Study week 3.

3.4.1 Low-Rate QAM-Type of Input Signals

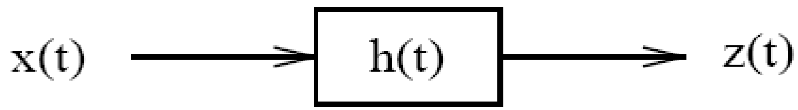


Figure 3.11: Bandpass filtering.

$$x(t) = x_I(t)\cos(\omega_c t) - x_Q(t)\sin(\omega_c t) = Re\{\tilde{x}(t)e^{j\omega_c t}\} \qquad (3.103)$$

$$\tilde{x}(t) = x_I(t) + jx_Q(t) \qquad (3.104)$$

This complex signal contains the information!

$$x(t) = x_I(t)\cos(\omega_c t) - x_Q(t)\sin(\omega_c t) = Re\{\tilde{x}(t)e^{j\omega_c t}\} \qquad (3.103)$$

$$z(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau = \int_{-\infty}^{\infty} h(\tau)Re\{\tilde{x}(t-\tau)e^{j\omega_c(t-\tau)}\}d\tau =$$

$$= Re\left\{e^{j\omega_c t}\int_{-\infty}^{\infty} h(\tau)\tilde{x}(t-\tau)e^{-j\omega_c\tau}d\tau\right\} \qquad (3.105)$$

3 assumptions:

1) The duration of the impulse response $h(t)$ can be considered to be equal to T_h. This means that essentially all the energy in $h(t)$ is assumed to be contained within the time interval $0 \le t \le T_h$.

2) The input signal is assumed to be a QAM-type of signal with duration $T = T_s$:

$$x(t) = \begin{cases} 0 & , \quad t < 0 \\ A\cos(\omega_c t) - B\sin(\omega_c t) = \sqrt{A^2 + B^2}\cos(\omega_c t + \nu) & , \quad 0 \le t \le T_s \\ 0 & , \quad t > T_s \end{cases} \qquad (3.106)$$

3) $T_s > T_h$ ("low" signaling rate).

$$\tilde{x}(t) = \begin{cases} A + jB = \sqrt{A^2 + B^2}\ e^{j\nu} & , \quad 0 \le t \le T_s \\ 0 & , \quad \text{otherwise} \end{cases} \tag{3.108}$$

$\boldsymbol{T_h \le t \le T_s}$ **:**

$$\begin{aligned} z(t) &= Re\left\{ e^{j\omega_c t} \int_0^{T_h} h(\tau)\sqrt{A^2 + B^2}\ e^{j\nu} e^{-j\omega_c \tau} d\tau \right\} = \\ &= Re\{\sqrt{A^2 + B^2}\ e^{j\nu} \cdot H(f_c) e^{j\omega_c t}\} = \\ &= |H(f_c)|\sqrt{A^2 + B^2} \cos(\omega_c t + \nu + \phi(f_c)) = A_z \cos(\omega_c t) - B_z \sin(\omega_c t) \end{aligned} \tag{3.109}$$

Hence, a QAM-signal at the output in this time interval!

However, **attenuation and rotation** compared with the input!
Compare with the input x(t) in (3.106)!

$$\begin{aligned} A_z + jB_z &= (A + jB)H(f_c) = \sqrt{A^2 + B^2}|H(f_c)|e^{j(\nu + \phi(f_c))} = \\ &= (A + jB)(H_{Re}(f_c) + jH_{Im}(f_c)) \end{aligned} \tag{3.110}$$

$$A_z + jB_z = (A + jB)H(f_c) = \sqrt{A^2 + B^2}|H(f_c)|e^{j(\nu+\phi(f_c))} =$$
$$= (A + jB)(H_{Re}(f_c) + jH_{Im}(f_c)) \quad (3.110)$$

A COMPACT MODEL WITH A COMPLEX CHANNEL PARAMETER!!

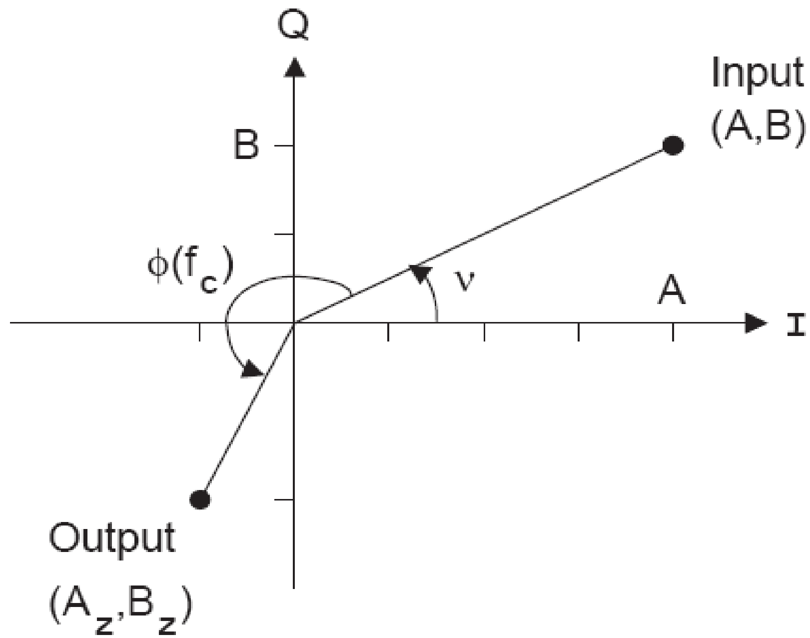


Figure 3.13: Illustrating that the input I-Q amplitudes (A,B) are scaled and rotated by the channel $H(f)$, see (3.109) and (3.110).

$$
z(t) = \begin{cases} 0 & , \quad t < 0 \\ \text{“non-stationary transient” starting interval} & , \quad 0 \leq t \leq T_h \\ |H(f_c)|\sqrt{A^2 + B^2}\ \cos(\omega_c t + \nu + \phi(f_c)) & , \quad T_h \leq t \leq T_s \\ \text{“non-stationary transient” ending interval} & , \quad T_s \leq t \leq T_s + T_h \\ 0 & , \quad t > T_s + T_h \end{cases}
$$

$$
\text{and within } T_h \leq t \leq T_s,\ A_z + jB_z = (A + jB)H(f_c) \tag{3.111}
$$

An important result here is that the input QAM signal $x(t)$ in (3.106) is changed to a new QAM signal by $|H(f_c)|$ and $\phi(f_c)$ in the interval $T_h \leq t \leq T_s$, see also Figure 3.13 and (3.110) how the I-Q components are changed. Furthermore, in OFDM applications the signaling rate $1/T_s$ is low such that $T_s \gg T_h$, and many QAM signals with different carrier frequencies are sent in parallel. ***Due to linearity, the result in (3.111) can be applied to each QAM signal in the OFDM signal by replacing f_c with f_n.*** In OFDM applications the receiver uses the time interval $\Delta_h \leq t \leq T_s$ for detection of the output QAM signals, and the duration of this observation interval is denoted $T_{obs} = T_s - \Delta_h$ (compare with (2.110) on page 51, and $T_h \leq \Delta_h$).

So, the n:th QAM signal constellation in a sent OFDM signal is attenuated and rotated by H(fn) which is the value of the channel transfer function H(f) at the carrier frequency fn.

3.4.3 N-Ray Channel Model

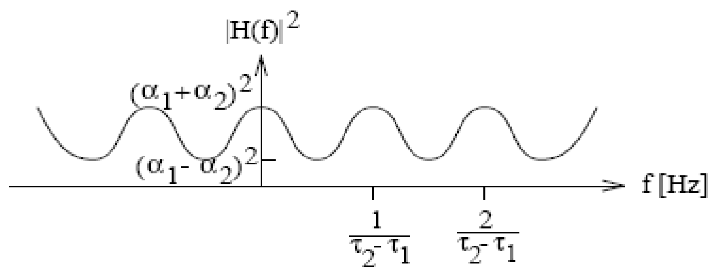
$$z(t) = x(t) * \underbrace{\left(\sum_{i=1}^{N} \alpha_i \delta(t - \tau_i) \right)}_{\text{Impulse response } h(t)} = \sum_{i=1}^{N} \alpha_i x(t - \tau_i) \qquad (3.126)$$

$$H(f) = \mathcal{F}\{h(t)\} = \sum_{i=1}^{N} \alpha_i e^{-j2\pi f \tau_i} \qquad (3.128)$$

So, **H(fc)** is easy to find!

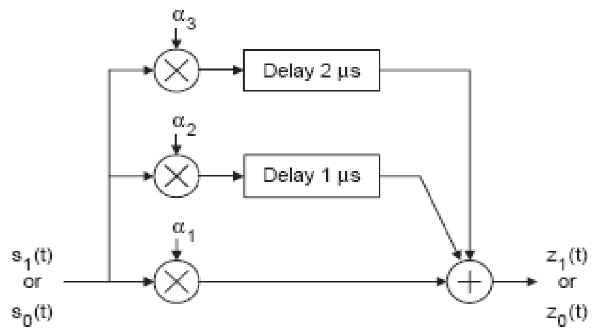
EXAMPLE 3.20

Rough sketch:



It is seen in this figure that the two signal paths add constructively or destructively (fading) depending on the frequency. Furthermore, if $\alpha_1 \approx \alpha_2$ then $|H(f)|$ is very close to zero at certain frequencies (so-called deep fades)!

EXAMPLE 3.19



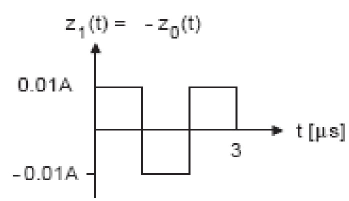
The signal $z_i(t) = s_i(t) * h(t)$ *is the output signal corresponding to the input signal* $s_i(t)$, $i = 0, 1$. *Determine and sketch* $z_0(t)$ *and* $z_1(t)$ *if* $\alpha_1 = 0.01$, $\alpha_2 = -0.01$, *and* $\alpha_3 = 0.01$.

Your conclusions concerning choice of bit rate to avoid overlapping signal alternatives after the channel?

Solution:

$$z_\ell(t) = \sum_{i=1}^{3} \alpha_i s_\ell(t - \tau_i) = 0.01 s_\ell(t) - 0.01 s_\ell(t - 10^{-6}) + 0.01 s_\ell(t - 2 \cdot 10^{-6}), \ \ell = 0, 1$$

yields,



Observe that the signal alternatives are changed significantly by the channel (filtering), and that the duration of both signal alternatives is increased from 1 [μs] before the channel, to 3[μs] after the channel!

If the bit rate is reduced to at most $10^6/3$ bps, *then no overlap of signal alternatives will exist after the channel.* □

5.34 Consider a communication system where N_t M-ary QAM signals are sent simultaneously (from N_t antennas). The n:th transmitted M-ary QAM signal is denoted $s_n(t)$,

$$s_n(t) = A(n)g(t)\cos(\omega_c t) - B(n)g(t)\sin(\omega_c t) \tag{5.133}$$

for $n = 1, 2, \ldots, N_t$. Note that the same carrier frequency is used for all N_t transmitted QAM signals!

The receiver is assumed to have N_r receiving antennas. The received signal $r_k(t)$ at the k:th receiving antenna is here modelled as

See (3.109)-(3.110)!

$$\begin{aligned} r_k(t) = \sum_{n=1}^{N_t} \Big(\big[H_{k,n}^{Re} A(n) - H_{k,n}^{Im} B(n)\big]\, g(t)\cos(\omega_c t) - \\ - \big[H_{k,n}^{Re} B(n) + H_{k,n}^{Im} A(n)\big]\, g(t)\sin(\omega_c t)\Big) + w_k(t) \end{aligned} \tag{5.134}$$

for $k = 1, 2, \ldots, N_r$. The variables $H_{k,n}^{Re}$ and $H_{k,n}^{Im}$ models how the n:th transmitted QAM signal is received at the k:th receiving antenna (attenuation and rotation of the I-Q components).

After I and Q demodulation of $r_k(t)$ to baseband, the receiver obtains the noisy signal space coordinates, here collected in r_k as

$$\begin{aligned} r_k = \underbrace{\sum_{n=1}^{N_t} \left(H_{k,n}^{Re} A(n) - H_{k,n}^{Im} B(n)\right)}_{\text{received } I \text{ component}} + j \underbrace{\sum_{n=1}^{N_t} \left(H_{k,n}^{Re} B(n) + H_{k,n}^{Im} A(n)\right)}_{\text{received } Q \text{ component}} \\ + \underbrace{(w_k^{Re} + j w_k^{Im})}_{\text{due to AWGN}} \end{aligned} \tag{5.135}$$

Note that complex notation ($j^2 = -1$) is used in (5.135)!

Let us now introduce the complex notations:

See (3.110)!

$$\begin{aligned} d_n &= A(n) + jB(n) \\ \alpha_{k,n} &= H_{k,n}^{Re} + jH_{k,n}^{Im} \\ w_k &= w_k^{Re} + jw_k^{Im} \end{aligned} \tag{5.136}$$

Then (5.135) can be formulated as,

$$r_k = \sum_{n=1}^{N_t} \alpha_{k,n} d_n + w_k \quad , k = 1, 2, \ldots, N_r \tag{5.137}$$

A compact formulation is now obtained as

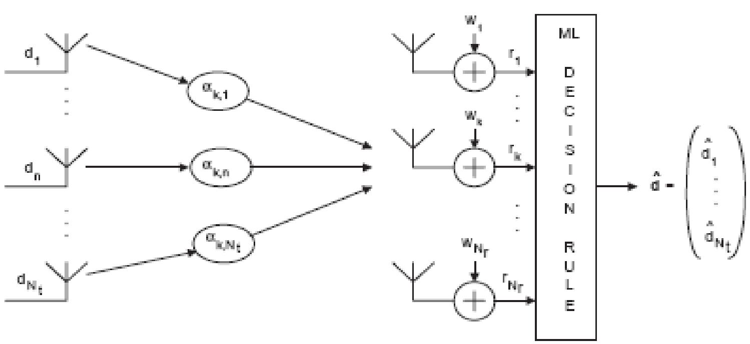
$$\boldsymbol{r} = \begin{pmatrix} r_1 \\ \vdots \\ r_{N_r} \end{pmatrix} = \boldsymbol{A} \begin{pmatrix} d_1 \\ \vdots \\ d_{N_t} \end{pmatrix} + \begin{pmatrix} w_1 \\ \vdots \\ w_{N_r} \end{pmatrix} = \boldsymbol{A}\boldsymbol{d} + \boldsymbol{w} \tag{5.138}$$

Very important!

where the $N_r \times N_t$ matrix $\boldsymbol{A}$ contains the channel coefficients $\{\alpha_{k,n}\}$. The relationship in (5.138) is a basic model in so-called multiple-input multiple-output (MIMO) systems.

The MIMO model is illustrated in the figure below,

MIMO MODEL



SISO
SIMO
MISO
MIMO
Diversity gain
Spatial multiplexing gain

$$r_k = \sum_{n=1}^{N_t} \alpha_{k,n} d_n + w_k$$

$$\boldsymbol{r} = \begin{pmatrix} r_1 \\ \vdots \\ r_{N_r} \end{pmatrix} = \boldsymbol{A} \begin{pmatrix} d_1 \\ \vdots \\ d_{N_t} \end{pmatrix} + \begin{pmatrix} w_1 \\ \vdots \\ w_{N_r} \end{pmatrix} = \boldsymbol{A}\boldsymbol{d} + \boldsymbol{w}$$

z=Ad is the received signalpoint and w is the additive noise vector.

64-QAM+Nt=8 (48bits): ML symbol decision rule ....?

Chapter 8

Trellis-coded Signals

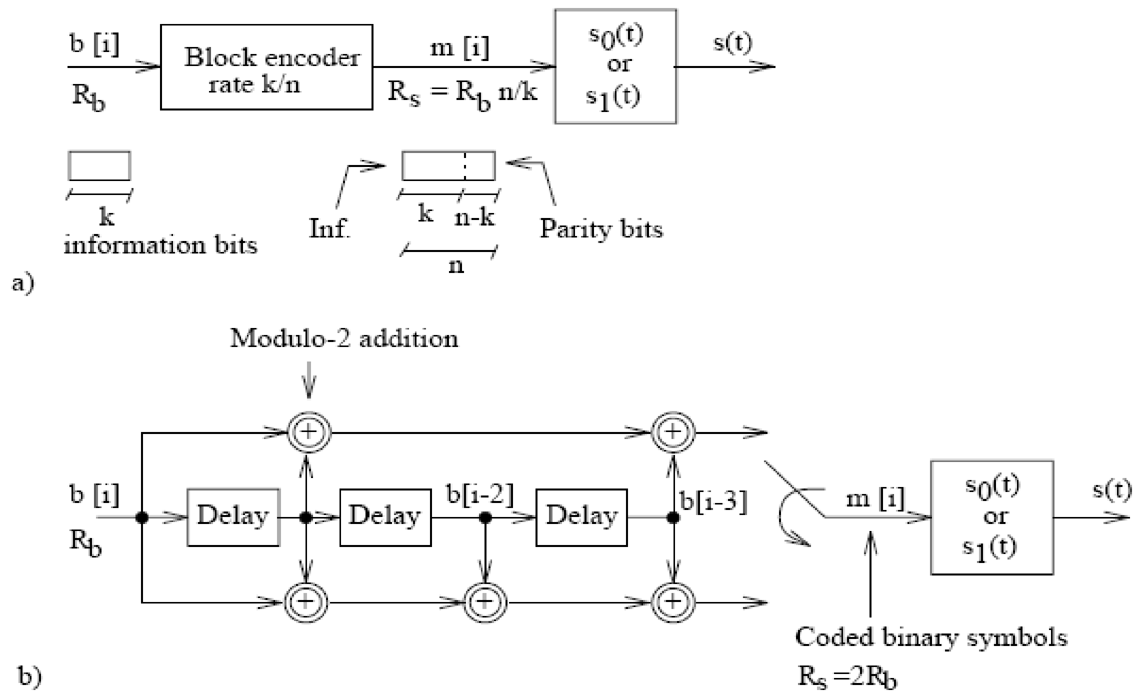


Figure 8.1: a) Block coding, $r_c = k/n$. b) Convolutional coding, $r_c = 1/2$.

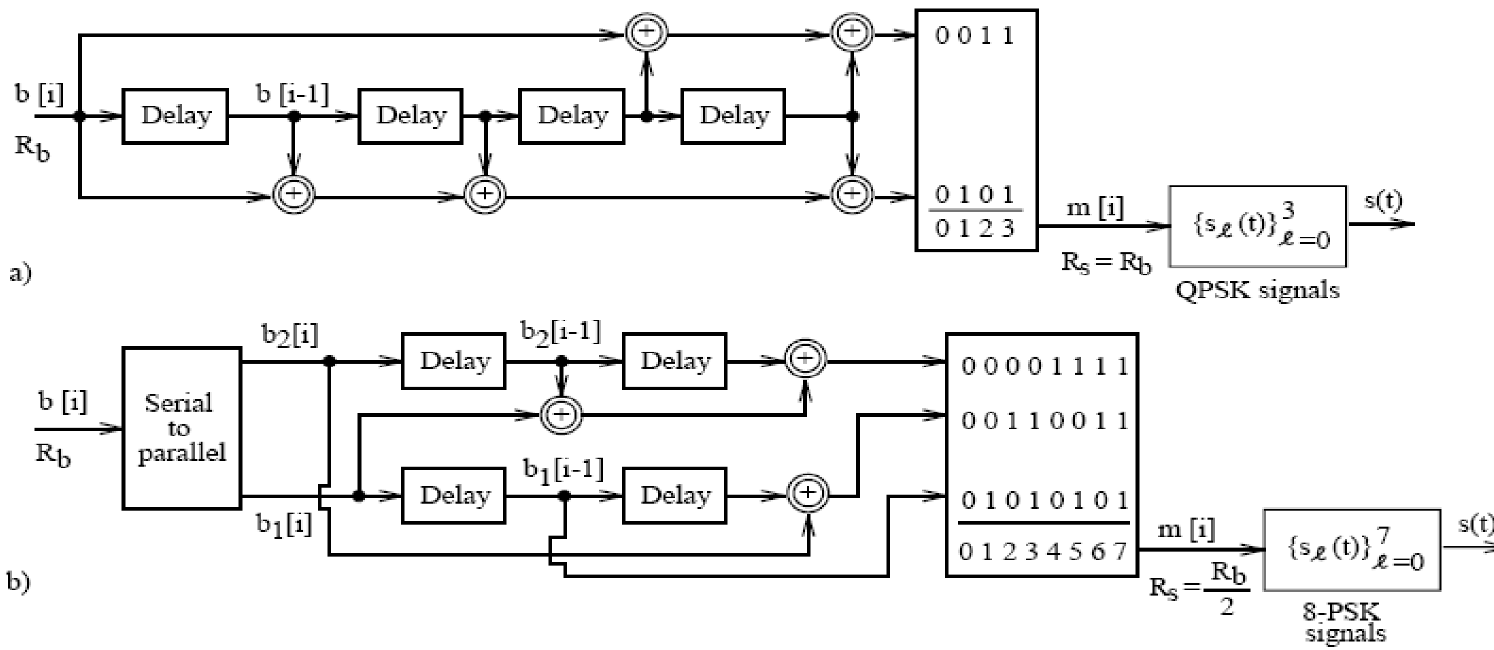
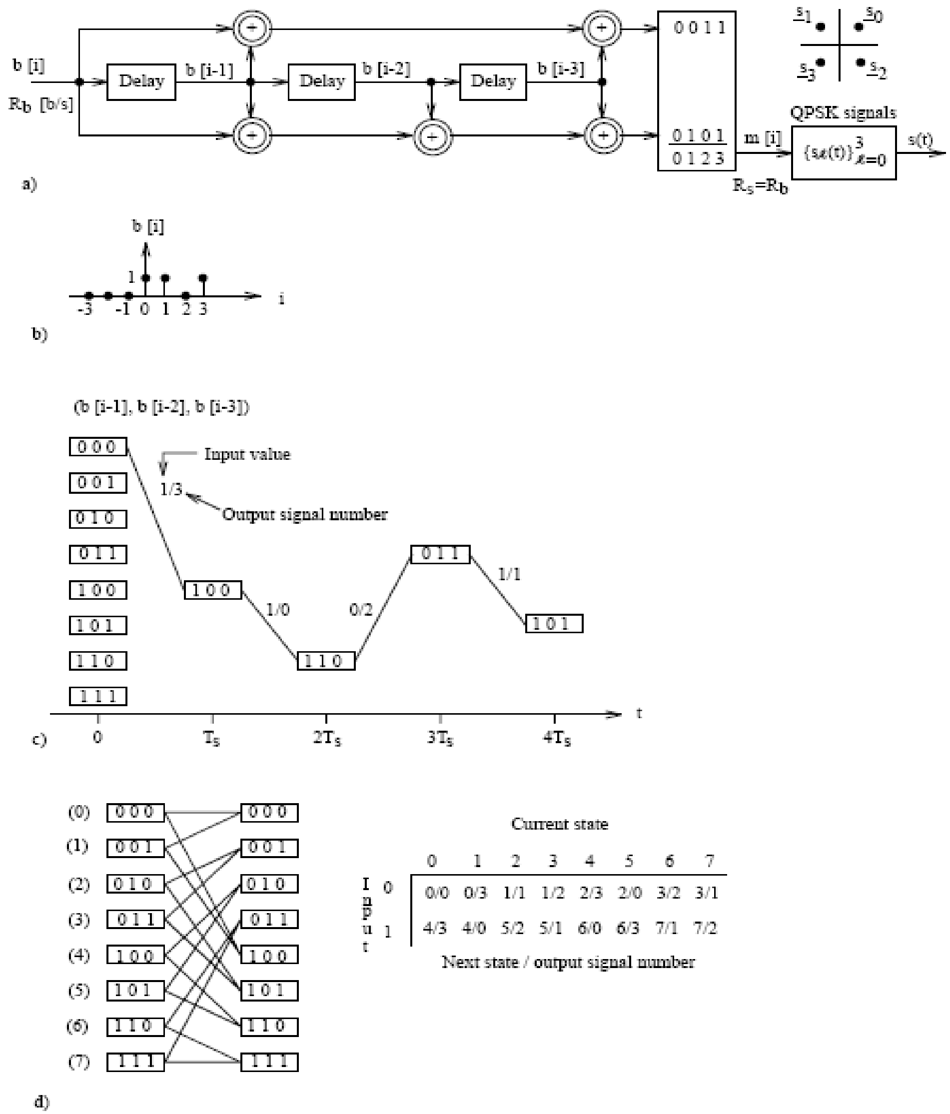
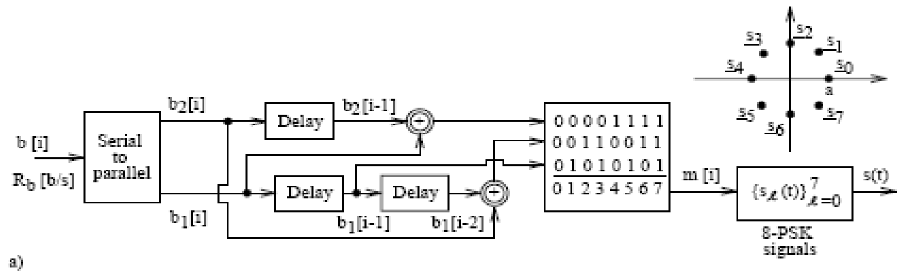


Figure 8.2: a) Rate $r_c = 1/2$ convolutional encoder combined with QPSK; b) Rate $r_c = 2/3$ convolutional encoder combined with 8-PSK, from [63], [64].



Input	0	1	2	3	4	5	6	7
0	0/0	0/3	1/1	1/2	2/3	2/0	3/2	3/1
1	4/3	4/0	5/2	5/1	6/0	6/3	7/1	7/2

Figure 8.4: a) A rate 1/2 convolutional encoder combined with QPSK signal alternatives; b) A specific input sequence $b[i]$; c) The corresponding path in the trellis; d) A trellis section, and a table containing all relevant parameters.



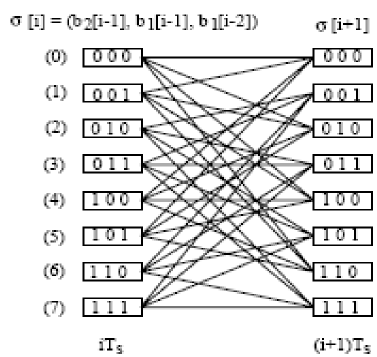
a)

Current state σ [i]

INPUT $\begin{pmatrix} b_2[i] \\ b_1[i] \end{pmatrix}$	(000) 0	(001) 1	(010) 2	(011) 3	(100) 4	(101) 5	(110) 6	(111) 7
$\binom{0}{0}$	0/0	0/2	1/1	1/3	0/4	0/6	1/5	1/7
$\binom{0}{1}$	2/4	2/6	3/5	3/7	2/0	2/2	3/1	3/3
$\binom{1}{0}$	4/2	4/0	5/3	5/1	4/6	4/4	5/7	5/5
$\binom{1}{1}$	6/6	6/4	7/7	7/5	6/2	6/0	7/3	7/1

σ [i+1] / m [i]

b)



c)

Figure 8.6: a) An example of TCM, from [63]–[64]; b) The mappings $F(\cdot,\cdot)$ and $G(\cdot,\cdot)$; c) A trellis section.

Memory (redundancy, dependancy) is introduced among the sent signal alternatives!

This gives us some new properties like, e.g.,:

8.10

Which of the following signal sequences are impossible?

1. $s_3(t), s_2(t-T_b), s_1(t-2T_b), s_1(t-3T_b)$
2. $s_3(t), s_2(t-T_b), s_2(t-2T_b), s_1(t-3T_b)$
3. $s_3(t), s_1(t-T_b), s_0(t-2T_b), s_2(t-3T_b)$
4. $s_3(t), s_1(t-T_b), s_3(t-2T_b), s_1(t-3T_b)$

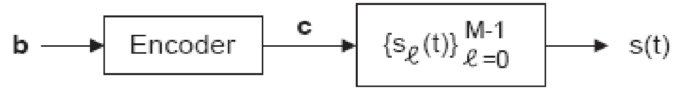
Note: In the uncoded case all signal sequences are possible.

Find the "missing" signal, in the sequence below,

$$s_1(t), s_3(t-T_b), \ ? \ , s_2(t-3T_b), s_3(t-4T_b), s_0(t-5T_b)$$

Note: This is not possible to do in the uncoded case!

2.32 Let us here study adaptive coding and modulation according to the block diagram below.



$$\bar{E}_{sent} = r_c \log_2(M) E_{b,sent} = \frac{k}{n} \log_2(M) E_{b,sent} \tag{8.4}$$

$$R_s = 1/T_s = \frac{1}{r_c} \cdot \frac{1}{\log_2(M)} \cdot R_b = \frac{1}{k/n} \cdot \frac{1}{\log_2(M)} \cdot R_b \tag{8.5}$$

$$W = c \cdot R_s \tag{8.6}$$

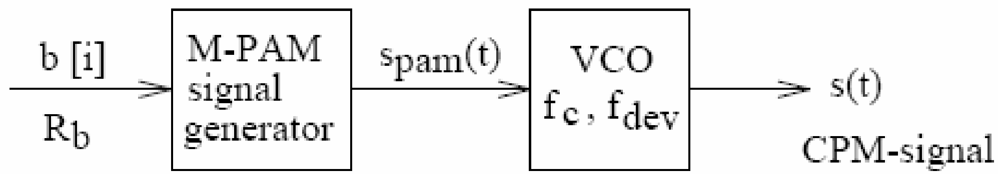
Typically, the bandwidth W is fixed and given but:
 the rate of the encoder
 the number of signal alternatives
 and the bit rate can be **ADAPTIVE**, see (8.5)-(8.6)!

We have memory in the sequence of sent signal alternatives!

Some sequences are impossible, see problem!

Only ”good” sequences are sent!

8.2.1 Continuous Phase Modulation (CPM)



(GSM, Bluetooth)

$$s_{pam}(t) = \sum_{n=-\infty}^{\infty} \alpha_n g(t - nT_s), \quad -\infty \leq t \leq \infty \tag{8.7}$$

$$\boxed{\begin{aligned} s(t) &= \sqrt{\frac{2E}{T_s}} \cos(\omega_c t + \theta(t)) \\ \theta(t) &= 2\pi h \sum_{n=-\infty}^{\infty} \alpha_n q(t - nT_s) + \theta_0 \end{aligned}} \quad , \quad \text{CPM} \tag{8.20}$$

Instantaneous frequency (”local frequency”):

$$\boxed{f_{ins}(t) = f_c + \frac{1}{2\pi} \frac{d\theta(t)}{dt} = f_c + f_{dev} s_{pam}(t) = f_c + f_{dev} \sum_{n=-\infty}^{\infty} \alpha_n g(t - nT_s)} \tag{8.12}$$

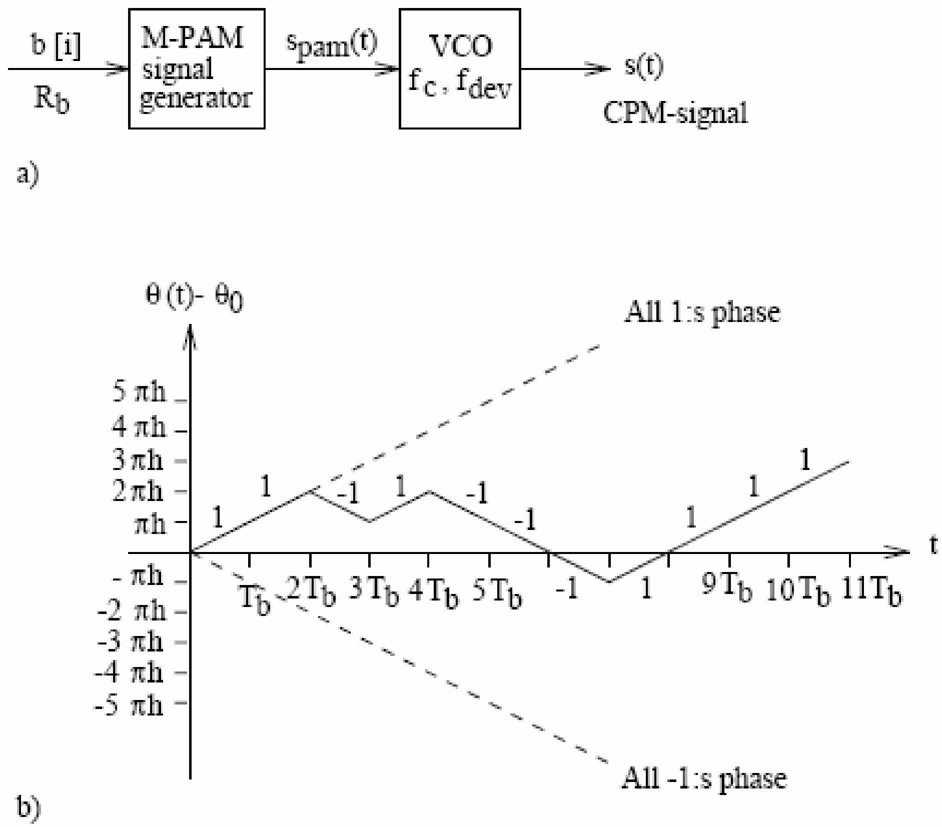


Figure 8.7: a) Generation of a CPM signal; b) Binary CPFSK, examples of phase functions.

Note: Continuous phase implies that there is <u>*memory*</u> in the signal!!

Examples of pulse shapes:

$$g_{rec}(t) = \begin{cases} 1/2LT_s & , \quad 0 \leq t \leq LT_s \\ 0 & , \quad \text{otherwise} \end{cases} \tag{8.9}$$

$$g_{rc}(t) = \begin{cases} [1 - \cos(2\pi t/LT_s)]/2LT_s & , \quad 0 \leq t \leq LT_s \\ 0 & , \quad \text{otherwise} \end{cases} \tag{8.10}$$

In these expressions, L is a positive integer which is greater or equal to 1. If $L = 1$, then the method is referred to as **full response** signaling, and if $L \geq 2$ then we have so-called **partial response** signaling, [2], [43].

<u>Important special cases:</u>

M-ary CPFSK **(continuous phase frequency shift keying) means L=1 and a rectangular pulse shape.**

MSK **(minimum shift keying)**

Means h=1/2 + binary CPFSK

$$\theta(t) = 2\pi f_{dev} \sum_{n=-\infty}^{\infty} \alpha_n q(t - nT_s) + \theta_0 \qquad (8.15)$$

$$q(t) = \int_0^t g(x)dx \qquad (8.14)$$

$$q_{rec}(t) = \int_0^t g_{rec}(x)dx = \begin{cases} 0 & , \quad t \leq 0 \\ t/2LT_s & , \quad 0 \leq t \leq LT_s \\ 1/2 & , \quad t \geq LT_s \end{cases} \qquad (8.16)$$

$$q_{rc}(t) = \int_0^t g_{rc}(x)dx = \begin{cases} 0 & , \quad t \leq 0 \\ \frac{t}{2LT_s} - \frac{1}{4\pi} \sin(2\pi t/LT_s) & , \quad 0 \leq t \leq LT_s \\ 1/2 & , \quad t \geq LT_s \end{cases} \qquad (8.17)$$

$$\boxed{\begin{aligned} s(t) &= \sqrt{\frac{2E}{T_s}}\cos(\omega_c t + \theta(t)) \\ \theta(t) &= 2\pi h \sum_{n=-\infty}^{\infty} \alpha_n q(t - nT_s) + \theta_0 \end{aligned}} \quad , \quad \text{CPM} \qquad (8.20)$$

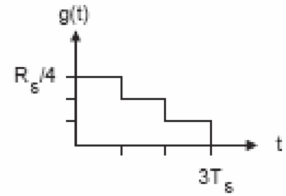
$$\boxed{f_{ins}(t) = f_c + \frac{1}{2\pi}\,\frac{d\theta(t)}{dt} = f_c + f_{dev} s_{pam}(t) = f_c + f_{dev} \sum_{n=-\infty}^{\infty} \alpha_n g(t - nT_s)} \qquad (8.12)$$

Instantaneous frequency (”local frequency”) within a symbol interval:

$$\begin{aligned} f_{ins}(t) &= f_c + h \sum_{n=-\infty}^{\infty} \alpha_n g(t - nT_s) \overset{\substack{\text{CPFSK} \\ \downarrow}}{=} f_c + \alpha_i \frac{hR_s}{2} = \\ &= f_c + \alpha_i \frac{hR_b}{2\log_2(M)} \, , \quad iT_s \le t \le (i+1)T_s \qquad (8.22) \end{aligned}$$

EXAMPLE 8.7

Assume binary CPM, where the pulse $g(t)$ is,



Which frequencies are possible in a symbol interval if the bit rate is 10 kbit/s and $h = 1/4$?

Solution:

Let us study the symbol interval $iT_s \leq t \leq (i+1)T_s$:

$$\begin{aligned} f_{ins}(t) &\overset{(8.22)}{=} f_c + h \sum_{n=-\infty}^{\infty} \alpha_n g(t - nT_s) = \\ &= f_c + h\,\frac{R_s}{4}\left(\frac{\alpha_{i-2}}{3} + \alpha_{i-1}\,\frac{2}{3} + \alpha_i\right) = f_c + \frac{10^4}{16} \cdot x \end{aligned}$$

where

$$x = \frac{\alpha_{i-2}}{3} + \alpha_{i-1}\,\frac{2}{3} + \alpha_i = \begin{cases} \begin{array}{rcrrrl} -2 & , & -1 & -1 & -1 & (\alpha_{i-2}, \alpha_{i-1}, \alpha_i) \\ 0 & , & -1 & -1 & 1 & \\ -2/3 & , & -1 & 1 & -1 & \\ 4/3 & , & -1 & 1 & 1 & \\ -4/3 & , & 1 & -1 & -1 & \\ 2/3 & , & 1 & -1 & 1 & \\ 0 & , & 1 & 1 & -1 & \\ 2 & , & 1 & 1 & 1 & \end{array} \end{cases}$$

Hence, the possible frequencies in a symbol interval $iT_s \leq t \leq (i+1)T_s$ are:

$f_c - 1250\ Hz$
$f_c - 833.33\ Hz$
$f_c - 416.67\ Hz$
f_c
$f_c + 416.67\ Hz$
$f_c + 833.33\ Hz$
$f_c + 1250\ Hz$ $\square$

$$\theta(t) = 2\pi f_{dev} \sum_{n=-\infty}^{\infty} \alpha_n q(t - nT_s) + \theta_0 \qquad (8.15)$$

The phase within the i:th symbol interval:

Phase = phase continuity + due to pulseoverlap + due to the current input datasymbol

$$\theta(t) = \pi h \sum_{n=-\infty}^{i-L} \alpha_n + \underbrace{2\pi h \sum_{n=i-L+1}^{i-1} \alpha_n q(t - nT_s)}_{\text{only if } L \geq 2} + 2\pi h \alpha_i q(t - iT_s) + \theta_0 \ ,$$

$$iT_s \leq t \leq (i+1)T_s \qquad (8.23)$$

The state of the CPM signal:

$$\boxed{\sigma[i] = \left(\left\{ \pi h \sum_{n=-\infty}^{i-L} \alpha_n \right\}_{\text{mod}2\pi} , \ \underbrace{\alpha_{i-L+1}, \alpha_{i-L+2}, \ldots, \alpha_{i-1}}_{\text{only if } L \geq 2} \right)} \qquad (8.25)$$

$$\sigma[i+1] = \left(\left\{ \pi h \sum_{n=-\infty}^{i-L} \alpha_n + \pi h \alpha_{i+1-L} \right\}_{\text{mod}2\pi} , \ \alpha_{i-L+2}, \alpha_{i-L+3}, \ldots, \alpha_i \right) \qquad (8.26)$$

define the current state $\sigma_z[i]$ of the received signal $z(t)$ at $t = iT_s$ as,

$$\boxed{\sigma_z[i] = \{\sigma_{tra}[i - L_x + 1],\ \underbrace{b[i - L_x + 1],\ b[i - L_x + 2], \ldots, b[i-1]}_{\text{only if } L_x \geq 2}\}} \tag{8.35}$$

In this expression, $b[\ell]$ denotes the current k-tuple of information bits to the transmitter at time ℓT_s. So, in (8.35), the state $\sigma_z[i]$ consists of the state of the transmitter at time $t = (i - L_x + 1)T_s$, together with the $(L_x - 1)$ previous k-tuples of information bits. Hence, the number of states $\mathcal{S}_z$ is,

$$\mathcal{S}_z = \mathcal{S}_{tra} 2^{k(L_x - 1)} \tag{8.36}$$