ETSN10 Formula Sheet

Probability

$$P(\bar{A}) = 1 - P(A)$$
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

A, B mutually exclusive:

$$P(A \cap B) = 0$$

A, B independent:

$$P(A \cap B) = P(A)P(B)$$

CDF:

$$F(x) = P(X \leq x)$$

PDF:

$$p_k = p(k) = P(X = k)$$

Discrete random variables

Mean:

$$\mu = \sum kp_k$$

Variance:

$$Var(X) = \sum (k - \mu)^2 p_k = E[X^2] - (E[X])^2$$

Standard deviation:

$$\sigma = \sqrt{(Var(X))}$$

$$E[aX + b] = aE[X] + bVar(aX + b) = a^2 Var(X)$$

Bernoulli random variable:

$$p_0 = 1 - p$$
$$p_1 = p$$
$$\mu = p$$
$$\sigma^2 = p(1 - p)$$

Geometric:

$$p_k = (1 - p)^{k-1} p$$
$$\mu = \frac{1}{p}$$
$$Var(X) = \frac{1 - p}{p^2}$$

Poisson random variable:

$$p_k = e^{-\lambda} \frac{\lambda^k}{k!}$$
$$\mu = \lambda$$
$$\sigma^2 = \lambda$$

Continuous random variables

Mean:

$$\mu = \int xp(x)\mathrm{d}x$$

Variance:

$$Var(X) = \int (x - \mu)^2 p(x)\mathrm{d}x$$

Negative exponential:

$$p(x) = \alpha e^{-\alpha x}$$
$$P(X \leq x) = 1 - e^{-\alpha x}$$
$$\mu = \frac{1}{\alpha}$$
$$Var(X) = \left(\frac{1}{\alpha}\right)^2$$

Gaussian:

$$p(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

Multiple random variables

Joint CDF:

$$F(x, y) = P(X \leq x, Y \leq y)$$

X, Y independent:

$$F(x, y) = F(x)F(y)$$

$$E[X + Y] = E[X] + E[Y]$$
$$E[XY] = E[X]E[Y]$$

$$Var(X + Y) = Var(X) + Var(Y)$$

Properties of random variables X, Y, constants a, b:

$$E[aX + b] = aE[X] + b$$
$$Var(aX + b) = a^2 Var(X)$$
$$E[X + Y] = E[X] + E[Y]$$

Covariance:

$$Cov(X, Y) = E[(X - E[X])(Y - E[Y])] = E[XY] - E[X]E[Y]$$

Correlation coefficient:

$$r(X, Y) = \frac{Cov(X, Y)}{\sigma_x \sigma_y}$$

Stochastic Processes

Autocorrelation:

$$R(t_1, t_2) = E[x(t_1)x(t_2)]$$

Autocovariance:

$$C(t_1, t_2) = Cov(x(t_1), x(t_2))$$

Sampling and random numbers

Sample mean:

$$\bar{z} = \frac{1}{n}\sum_{i=1}^{n} z_i$$

Sample variance:

$$\hat{V} = \frac{1}{n}\sum_{i=1}^{n}(z_i - \bar{z})^2$$

Unbiased sample variance (Bessel's correction):

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(z_i - \bar{z})^2$$

Confidence intervals:

$$\text{Confidence} = Pr(|\bar{z} - \mu| \leq \alpha \times \frac{\sigma}{\sqrt{n}}$$

$\alpha = 1.96$ gives confidence of 95%.
Inverse method:

$$X = F^{-1}(Y)$$

Medium Access Control

FDMA/TDMA rate of work:

$$\eta = \frac{1}{n \times \nu}\sum_{i=1}^{n}\rho_i$$

Polling efficiency:

$$E = \frac{T_t}{T_t + T_{idle} + T_{poll}}$$

ALOHA throughput:

$$S = Ge^{-G}$$

Slotted ALOHA throughput:

$$S = Ge^{-2G}$$

1-persistent CSMA throughput:

$$S = \frac{Ge^{-G}(1+G)}{G + e^{-G}}$$

Non-persistent CSMA throughput:

$$S = \frac{G}{1+G}$$

CSMA utilisation:

$$A = kp(1-p)^{k-1}$$

$$E[w] = \frac{1-A}{A}$$

$$U = \frac{1}{1 + 2a + a(1-A)/A}$$

Queueing Systems

Kendall Notation parameters:

1. Arrival distribution
2. Service distribution
3. Number of servers
4. Total capacity (default: infinite)
5. Population size (default: infinite)
6. Service disciplien (default: FIFO)

Little's Law:

$$E[R] = \lambda E[T_R]$$

Occupancy:

$$\rho = \frac{\lambda}{\mu}$$

M/M/1

Number of items in the system:

$$E[R] = \frac{\rho}{1-\rho}$$

Total time in system:

$$E[T_R] = \frac{1}{\mu(1-\rho)}$$

Waiting time:

$$E[T_W] = \frac{\rho}{\mu(1-\rho)}$$

Delay bound:

$$Pr(T_R \leq t) = 1 - e^{-\mu(1-\rho)t}$$

M/M/2

$$E[T_R] = \frac{1}{\mu(1+\rho)(1-\rho)}$$

M/M/1/n

Blocking probability:

$$P_B = \frac{1-\rho}{1-\rho^{n+1}}\rho^n$$

Carried traffic:

$$\gamma = \lambda(1-P_B)$$

Little's Law:

$$E[R] = \gamma E[T_R]$$

Number of items in the system:

$$E[R] = \frac{1-\rho}{1-\rho^{n+1}}\sum_{i=0}^{n} i\rho^i$$

Erlang (M/M/n/n)

Offered traffic:

$$A = \lambda h$$

Service rate:

$$\mu = \frac{1}{h}$$

Erlang loss function:

$$E_n(A) = P_B = \frac{\frac{A^n}{n!}}{\sum_{j=0}^{n}\frac{A^j}{j!}}$$

Carried traffic:

$$A_c = A(1-P_B)$$

Lost traffic:

$$A - A_c$$

Jackson Networks

Probability a packet leaves the network from node i:

$$1 - \sum_{j=1}^{n} r_{ij}$$

Total arrival rate to node i:

$$\lambda_i = \gamma_i + \sum_{j=1}^{n}\lambda_j r_{ji}$$

Queueing Disciplines

Kleinrock Conservation Law:

$$\sum_{n=1}^{N}\rho_n q_n = C$$

Processor Sharing:

$$F_i^\alpha = S_i^\alpha + P_i^\alpha$$

$$S_i^\alpha = \max\{F_{i-1}^\alpha, A_i^\alpha\}$$

Generalised Processor Sharing:

$$F_i^\alpha = S_i^\alpha + \frac{P_i^\alpha}{w_\alpha}$$

Network Architectures

Packet reception rate:

$$PRR = \frac{S}{T}$$

Cellular frequency re-use:

$$D = R\sqrt{3K}$$

Congestion Control

Token bucket:

$$R = \rho T + \beta$$

Random Early Discard (RED)

$$P_b = P_{max}\frac{(avg - Th_{min}}{Th_{max} - Th_{min}}$$

$$P_a = \frac{P_b}{1 - count \times P_b}$$

TCP

Max bandwidth:

$$BW_{max} = \frac{MSS \times C}{RTT \times \sqrt{p}} \qquad C = \sqrt{\frac{3}{2}}$$

Normalised throughput:

$$S = \begin{cases} 1 & W \geq 2RD \\ \frac{W}{2RD} & W < 2RD \end{cases}$$

Expected (average) round trip time:

$$ERTT(K+1) = \frac{K}{K+1}ERTT(K) + \frac{1}{K+1}RTT(K+1)$$

Smoothed round trip time:

$$SRTT(K+1) = \alpha \times SRTT(K) + (1-\alpha) \times RTT(K+1)$$

Retransmission timeout with SRTT:

$$RTO(K+1) = \min\{UB, \max\{LB, \beta \times SRTT(K+1)\}\}$$

Van Jacobson's algorithm:

$$DRTT(K+1) = (1-\alpha) \times DRTT(K) + \alpha \times (SRTT - ERTT)$$

$$RTO = ERTT + 4 \times DRTT$$

Exponential backoff:

$$RTO_{i+1} = q \times RTO_i$$

Other useful formulas

Geometric series:

$$\sum_{k=0}^{n-1} ar^k = a\frac{1-r^n}{1-r}$$

$$\sum_{k=0}^{\infty} ar^k = \frac{a}{1-r}$$

Expectation of geometric series:

$$\sum_{k=0}^{\infty} kar^k = \frac{ar}{(1-r)^2}$$