3/3	3/1
1	4/2	4/0	5/3	5/1	6/0	6/2	7/1	7/3

8.3 ML Reception of Trellis-coded Signals in AWGN

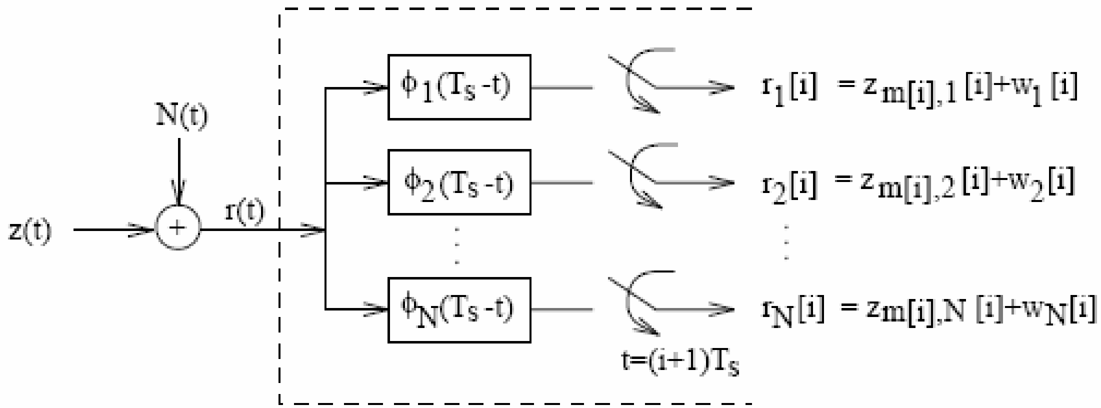


Figure 8.10: The first stage of the ML receiver.

Coherent ML decision rule: "Choose as the decision the "message" corresponding to the signal $z(t)$ that is closest to the received signal $r(t)$ in signal space".

The difference in this subsection, compared with the situation considered in Chapter 4, is that the received information carrying signal $z(t)$ in (8.30) now contains dependency. Consequently, the waveforms received in different symbol intervals are here dependent, and this means that the ML receiver should observe the received signal $r(t)$,

$$r(t) = z(t) + N(t) \quad , \quad -\infty \leq t \leq \infty \tag{8.41}$$

over several symbol intervals before making a decision. In the special case when no such dependency exists, it was found in Chapter 5 that the MAP receiver had to observe the received signal $r(t)$ only over the current symbol interval, to make an optimum decision of the transmitted message.

Correlator-Outputs:

$$\boxed{\boldsymbol{r}[i] = \begin{pmatrix} r_1[i] \\ r_2[i] \\ \vdots \\ r_N[i] \end{pmatrix} = \begin{pmatrix} z_{m[i],1}[i] \\ z_{m[i],2}[i] \\ \vdots \\ z_{m[i],N}[i] \end{pmatrix} + \begin{pmatrix} w_1[i] \\ w_2[i] \\ \vdots \\ w_N[i] \end{pmatrix} = \boldsymbol{z}_{m[i]}[i] + \boldsymbol{w}[i]} \quad (8.44)$$

$$\ldots, \boldsymbol{r}[i-1], \boldsymbol{r}[i], \boldsymbol{r}[i+1], \ldots \quad (8.45)$$

ML: The received sequence of noisy signal points should be compared to all possible sequences

$$\ldots, \boldsymbol{z}_{m[i-1]}[i-1], \boldsymbol{z}_{m[i]}[i], \boldsymbol{z}_{m[i+1]}[i+1], \ldots \quad (8.46)$$

$$\begin{aligned} D^2_{\boldsymbol{r},\boldsymbol{z}} &= \sum_{n=-\infty}^{\infty} \left(\boldsymbol{r}[n] - \boldsymbol{z}_{m[n]}[n]\right)^{tr} \left(\boldsymbol{r}[n] - \boldsymbol{z}_{m[n]}[n]\right) = \\ &= \sum_{n=-\infty}^{\infty} \sum_{\ell=1}^{N} \left(r_\ell[n] - z_{m[n],\ell}[n]\right)^2 \end{aligned} \quad (8.47)$$

Up to time t = (i+1)Ts:

Define the **accumulated** squared Euclidean distance $D^2_{\boldsymbol{r},\boldsymbol{z}}[i]$ up to time $t = (i+1)T_s$ as,

$$D^2_{\boldsymbol{r},\boldsymbol{z}}[i] = \sum_{n=-\infty}^{i} \left(\boldsymbol{r}[n] - \boldsymbol{z}_{m[n]}[n]\right)^{tr} \left(\boldsymbol{r}[n] - \boldsymbol{z}_{m[n]}[n]\right) \tag{8.48}$$

Contribution over the i:th symbol interval:

Also define the squared Euclidean distance **increment** at time $t = (i+1)T_s$ as,

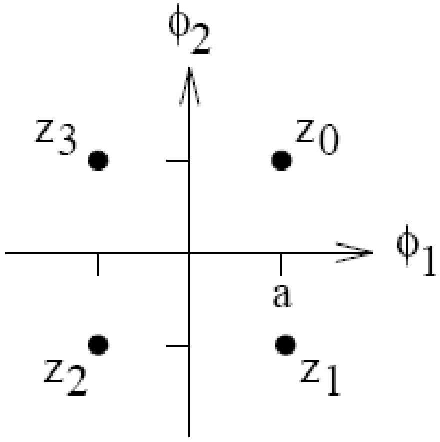
$$D^2_{inc}[i] = \left(\boldsymbol{r}[i] - \boldsymbol{z}_{m[i]}[i]\right)^{tr} \left(\boldsymbol{r}[i] - \boldsymbol{z}_{m[i]}[i]\right) = \sum_{\ell=1}^{N} \left(r_\ell[i] - z_{m[i],\ell}[i]\right)^2 \tag{8.49}$$

Hence, $D^2_{inc}[i]$ is the squared Euclidean distance contribution obtained in the i:th symbol interval $iT_s \leq t \leq (i+1)T_s$.

Recursive calculation:

$$\boxed{D^2_{\boldsymbol{r},\boldsymbol{z}}[i] = D^2_{\boldsymbol{r},\boldsymbol{z}}[i-1] + D^2_{inc}[i]} \tag{8.50}$$

EXAMPLE 8.14



$$\boldsymbol{r}[i-2] = \begin{pmatrix} a/2 \\ -a/4 \end{pmatrix}, \boldsymbol{r}[i-1] = \begin{pmatrix} -a/2 \\ -3a/4 \end{pmatrix}, \boldsymbol{r}[i] = \begin{pmatrix} 3a/2 \\ a/4 \end{pmatrix}$$

Calculate the squared Euclidean distance increments associated with the candidate sequence $\boldsymbol{z}_0[i-2]$, $\boldsymbol{z}_3[i-1]$, $\boldsymbol{z}_2[i]$.

$$\begin{aligned} D^2_{inc}[i-2] &= \sum_{\ell=1}^{2} (r_\ell[i-2] - z_{0,\ell}[i-2])^2 = \\ &= \left(\frac{a}{2} - a\right)^2 + \left(-\frac{a}{4} - a\right)^2 = \frac{29}{16}\ a^2 \end{aligned}$$

$$\boxed{D^2_{\boldsymbol{r},\,\boldsymbol{z}}[i] = D^2_{\boldsymbol{r},\,\boldsymbol{z}}[i-1] + D^2_{inc}[i]} \tag{8.50}$$

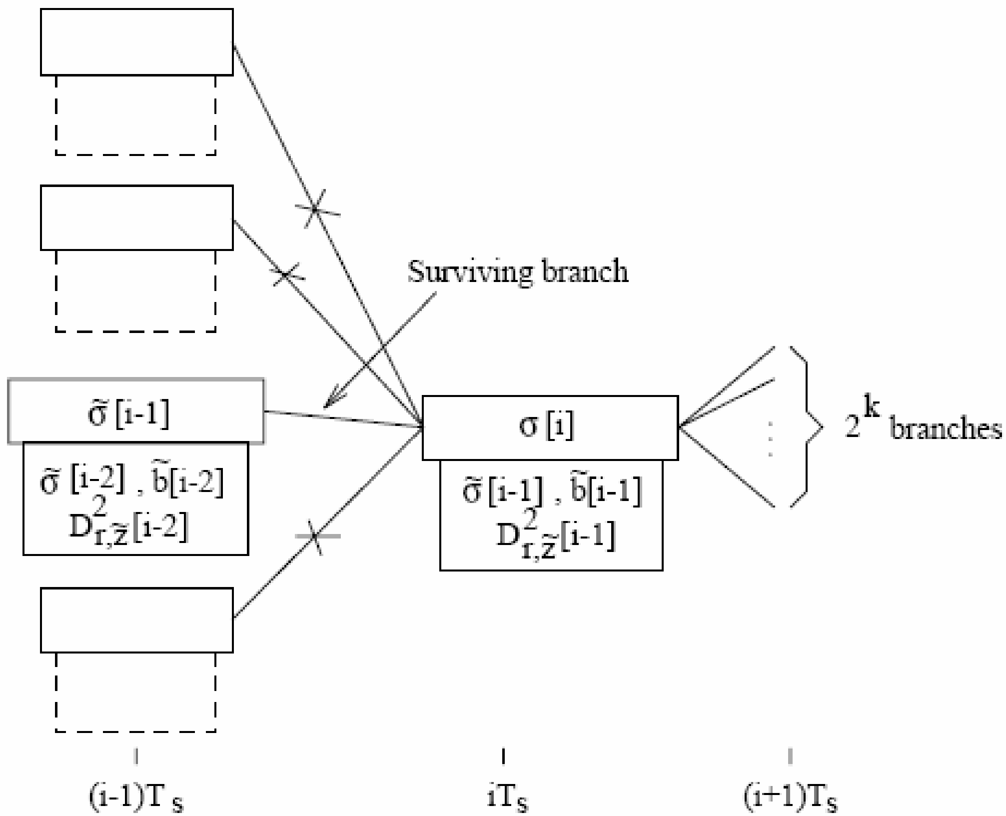
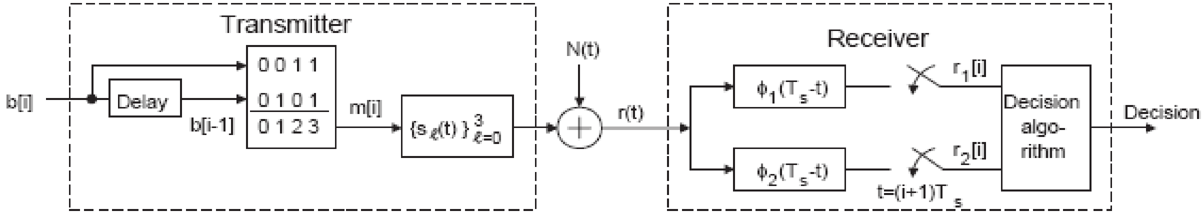
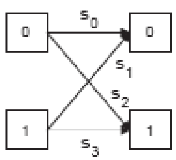


Figure 8.11: Illustrating how branches in the trellis are deleted (x) by the Viterbi algorithm.

The result of the Viterbi algorithm, at time $t = iT_s$, is exactly $\mathcal{S}$ surviving **candidate sequences**, one for each state $\sigma[i]$. These sequences can be found, i.e., traced back in the trellis by using the saved information at each state. *Note that the most likely sequence, up to time $t = iT_s$, must be one of these $\mathcal{S}$ candidate sequences!*

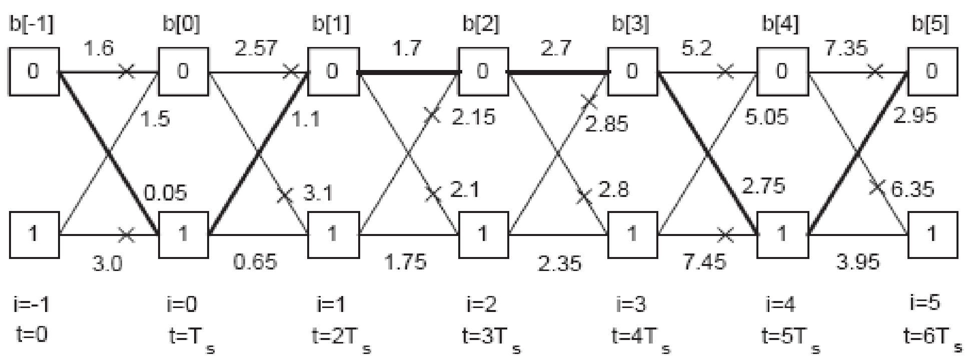
EXAMPLE 8.16





The table below gives the squared Euclidean distance increments $D^2_{inc}[i] = (r_1[i] - s_{j,1})^2 + (r_2[i] - s_{j,2})^2$, obtained between $\mathbf{r}[i]$ and $\mathbf{s}_j$, $j = 0, 1, 2, 3$. The noise $N(t)$ is AWGN.

i :	0	1	2	3	4	5
$\mathbf{s}_0$	1.6	1.07	0.6	1.0	2.5	2.3
$\mathbf{s}_1$	1.5	1.05	1.5	1.1	2.7	0.2
$\mathbf{s}_2$	0.05	1.6	1.0	1.1	0.05	1.3
$\mathbf{s}_3$	3.0	0.6	1.1	0.6	5.1	1.2



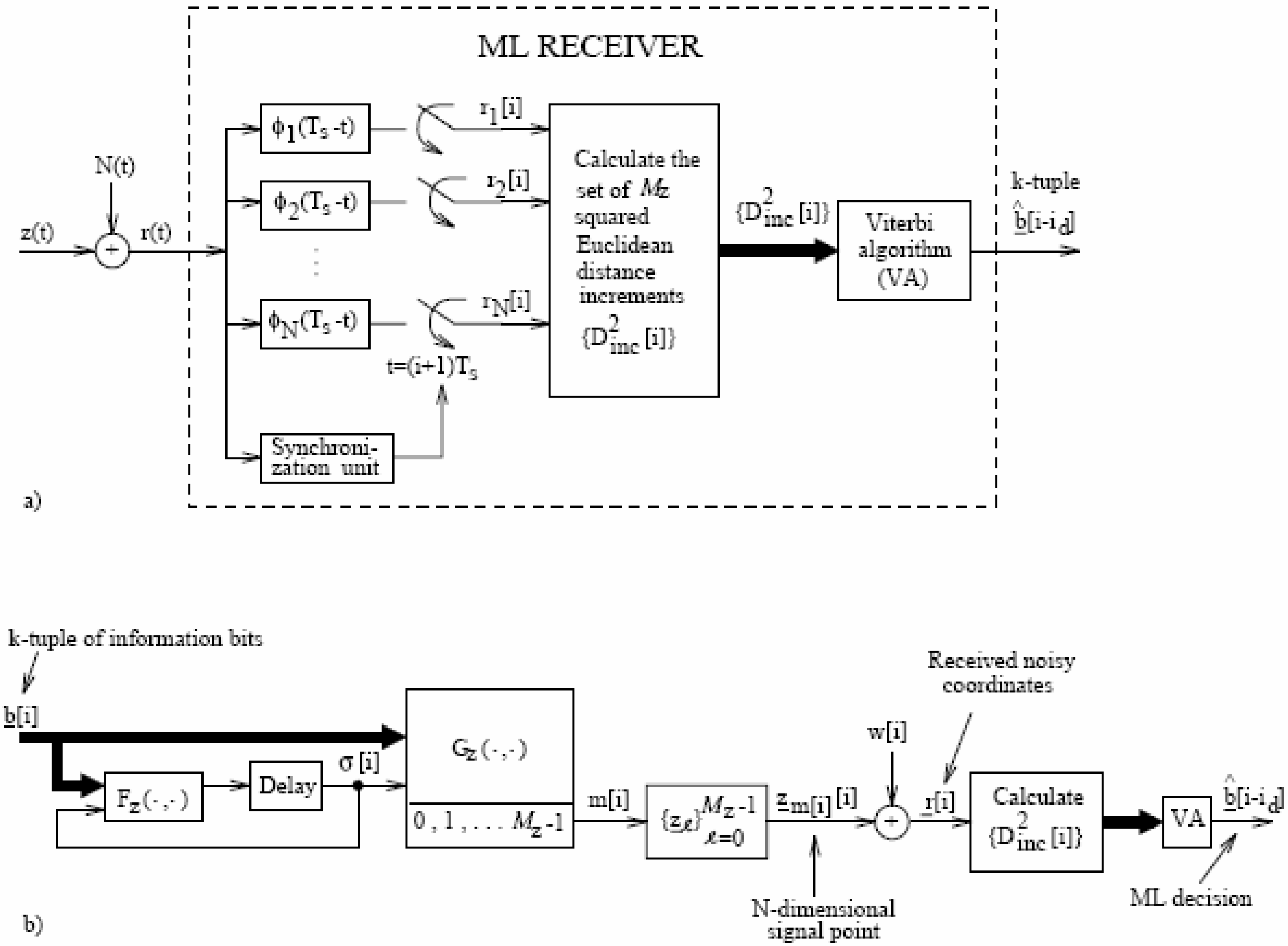


Figure 8.12: a) The coherent ML (sequence) receiver for trellis-coded signals in AWGN; b) A discrete-time model in signal space of the overall digital communication system.

8.3.1 The ML Receiver - An Alternative Approach

In this subsection, an alternative implementation of the coherent ML receiver for trellis coded signals in AWGN is derived. With this implementation the number of matched filters in the receiver is significantly reduced compared with Figure 8.12a, making it particularly interesting in many applications.

$$r(t) = z(t) + N(t) = \sum_{n=-\infty}^{\infty} x_{m[n]}(t - nT_s) + N(t), \quad -\infty \leq t \leq \infty \qquad (8.54)$$

$$x_\ell(t) = s_\ell(t) * h(t), \quad \ell = 0, 1, \ldots, \mathcal{M}_{tra} - 1 \qquad (8.55)$$

$$\int_{-\infty}^{\infty} (r(t) - z(t))^2 dt \qquad (8.56)$$

maximize the expression,

$$\int_{-\infty}^{\infty} \left(r(t)z(t) - \frac{z^2(t)}{2} \right) dt \qquad (8.57)$$

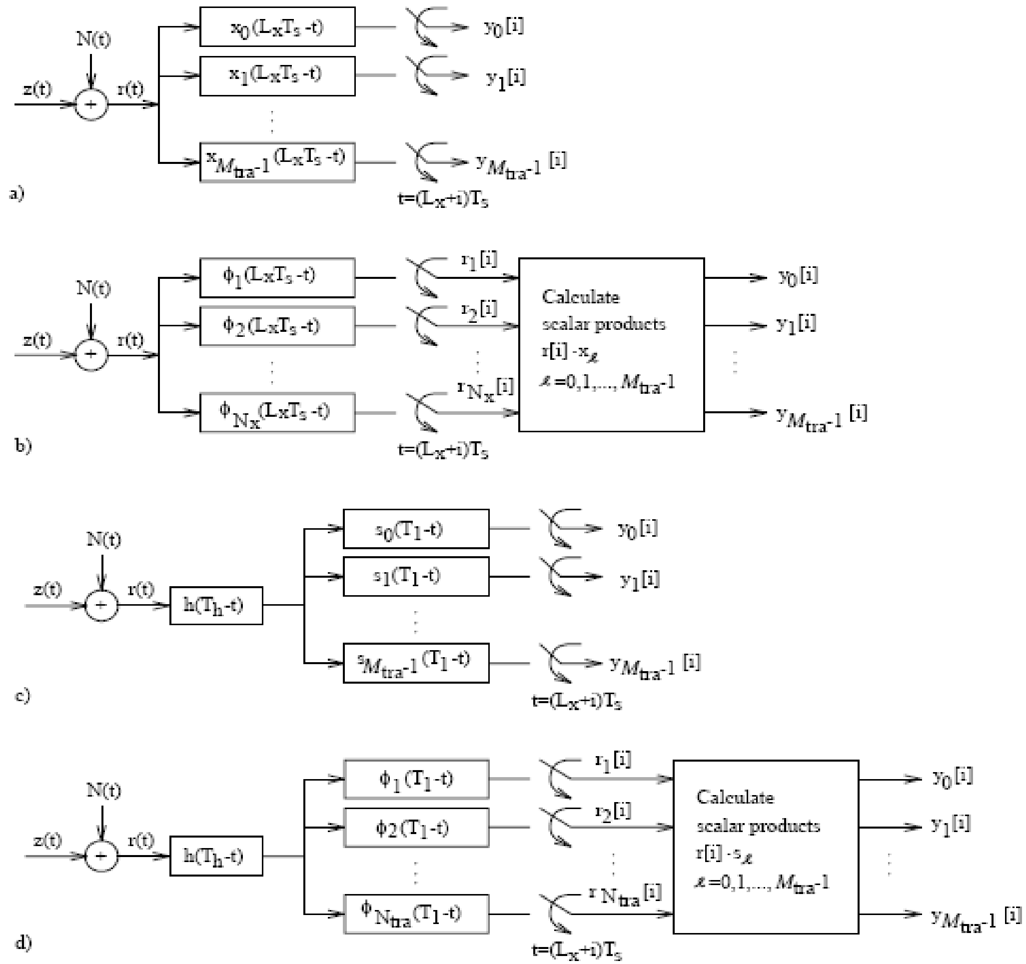


Figure 8.13: The first stage in the ML receiver. a) Filters matched to the signals $x_\ell(t)$; b) Filters matched to the N_x ($N_x \leq \mathcal{M}_{tra}$) basis functions of $\{x_\ell(t)\}_{\ell=0}^{\mathcal{M}_{tra}-1}$; c) Filters matched to $h(t)$ and to $s_\ell(t)$; d) Filters matched to $h(t)$ and to the N_{tra} basis functions of $\{s_\ell(t)\}_{\ell=0}^{\mathcal{M}_{tra}-1}$.

8.4 Bit Error Probability for the ML Receiver in AWGN

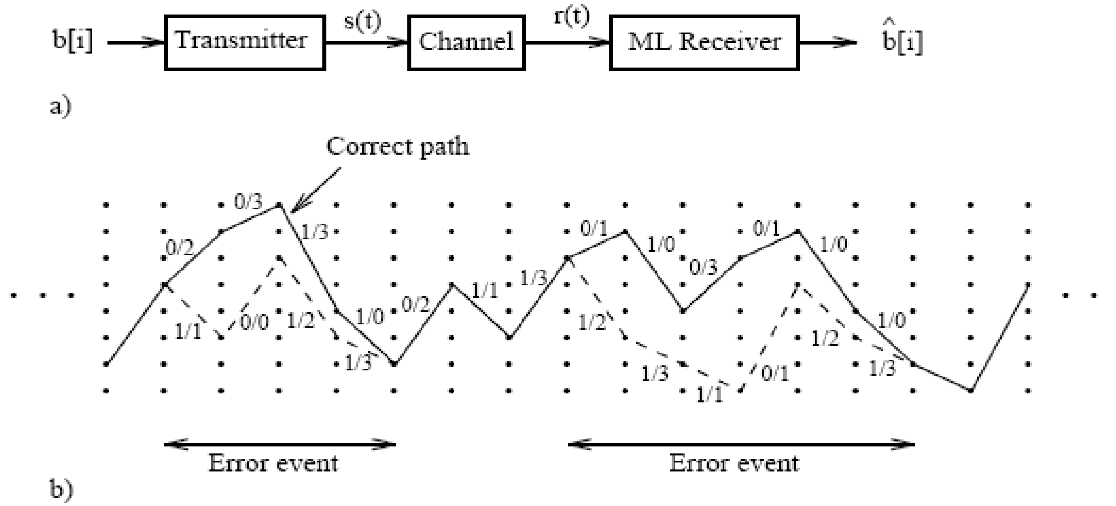


Figure 8.19: a) The digital communication system. b) Parts of the correct path (solid) and the decoded path (dashed) in the trellis.

Observe that by using trellis-coding, longer error events than in the uncoded case are possible. ***So, with trellis-coding there is a potential for larger Euclidean distances than in the uncoded case.***

It can be shown that as a first approximation, at high signal-to-noise ratios, the bit error probability can be approximated with the expression

$$P_b \approx cQ\left(\sqrt{\frac{D_{\min}^2}{2N_0}}\right) = cQ\left(\sqrt{d_{\min}^2 \mathcal{E}_b / N_0}\right) \qquad (8.102)$$

where $D_{\min}$ is the smallest Euclidean distance *in the set of all error events, and* $d_{\min}^2 = D_{\min}^2 / 2\mathcal{E}_b$.

Chapter 9

An Introduction to Time-varying Multipath Channels

$$\boxed{z(t) = \sum_n \alpha_n(t) s(t - \tau_n(t))} \qquad (9.1)$$

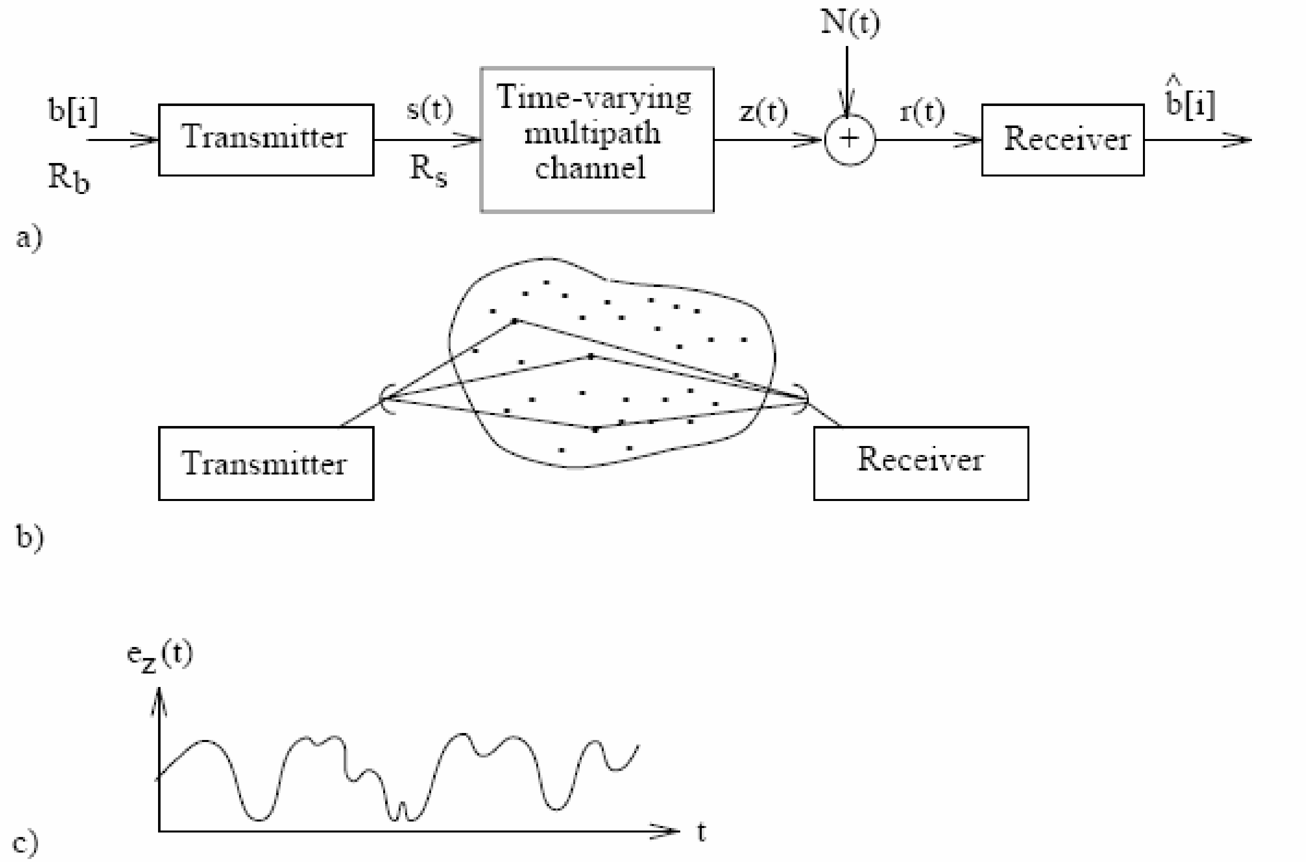


Figure 9.1: a) The digital communication system; b) A scattering medium; c) Illustrating the fading envelope $e_z(t)$.

$$s(t) = \cos((\omega_c + \omega_1)t) \ , \quad -\infty \leq t \leq \infty \qquad (9.2)$$

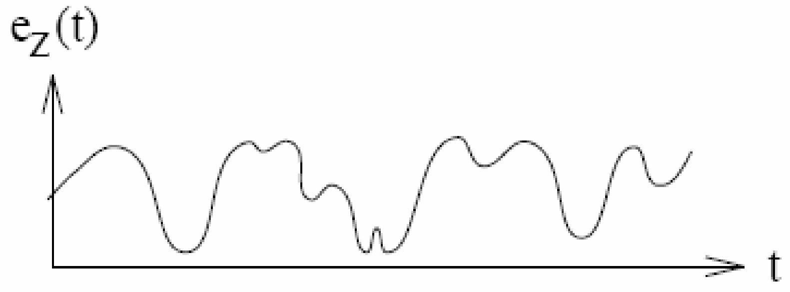
$$\begin{aligned} z(t) &= \sum_n \alpha_n(t) \cos((\omega_c + \omega_1)(t - \tau_n(t))) = \\ &= \underbrace{\left[\sum_n \alpha_n(t) \cos((\omega_c + \omega_1)\tau_n(t))\right]}_{z_I(t) = \tilde{H}_{Re}(f_1,t)/2} \cos((\omega_c + \omega_1)t) - \\ &\quad - \underbrace{\left[\sum_n \alpha_n(t) \sin(-(\omega_c + \omega_1)\tau_n(t))\right]}_{z_Q(t) = \tilde{H}_{Im}(f_1,t)/2} \sin((\omega_c + \omega_1)t) \\ &= z_I(t) \cos((\omega_c + \omega_1)t) - z_Q(t) \sin((\omega_c + \omega_1)t) \\ &= e_z(t) \cos((\omega_c + \omega_1)t + \theta_z(t)) \qquad (9.3) \end{aligned}$$

Compare with the time-invariant QAM-result:

$$\begin{aligned} A_z + jB_z = (A + jB)H(f_c) &= \sqrt{A^2 + B^2}|H(f_c)|e^{j(\nu + \phi(f_c))} = \\ &= (A + jB)(H_{Re}(f_c) + jH_{Im}(f_c)) \qquad (3.110) \end{aligned}$$

$$s(t) = \cos((\omega_c + \omega_1)t) \ , \quad -\infty \le t \le \infty \tag{9.2}$$

$$\begin{aligned} z(t) &= \sum_n \alpha_n(t) \cos((\omega_c + \omega_1)(t - \tau_n(t))) = \\ &= e_z(t) \cos((\omega_c + \omega_1)t + \theta_z(t)) \end{aligned} \tag{9.3}$$



Observe that the quadrature components $z_I(t)$ and $z_Q(t)$ in (9.3) are *time-varying*. Hence, the output signal $z(t)$ is *not* a pure sine wave with frequency $f_c + f_1$. ***This is a significant difference compared with the linear time-invariant channel.*** It is seen in (9.3) that the quadrature components depend

$$
\begin{aligned}
z(t) &= \sum_n \alpha_n(t)\cos((\omega_c+\omega_1)(t-\tau_n(t))) = \\
&= z_I(t)\cos((\omega_c+\omega_1)t) - z_Q(t)\sin((\omega_c+\omega_1)t) \\
&= e_z(t)\cos((\omega_c+\omega_1)t+\theta_z(t))
\end{aligned}
$$

Throughout this chapter it is assumed that $z_I(t)$ and $z_Q(t)$ may be modelled as baseband zero-mean wide-sense-stationary (WSS) *Gaussian random processes* (with variances $\sigma_I^2 = \sigma_Q^2 = \sigma^2$). This is a commonly used assumption when the number of scatterers is large, implying that central limit theorem arguments can be used [43], [65], [68], [39]. For a fixed value of t, this assumption leads to a Rayleigh-distributed envelope $e_z(t)$,

$$e_z(t) = \sqrt{z_I^2(t) + z_Q^2(t)} \tag{9.4}$$

$$p_{e_z}(x) = \frac{2x}{b}\, e^{-x^2/b}, \quad x \geq 0\ , \text{ Rayleigh distr.} \tag{9.5}$$

$$b = E\{e_z^2(t)\} = 2\sigma^2 = 2P_z \tag{9.6}$$

and a uniformly distributed phase $\theta_z(t)$ (over a 2π interval). The zero-mean assumption means that there is no deterministic signal path present in $z(t)$. If a

9.1.1 Doppler Power Spectrum and Coherence Time

$$\boxed{\begin{aligned} R_{\mathcal{D}}(f) &= \mathcal{F}(\tilde{c}_z(\tau)) \\ \tilde{c}_z(\tau) &= \frac{1}{2}\, E\{[z_I(t+\tau) + jz_Q(t+\tau)]\ [z_I(t) - jz_Q(t)]\} \\ R_z(f) &= \frac{1}{2}\ (R_{\mathcal{D}}(f + f_c + f_1) + R_{\mathcal{D}}(f - f_c - f_1)) \end{aligned}} \qquad (9.7)$$

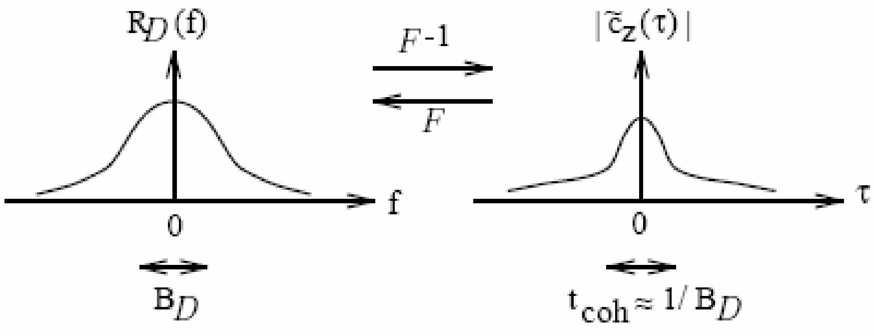


Figure 9.2: Illustrating the Fourier transform pair $\tilde{c}_z(\tau) \longleftrightarrow R_{\mathcal{D}}(f)$.

$$t_{coh} \approx 1/B_{\mathcal{D}} \qquad (9.8)$$

If the channel is slowly changing, then the coherence time is large. Note that $z_I(t+\tau)$ and $z_I(t)$ (also $z_Q(t+\tau)$ and $z_Q(t)$) are correlated over time-intervals τ (much) smaller than the coherence time t_{coh}. Hence, input signals within such intervals are therefore affected similarly by the fading channel. On the other hand, input signals that are separated in time by (much) more than t_{coh}, are affected differently by the channel, and at the output of the channel they become essentially independent of each other. If the former case apply (time flat fading), for a given time-interval, then we say that the channel is **time-nonselective**, and if the latter case apply, then the channel is said to be **time-selective**.

9.1.2 Coherence Bandwidth and Multipath Spread

$$z(t) = z(f_1,t) = \underbrace{\frac{1}{2}\,\tilde{H}_{Re}(f_1,t)}_{z_I(t)}\cos((\omega_c+\omega_1)t) - \underbrace{\frac{1}{2}\,\tilde{H}_{Im}(f_1,t)}_{z_Q(t)}\sin((\omega_c+\omega_1)t) \tag{9.9}$$

What can be said about the output signal $z(t)$ if another frequency $f_2 = f_1 + f_\Delta$ is used, instead of f_1? Are different frequency-intervals, in the input signal spectrum, treated differently by the time-varying multipath channel? To answer these questions the correlation between $z(f_1,t)$ and $z(f_1+f_\Delta,t)$ can be found by

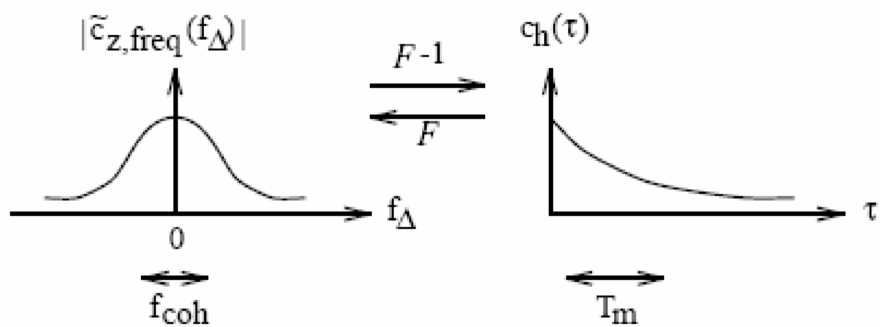


Figure 9.3: Illustrating the Fourier transform pair $c_h(\tau) \longleftrightarrow \tilde{c}_{z,freq}(f_\Delta)$.

The **coherence bandwidth** $\boldsymbol{f_{coh}}$ of the channel is defined as the width of the autocorrelation function $\tilde{c}_{z,freq}(f_\Delta)$, see Figure 9.3. Note that frequencies within a frequency-interval (much) smaller than the coherence bandwidth f_{coh} are correlated, and they are affected similarly by the fading channel. On the other hand, two frequencies that are separated by (much) more than f_{coh}, are affected differently by the channel, and they are essentially independent of each other. If the former case apply (frequency flat fading), for a given frequency-interval, then we say that the channel is **frequency-nonselective**, and if the latter case apply, then the channel is said to be **frequency-selective**.

$$\boxed{z(t) = \int_{-\infty}^{\infty} h(\tau, t) s(t - \tau) d\tau} \tag{9.10}$$

delay power spectrum $c_h(\tau)$ (also multipath intensity profile) of the time-varying impulse response $h(\tau, t)$,

$$c_h(\tau) = E\left\{\frac{h^2(\tau, t)}{2}\right\} = \frac{1}{2} E\{h_I^2(\tau, t) + h_Q^2(\tau, t)\} = \frac{1}{2} E\{\tilde{h}(\tau, t)\tilde{h}^*(\tau, t)\} \tag{9.15}$$

An example of the delay power spectrum $c_h(\tau)$ is illustrated in Figure 9.3. The width of the delay power spectrum is referred to as the **multipath spread** of the channel and it is denoted by $\boldsymbol{T_m}$. This is an important parameter since if T_m is too large, compared with e.g. the symbol time, then intersymbol interference can occur.

$$T_m \approx 1/f_{coh} \tag{9.16}$$

9.2 Frequency-Nonselective, Slowly Fading Channel

$$T_s \ll t_{coh} \tag{9.27}$$

or equivalently,

$$B_{\mathcal{D}} \ll R_s \tag{9.28}$$

This means that the channel is **slowly fading**, which imply that it can be treated as a time-invariant channel within the coherence time.

In this subsection a frequency-nonselective channel is investigated. To obtain this situation it is required that the bandwidth of the transmitted signal, denoted W, is much smaller than the coherence bandwidth f_{coh} of the channel,

$$W \ll f_{coh} \tag{9.29}$$

or equivalently,

$$T_m \ll 1/W \tag{9.30}$$

$$\boxed{\boxed{\tilde{z}(t) = \frac{1}{2}\int_{-\infty}^{\infty} \tilde{S}(f)\tilde{H}(f,t)e^{j2\pi ft}df}} \tag{9.26}$$

$$\boxed{z_I(t) + jz_Q(t) = \frac{1}{2}\int_{-\infty}^{\infty}[S_I(f) + jS_Q(f)]\ [H_I(f,t) + jH_Q(f,t)]e^{j2\pi ft}df} \tag{9.33}$$

$$z_I(t) + jz_Q(t) = \frac{1}{2}\int_{-\infty}^{\infty}[S_I(f) + jS_Q(f)] \cdot (H_I + jH_Q)e^{j2\pi ft}df \tag{9.36}$$

$$\begin{aligned} z_I(t) + jz_Q(t) &= \frac{1}{2}\ (s_I(t) + js_Q(t))(H_I + jH_Q) = \\ &= e_s(t)e^{j\theta_s(t)} \cdot ae^{j\phi} = e_z(t)e^{j\theta_z(t)} \end{aligned} \tag{9.37}$$

$$
\begin{aligned}
z_I(t) + jz_Q(t) &= \frac{1}{2}\,(s_I(t) + js_Q(t))(H_I + jH_Q) = \\
&= e_s(t)e^{j\theta_s(t)} \cdot ae^{j\phi} = e_z(t)e^{j\theta_z(t)} \qquad (9.37)
\end{aligned}
$$

$$
\boxed{\boxed{z(t) = ae_s(t)\cos(\omega_c t + \theta_s(t) + \phi)}} \qquad (9.38)
$$

$$
p_a(x) = \frac{2x}{b}\, e^{-x^2/b}, \quad x \geq 0 \quad \text{(Rayleigh distribution)} \qquad (9.39)
$$

where,

$$
\begin{aligned}
E\{a\} &= \frac{1}{2}\,\sqrt{\pi b} \qquad (9.40) \\
E\{a^2\} &= b \qquad (9.41)
\end{aligned}
$$

and,

$$
p_\phi(y) = \begin{cases} 1/2\pi & , \quad -\pi \leq y \leq \pi \\ 0 & , \quad \text{otherwise} \end{cases} \qquad (9.42)
$$

If we assume uncoded equally likely binary signals over a Rayleigh fading channel ($z_1(t) = as_1(t), z_0(t) = as_0(t)$), then the bit error probability of the ideal coherent ML receiver is ($0 < d^2 = \frac{D^2_{s_1,s_0}}{2E_{b,sent}} \leq 2$)

$$\boxed{\boldsymbol{P_b = \int_0^\infty \mathbf{Pr}\{\mathbf{error}|a\} p_a(x) dx = E\{\mathbf{Pr}\{\mathbf{error}|a\}\}}} \qquad (9.43)$$

$$\begin{aligned}
P_b &= \int_0^\infty Q(\sqrt{d^2 x^2 E_{b,sent}/N_0})\ \frac{2x}{b}\ e^{-x^2/b} dx = \\
&= -e^{-x^2/b}\ Q(x\sqrt{d^2 E_{b,sent}/N_0})\Big]_0^\infty - \int_0^\infty (-e^{-x^2/b}) \\
&\quad \left(\frac{-\sqrt{d^2 E_{b,sent}/N_0}}{\sqrt{2\pi}}\ e^{-\frac{x^2 d^2 E_{b,sent}/N_0}{2}} \right) dx = \\
&= \frac{1}{2} - \sqrt{d^2 E_{b,sent}/N_0} \cdot \beta \underbrace{\int_0^\infty \frac{e^{-x^2/2\beta^2}}{\beta\sqrt{2\pi}} dx}_{1/2}
\end{aligned} \qquad (9.44)$$

$$\mathcal{E}_b = E\{a^2\}E_{b,sent} = bE_{b,sent} \tag{9.45}$$

$$P_b = \frac{1}{2}\left(1 - \sqrt{\frac{d^2\mathcal{E}_b/N_0}{2 + d^2\mathcal{E}_b/N_0}}\right) = \frac{1}{2 + d^2\mathcal{E}_b/N_0 + \sqrt{2 + d^2\mathcal{E}_b/N_0}\sqrt{d^2\mathcal{E}_b/N_0}}$$

$$\begin{array}{c} \mathcal{E}_b/N_0 \text{ ``large''} \\ \downarrow \\ \approx \end{array} \quad \frac{1}{2d^2\mathcal{E}_b/N_0} \tag{9.46}$$

where $d^2 = 2$ for antipodal signals and $d^2 = 1$ for orthogonal signals. ***Observe the dramatic increase in P_b due to the Rayleigh fading channel. P_b is no longer exponentially decaying in $\mathcal{E}_b/N_0$, it now decays essentially as $(\mathcal{E}_b/N_0)^{-1}$!***

EXAMPLE 9.1

Assume that equally likely, binary orthogonal FSK signals, with equal energy, are sent from the transmitter. Hence, $s_i(t) = \sqrt{2E_{b,sent}/T_b}\cos(2\pi f_i t)$ *in* $0 \le t \le T_b$, $i = 0, 1$.

These signals are communicated over a Rayleigh fading channel, i.e. the received signal is (see (9.38)),

$$r(t) = a\sqrt{2E_{b,sent}/T_b}\cos(2\pi f_i t + \phi) + N(t)$$

Assume that the incoherent receiver in Figure 5.28 on page 397 is used. From (5.109) it is known that for a given value of a,

$$P_b = \frac{1}{2}\ e^{-a^2 E_{b,sent}/2N_0}$$

since $a^2E_{b,sent}$ *then is the average received energy per bit.*

For the Rayleigh fading channel, and the same receiver, P_b *can be calculated by using (9.43),*

$$\boxed{P_b = \int_0^\infty \Pr\{error|a = x\}p_a(x) = E\{\Pr\{error|a\}\}}$$

$$E\{\Pr\{error|a\}\} = E\left\{\frac{1}{2}\ e^{-a^2 E_{b,sent}/2N_0}\right\} =$$

$$E\left\{\frac{1}{2}\ e^{-a_1^2 E_{b,sent}/2N_0}\right\} \cdot E\left\{e^{-a_2^2 E_{b,sent}/2N_0}\right\}$$

$$P_b = \frac{1/2}{1 + \frac{E_{b,sent}}{N_0} \cdot \frac{E\{a^2\}}{2}} = \frac{1}{2 + \mathcal{E}_b/N_0}$$

Observe the dramatic increase in P_b due to the Rayleigh fading channel. P_b is no longer exponentially decaying in $\mathcal{E}_b/N_0$, it now decays essentially as $(\mathcal{E}_b/N_0)^{-1}$! *As an example, assuming $\mathcal{E}_b/N_0 = 1000$ (30 dB), we obtain*

$$P_b = \begin{cases} 0.5e^{-500} \approx 3.6 \cdot 10^{-218} & , \quad AWGN \\ (1002)^{-1} \approx 10^{-3} & , \quad Rayleigh{+}AWGN \end{cases}$$

DIVERSITY IS NEEDED!