ETSF15
Physical layer communication

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Physical layer

- Analog vs digital (Previous lecture)
- Transmission media
- Modulation
  - Represent digital data in a continuous world
- Disturbances, Noise and distortion
- Information

Transmission media

Guided media

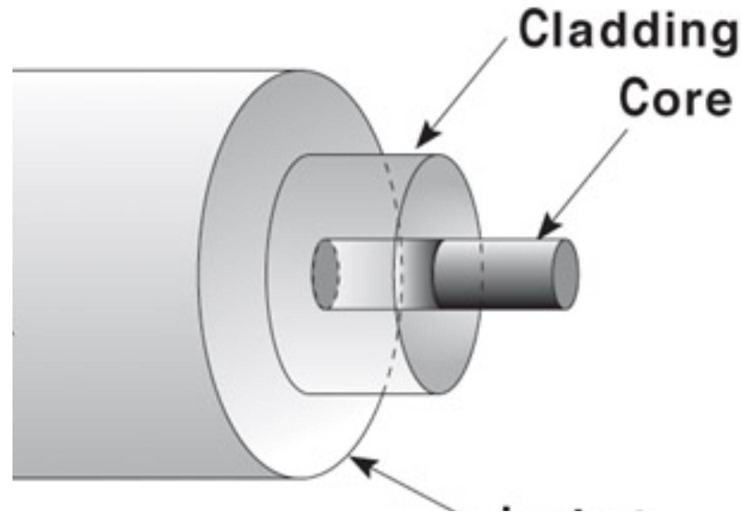
- Fibre optic cable
- Twisted pair copper cables
- Coax cable

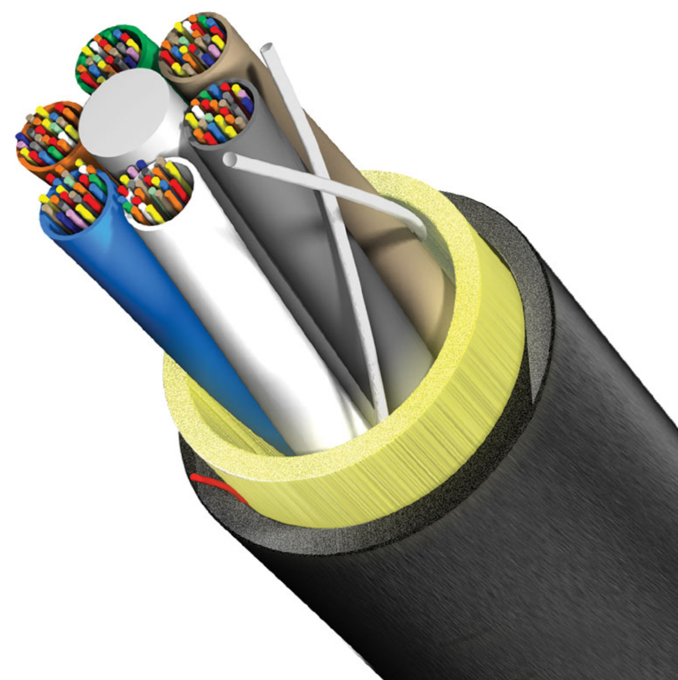
Unguided media

- Radio
- Microwave
- Infra red

Fibre optic

- Transmission is done by light in a glass core (very thin)
- Total reflection from core to cladding
  - Multi-mode (typ 50-100 um)
  - Single-mode (typ 5-10 um)
- Very high capacity
- Not disturbed by radio signals





Optical network architekture

Point to point

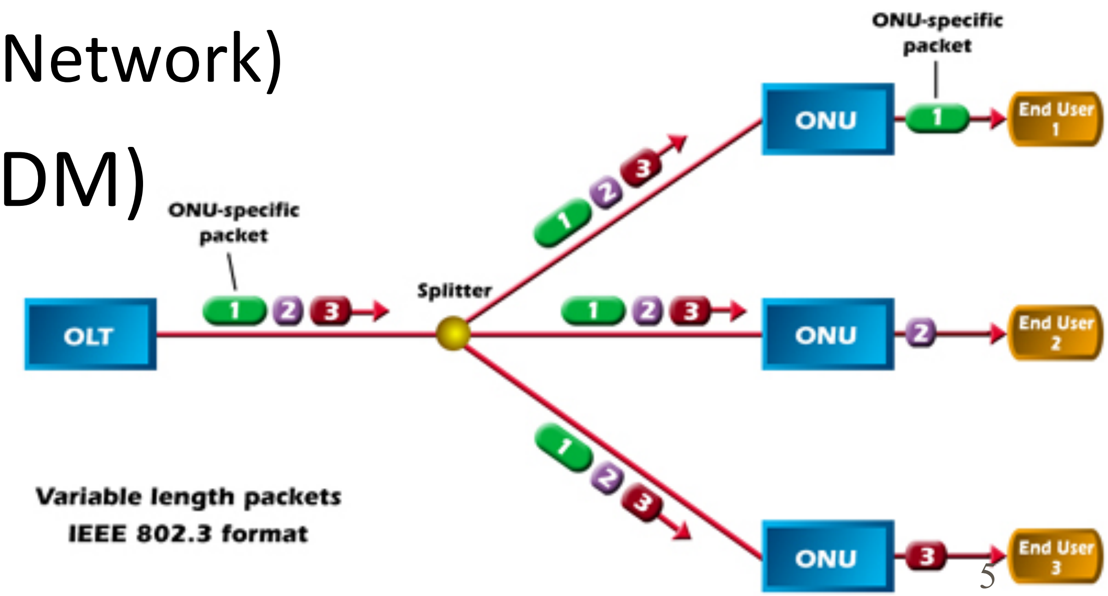
- Two nodes are connected by one dedicated fibre

Point to multi-point

- One point is connected to several end nodes
  - PON (Passive Optical Network)

Wavelength division (WDM)

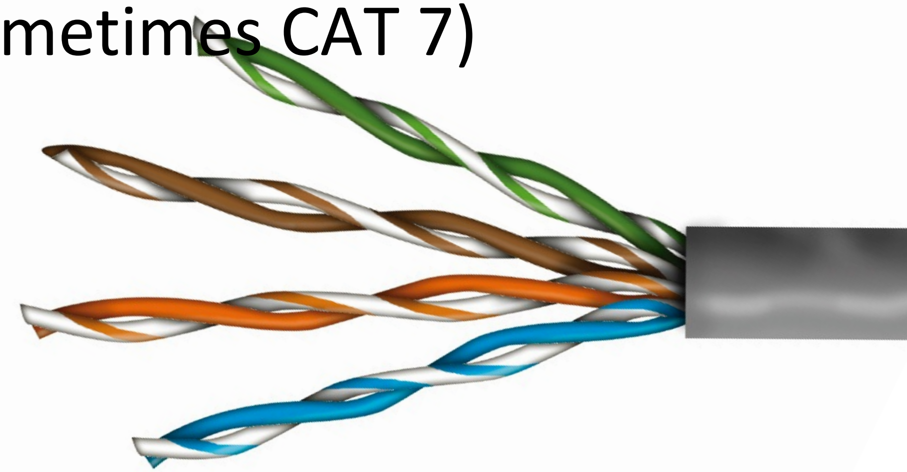
- Physical P2P
- Logical P2MP



Twisted pair copper cables

Two copper lines twisted around each other

- Twisting decreases disturbances (and emission)
- Used for
  - Telephony loop (CAT3)
  - Ethernet (CAT5, CAT6 and sometimes CAT 7)



Coax cable

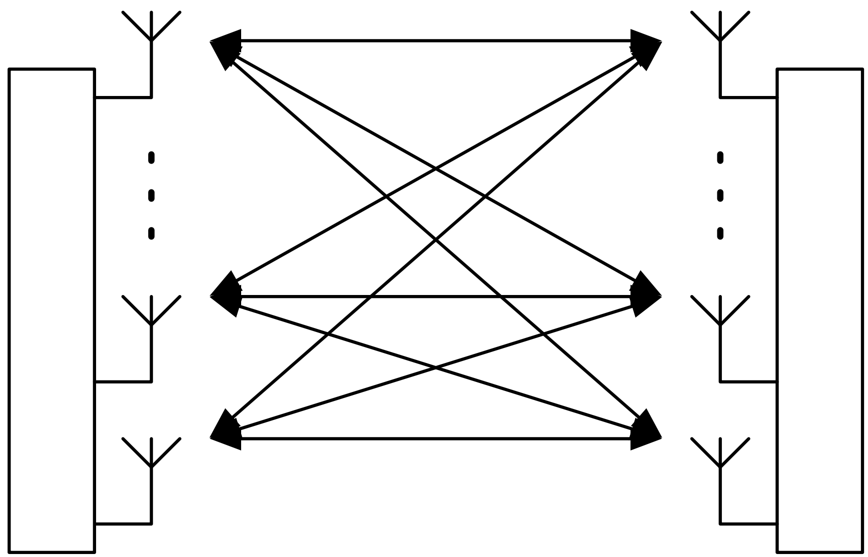
One conductor surrounded by a shield

- Used for
  - Antenna signals
  - Measurement instrumentations

Radio structures

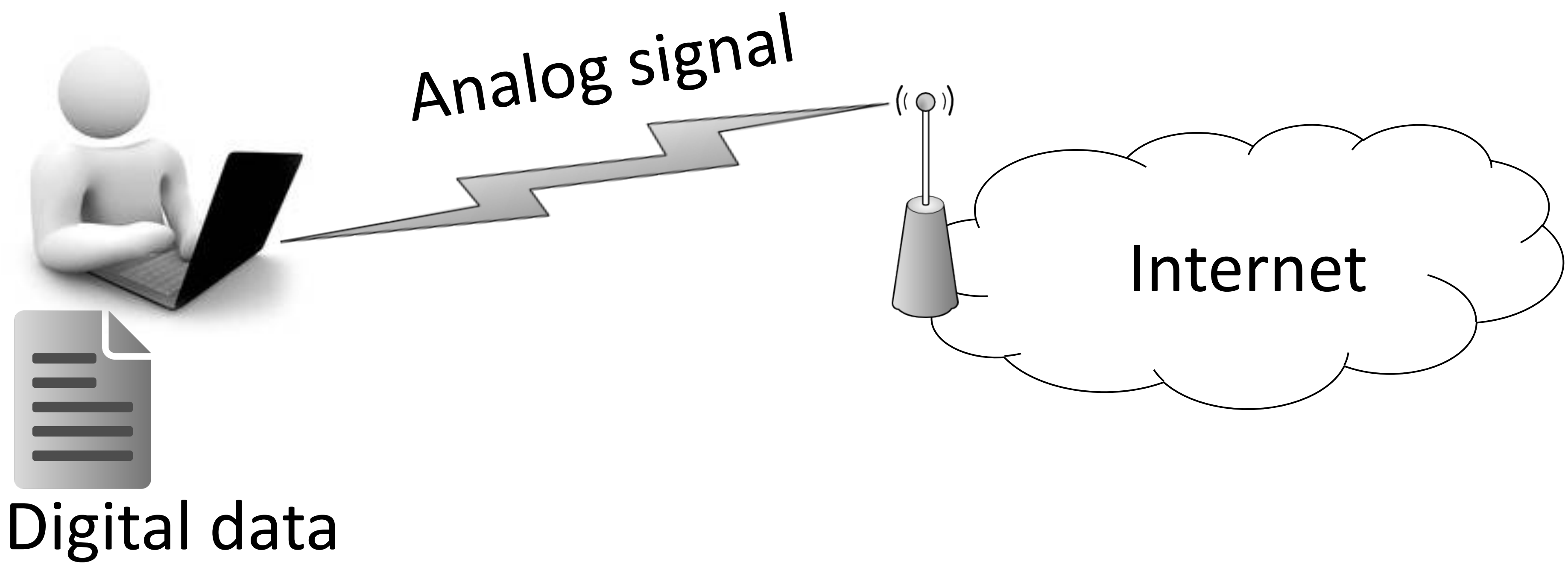
- Single antenna system

- MIMO (Multiple In Multiple Out)



From bits to signals

- Principles of digital communications

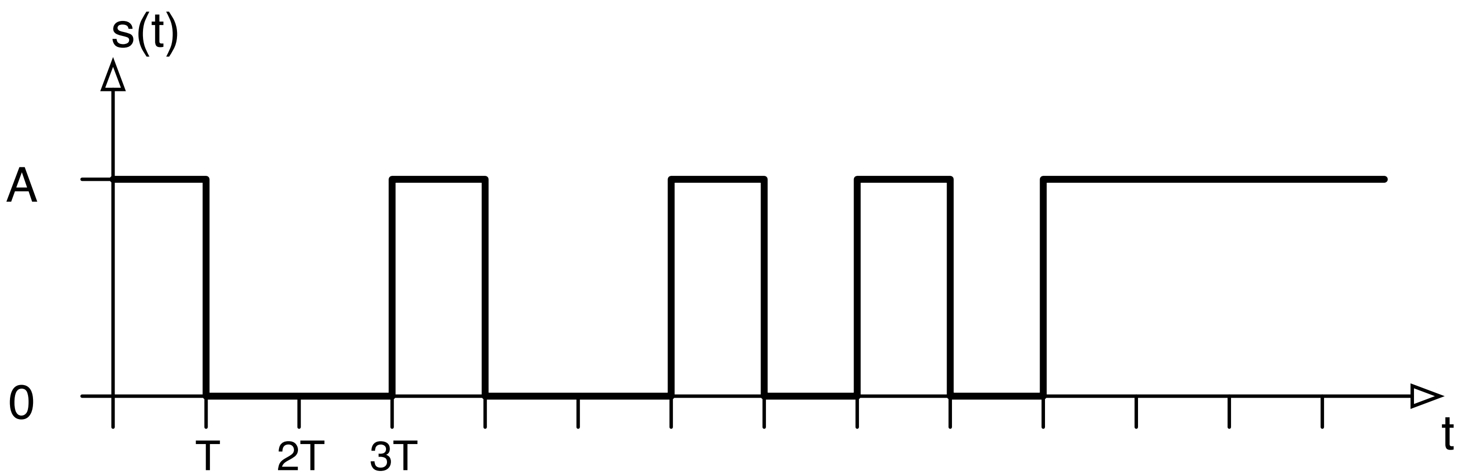


On-off keying

- Send one bit during T_b seconds and use two signal levels, "on" and "off", for 1 and 0.

$$a(t) = A \cdot x \qquad 0 \leq t \leq T_b$$

Ex. x=10010010101111100



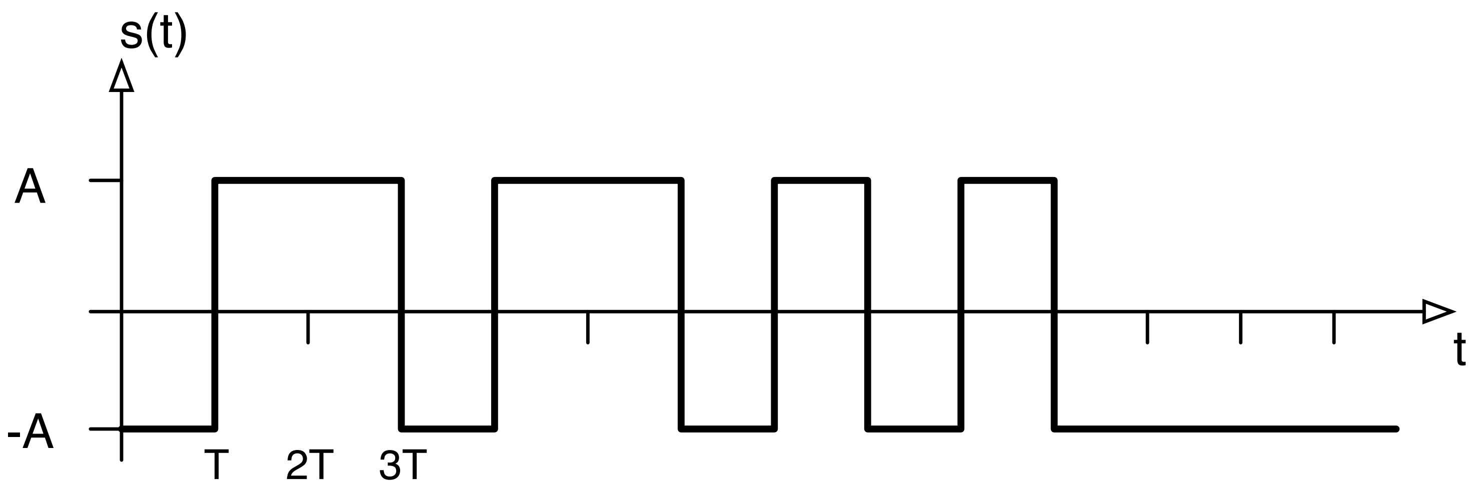
Non-return to zero (NRZ)

- Send one bit during T_b seconds and use two signal levels, *+A* and *-A*, for 0 and 1.

$$a(t) = A \cdot (-1)^x \quad 0 \le t \le T_b$$

Ex.

x=1001001010111100



Description of general signal

With the pulse form $g(t) = A, 0 \leq t < T_s$, the signals can be described as

$$s(t) = \sum_n a_n g(t - nT_s)$$

Two signal alternatives

On-off keying

$$a_n = x_n \qquad \Rightarrow s_0(t) = 0 \text{ and } s_1(t) = g(t)$$

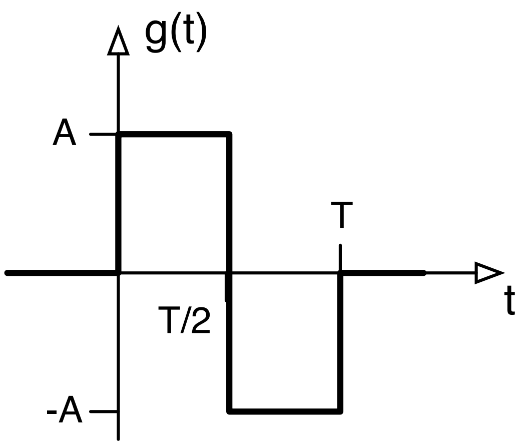
NRZ

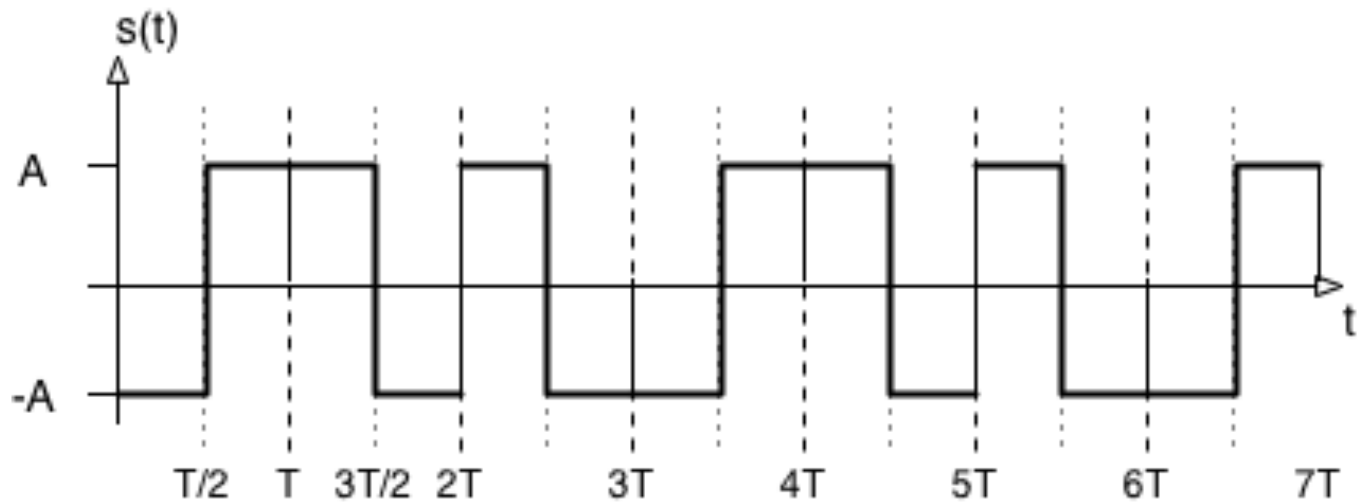
$$a_n = (-1)^{x_n} \Rightarrow s_0(t) = g(t) \text{ and } s_1(t) = -g(t)$$

Manchester coding

- To get a zero passing in each signal time, split the pulse shape *g(t)* in two parts and use +/- as amplitude.

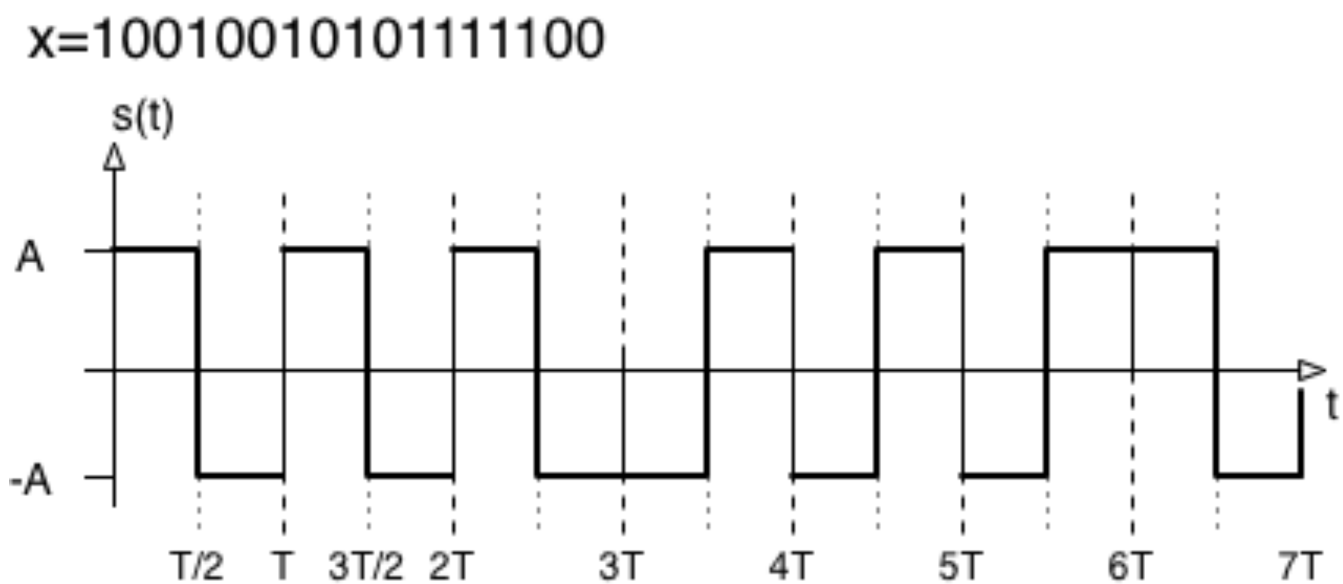
Ex.





Differential Manchester coding

- Use a zero transition at the start to indicate the data.
- For a transmitted 0 the same pulse as previous slot is used, while for a transmitted 1 the inverted pulse is used, i.e. $a_n = a_{n-1}(-1)^{x_n}$

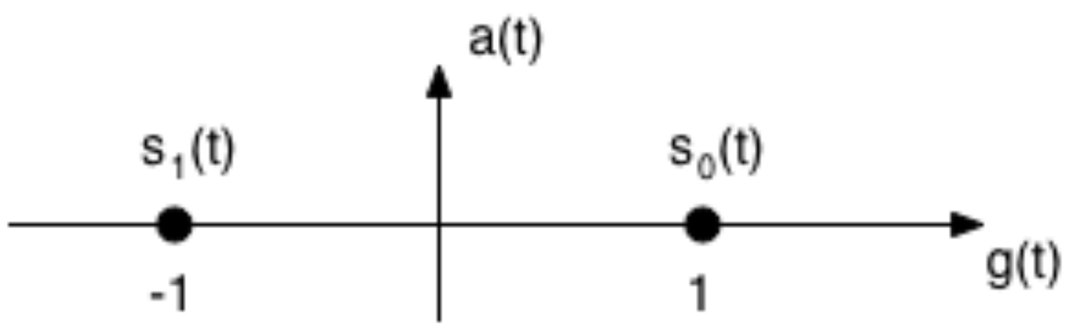


PAM (Pulse Amplitude Modulation)

- NRZ and Manchester are forms of binary PAM
- The data is stored in the amplitude and transmitted with a pulse shape $g(t)$

$$a(t) = a_n \cdot g(t) \qquad a_n = (-1)^x$$

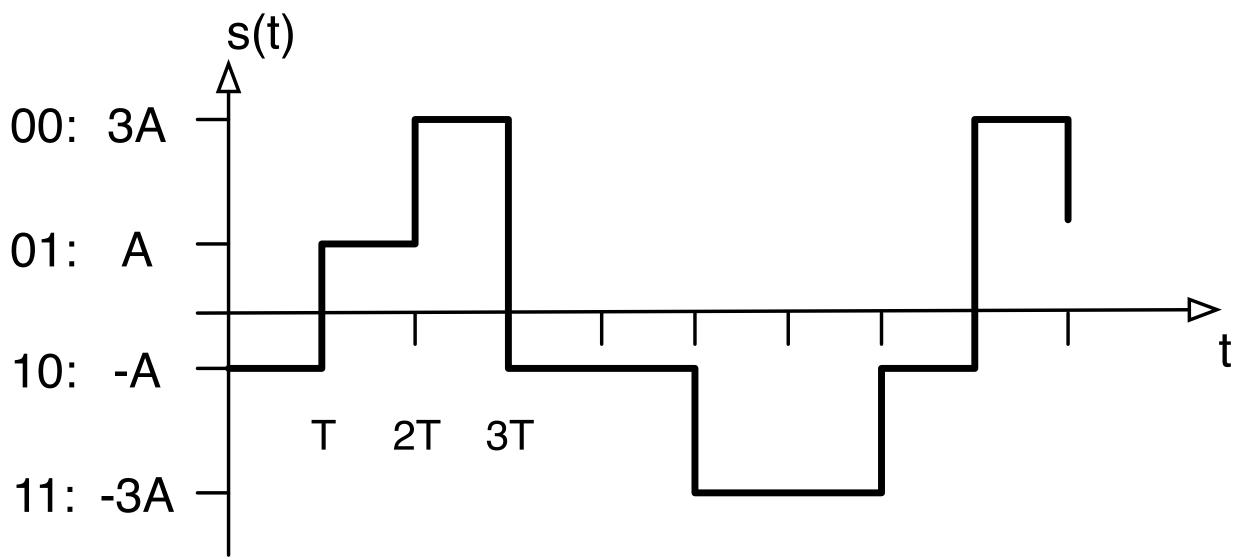
- Graphical representation



M-PAM

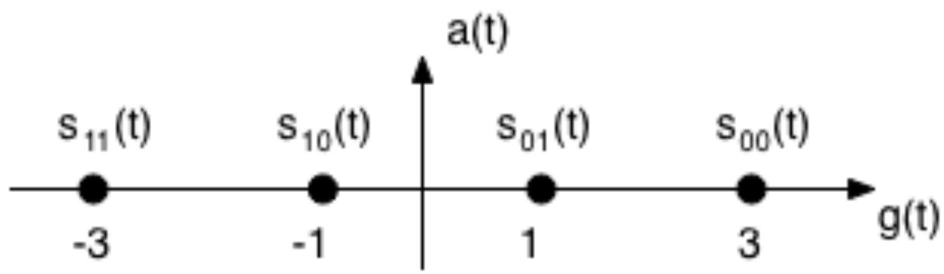
- Use *M* amplitude levels to represent $k=log_2(M)$ bits
- Ex. Two bits per signal (4-PAM)

x=10 01 00 10 10 11 11 10 00

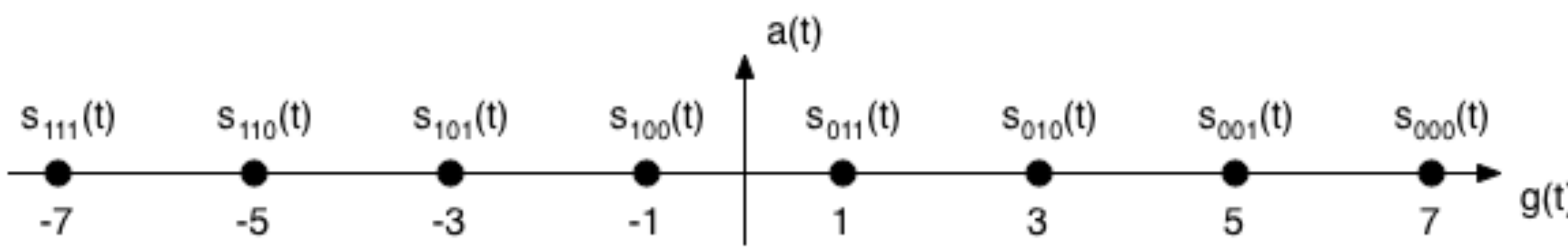


M-PAM

- Ex: 4-PAM

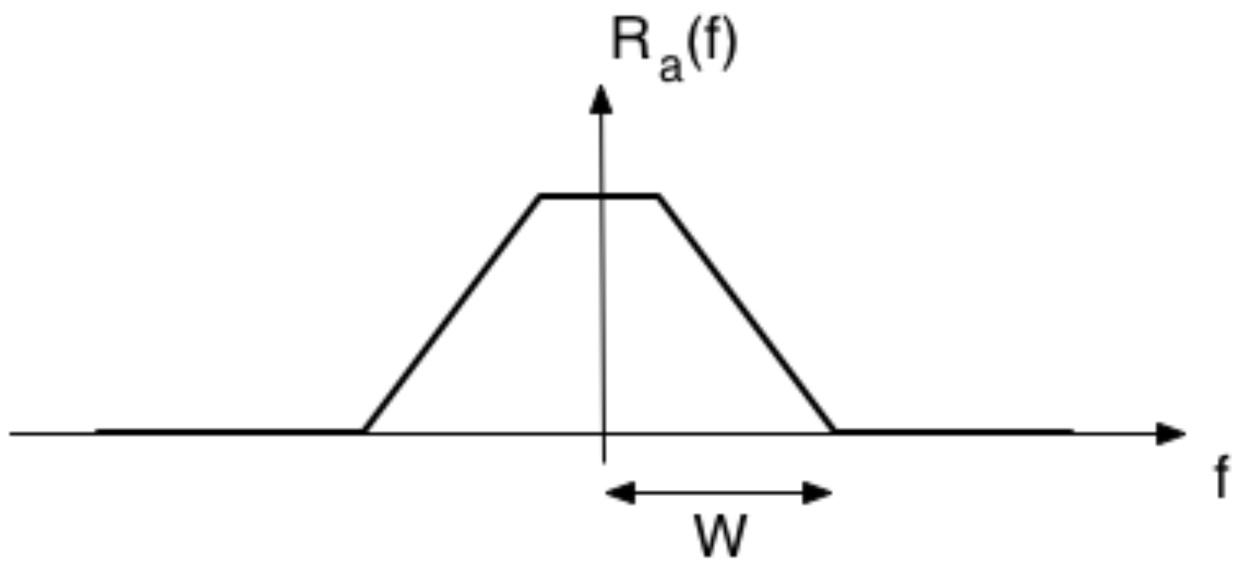


- Ex: 8-PAM



Bandwidth of signal

- The **bandwidth**, *W*, is the (positive side) frequency band occupied by the signal
- So far, only **base-band** signals (centered around *f=0*)



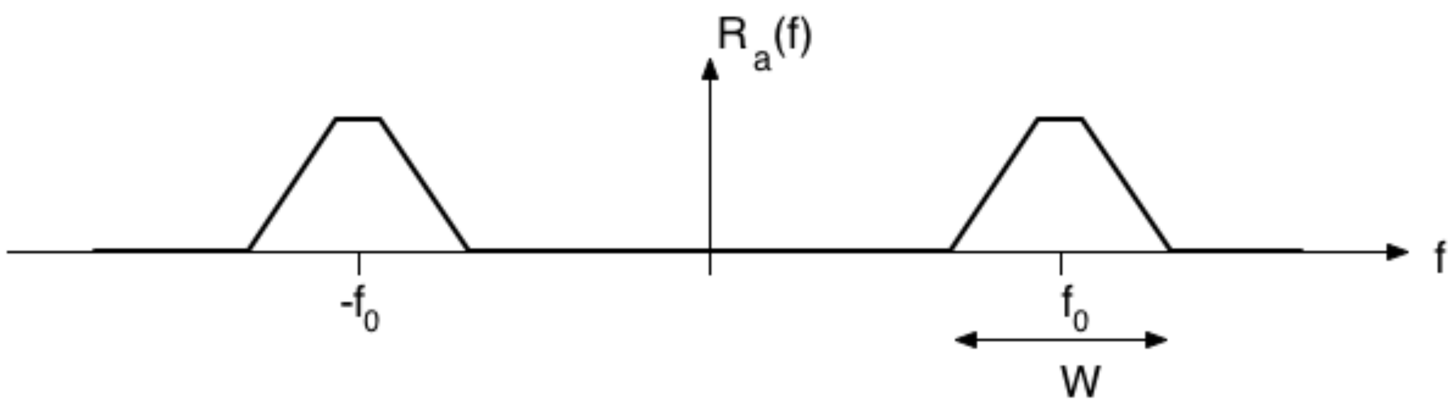
Pass-band signal

- The following multiplication centers the signal around the carrier frequency f_0

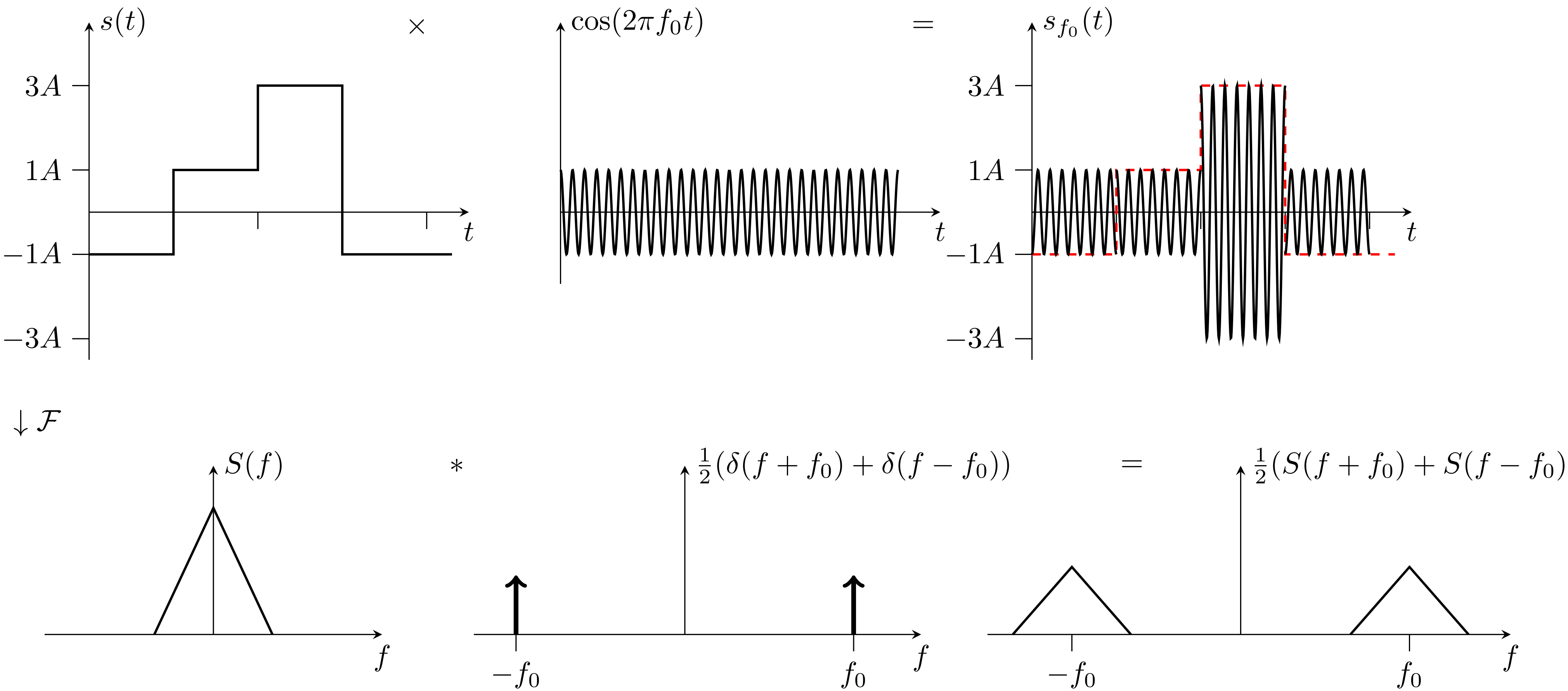
  $$s(t) = a(t) \cdot \cos(2\pi f_0 t)$$

  where $a(t)$ is a base-band signal

- Frequency modulate the signal to a carrier frequency f_0



Modulation in frequency

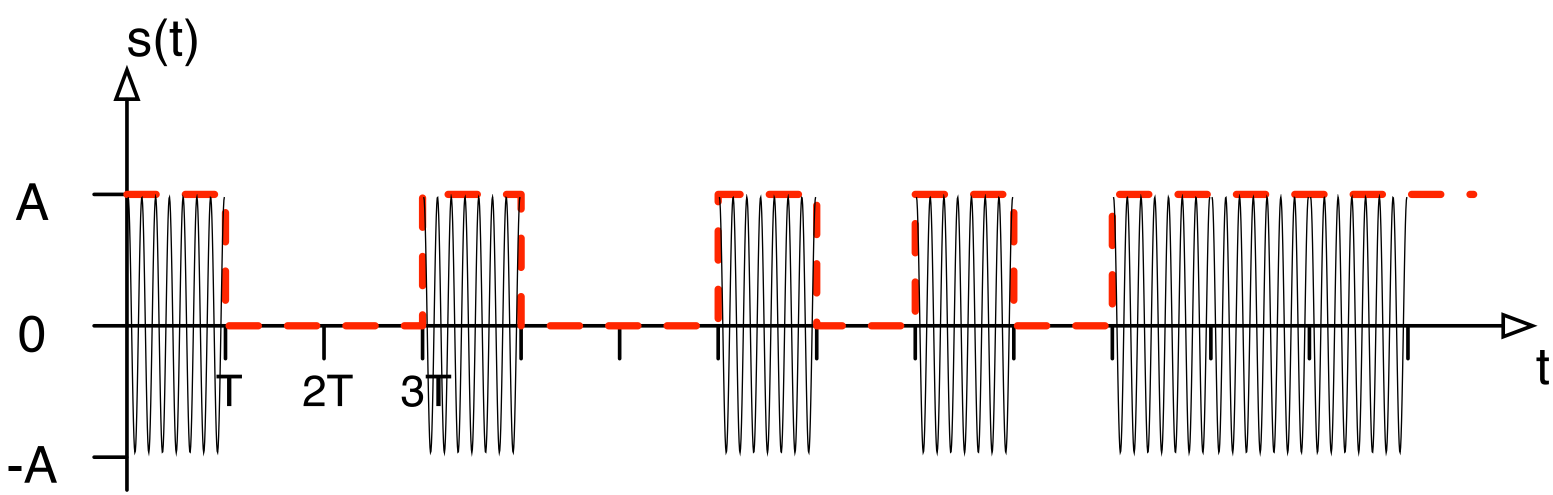


ASK (Amplitude Shift Keying)

- Use on-off keying at frequency f_0.

$$s(t) = \sum_n x_n g(t - nT)\cos(2\pi f_0 t)$$

- Ex. x=10010010101111100

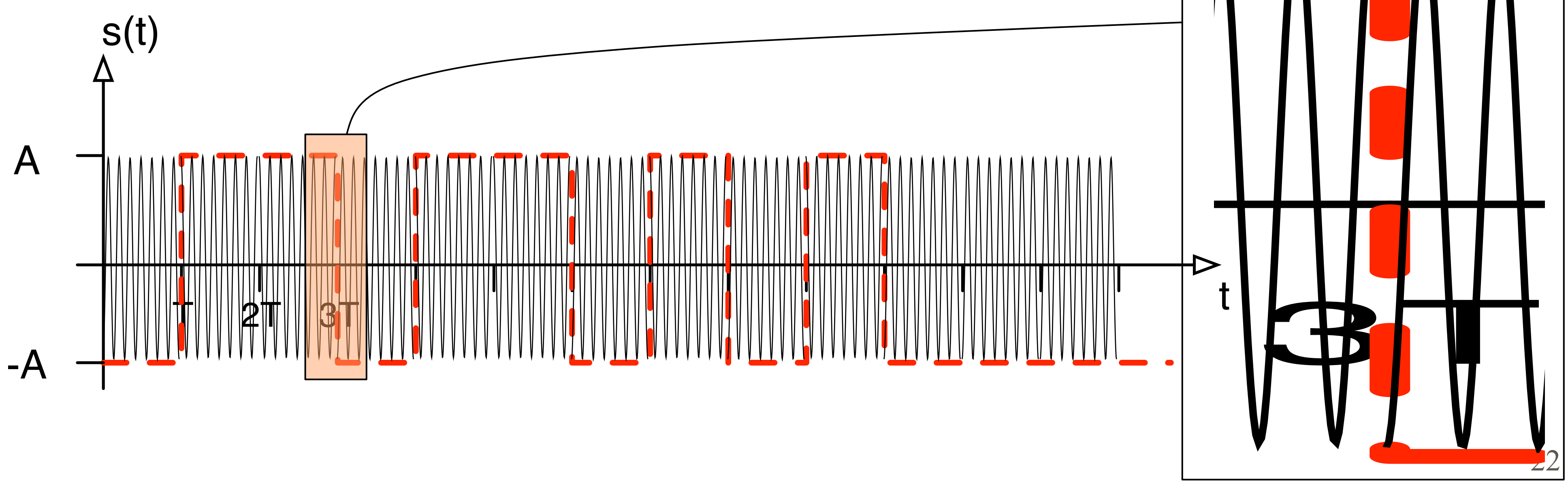


BPSK (Binary Phase Shift Keying)

- Use NRZ at frequency f_0, but view information in phase

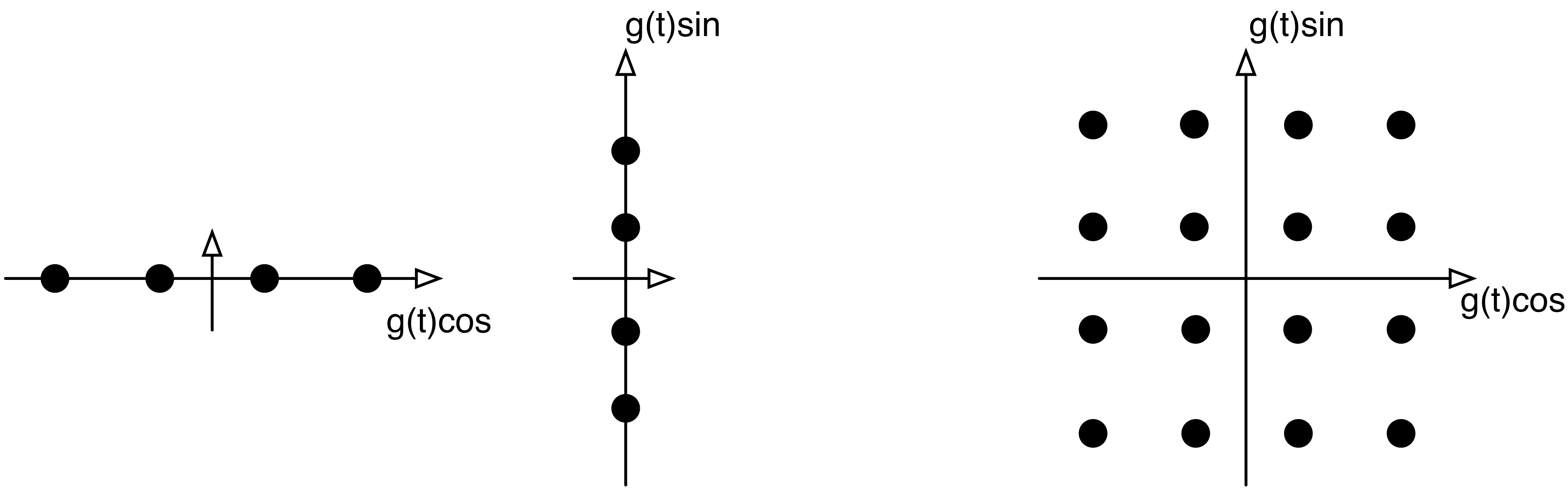
$$s(t) = \sum_n (-1)^{x_n} g(t - nT)\cos(2\pi f_0 t) = \sum_n g(t - nT)\cos(2\pi f_0 t + x_n \pi)$$

x=10010010101111100



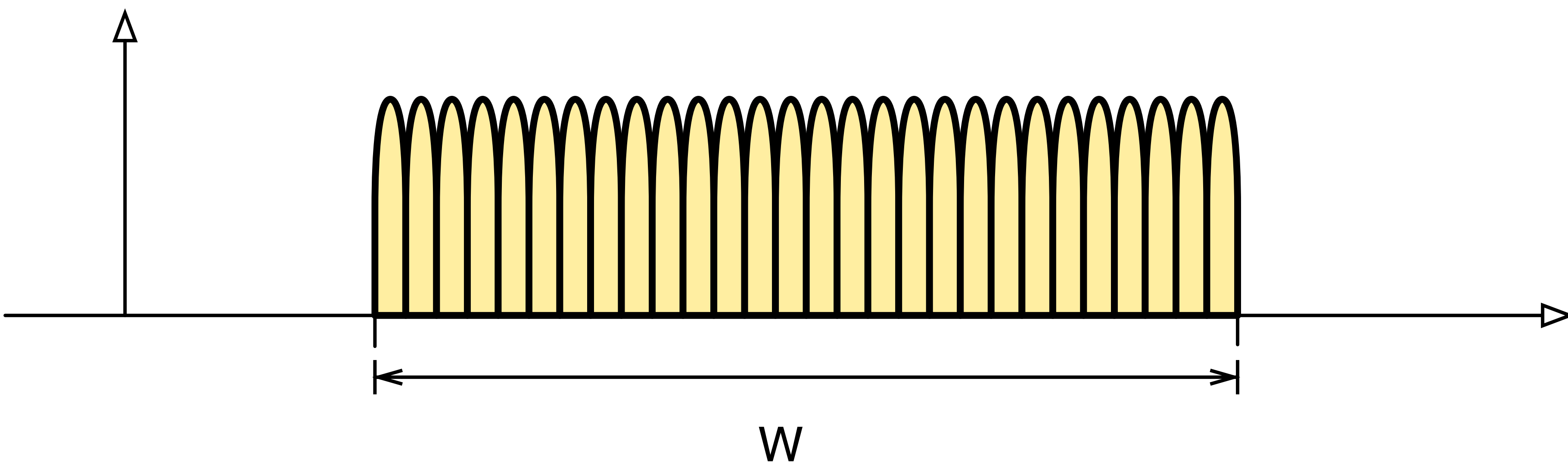
M-QAM
(Quadrature Amplitude Modulation)

Use that $\cos(2\pi f_0 t)$ and $\sin(2\pi f_0 t)$ are orthogonal (for high f_0) to combine two orthogonal M-PAM constellations

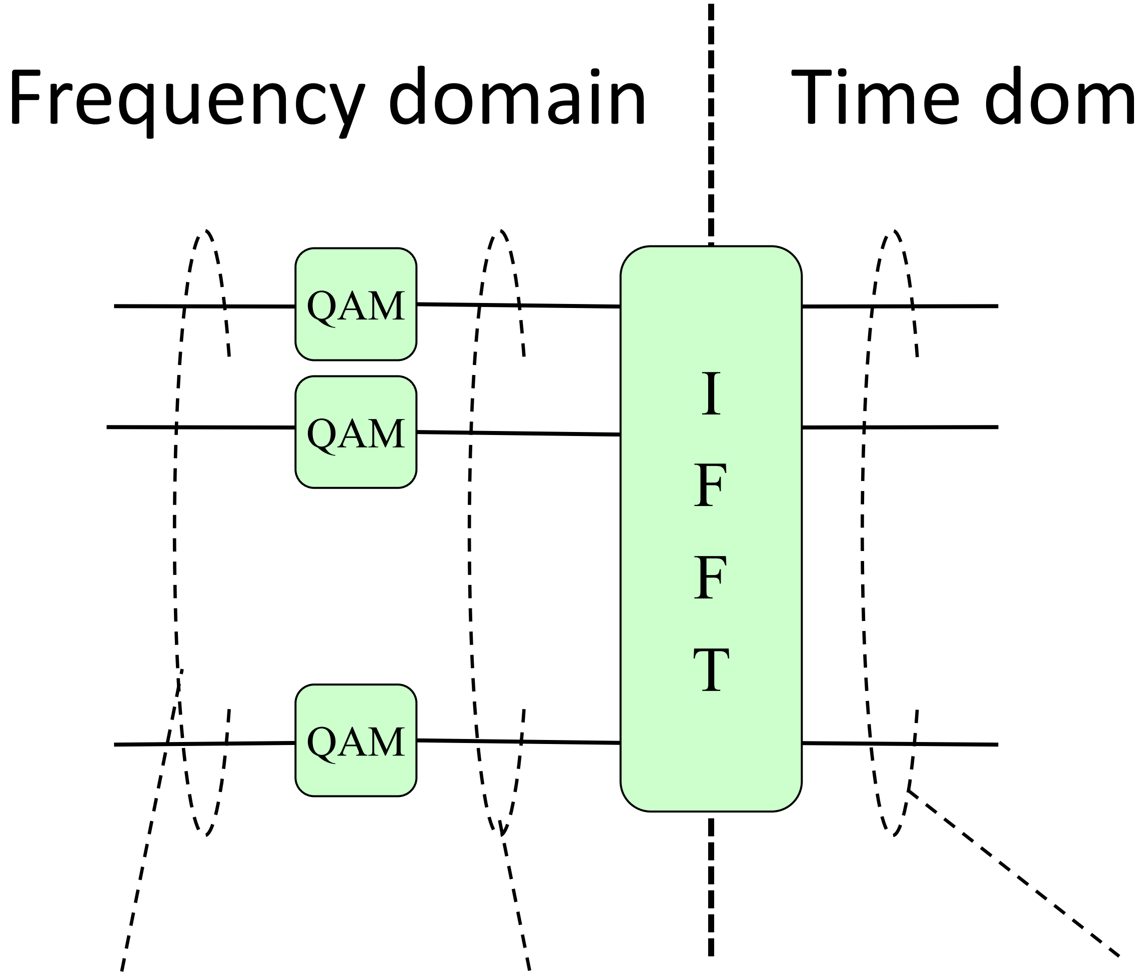


OFDM
Orthogonal Frequency Division Multiplexing

- N QAM signals combined in an orthogonal manner
- Used in e.g. xDSL, WiFi, DVB-C&T&H, LTE, etc

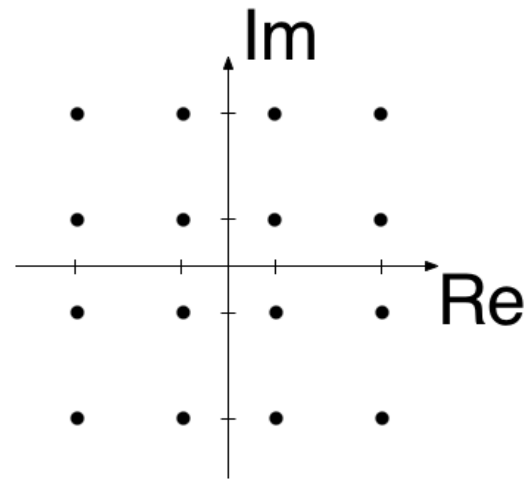


Idea of OFDM implementation



$(a_1,\ldots,a_N)\in Z_{16}{}^{N}$ $(x_1,\ldots,x_N)\in\mathbb{C}^N$ $(y_1,\ldots,y_N)=IFFT(\vec{x})$

QAM mapping



Some important parameters

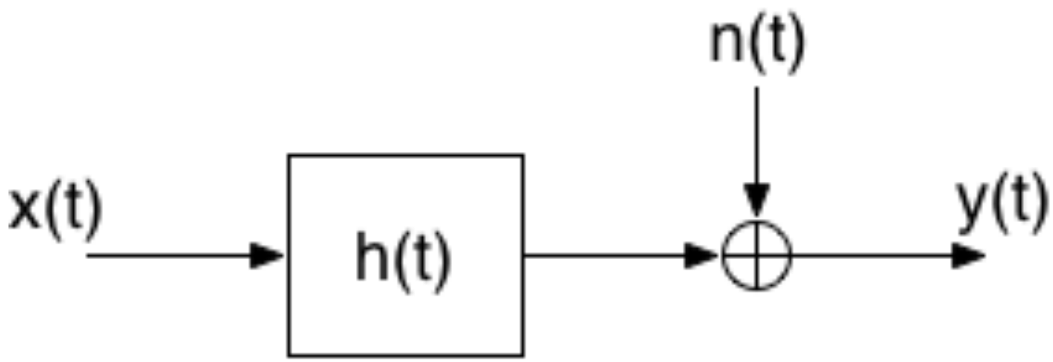
k=bit per symbol

- T_s time per symbol
- R_s symbol per second
- E_s energy per symbol
- $T_b=T_s/k$ time per bit
- $R_b=kR_s$ bit per second [bps]
- $E_b=E_s/k$ energy per bit
- SNR, Signal to noise ratio
  - average signal power relative to noise power
- W Bandwidth, frequency band occupied by signal
- Bandwidth efficiency: bits per second per Hz [bps/Hz]

$$\rho = \frac{R_b}{W}$$

Impairments on the communication channel (link)

- Attenuation
- Multipath propagation (fading)
- Noise



$$y(t) = x(t) * h(t) + n(t)$$

Noise disturbances

- Thermal noise (Johnson-Nyquist)
  - Generated by current in a conductor
  - -174 dBm/Hz (=$3.98*10^{-18}$ mW/Hz)
- Impulse noise (Often user generated, e.g. electrical switches)
- Intermodulation noise (From other systems)
- Cross-talk (Users in the same system)
- Background noise (Misc disturbances)

https://en.wikipedia.org/wiki/Johnson-Nyquist_noise

Some Information Theory
Entropy

- Discrete case: *X* discrete random variable

$$H(X) = E[-\log_2 p(X)] = -\sum_x p(x) \log_2 p(x)$$

Entropy is uncertainty of outcome (for discrete case)

- Continuous case: *X* continuous random variable

$$H(X) = E[-\log_2 f(X)] = -\int_R f(x) \log_2 f(x) dx$$

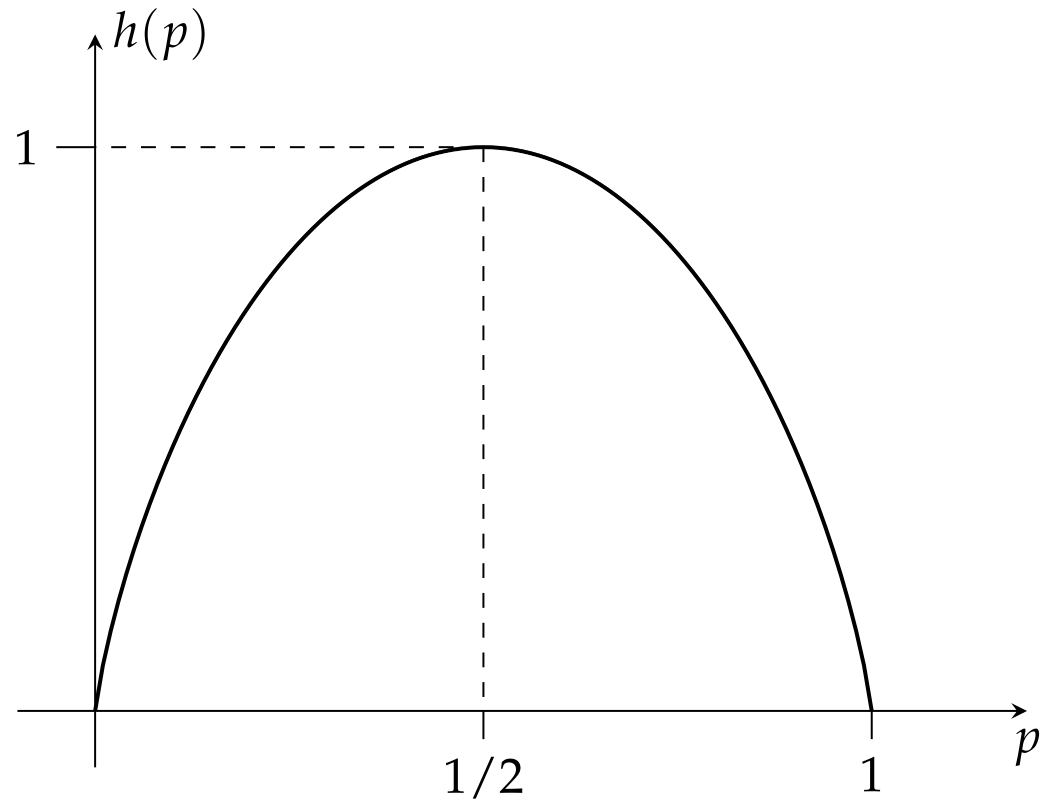
Example Entropy

Let *X* be a binary random variable with

P(X=0)=p

P(X=1)=1-p.

The binary entropy function is



$$h(p) = -p\log_2 p - (1-p)\log_2(1-p)$$

Compression

The entropy sets a limit on the compression ratio

- Consider a source for *X* with *N* symbols and the distribution *P(N)*. In average a symbol must be represented by at least *H(P)* bits.

- Well known compression algorithms are *zip*, *gz*, *png*, *Huffman*
- Lossy compression e.g. *jpeg* and *mpeg*

Some more Information Theory
Mutual information

- Let X and Y be two random variables
- The information about X by observing Y is given by

$$I(X;Y) = E\left[\log_2 \frac{P(X,Y)}{P(X)P(Y)}\right]$$

- This gives

$$I(X;Y) = H(X) + H(Y) - H(X,Y)$$

Example Mutual Information

The random variables *X* and *Y* has the joint distribution

P(X,Y)	Y=0	Y=1
X=0	0	3/4
X=1	1/8	1/8

That gives

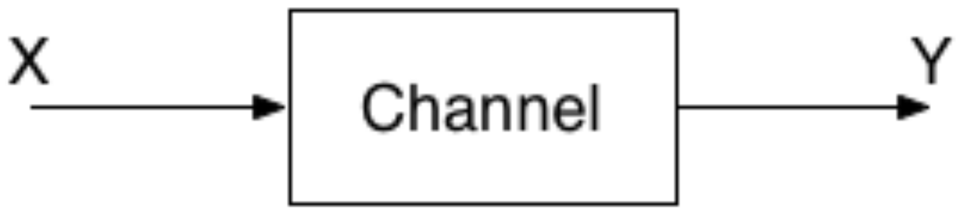
$$P(X=0)=3/4 \quad \text{and} \quad P(X=1)=1/4$$
$$P(Y=0)=1/8 \quad \text{and} \quad P(Y=1)=7/8$$

Entropies:

$$H(X)=h(\tfrac{1}{4})=0.8114$$
$$H(Y)=h(\tfrac{1}{8})=0.5436$$
$$H(X,Y)=-\tfrac{3}{4}\log\tfrac{3}{4}-\tfrac{1}{8}\log\tfrac{1}{8}-\tfrac{1}{8}\log\tfrac{1}{8}=1.0613$$

Information: $I(X;Y)=H(X)+H(Y)-H(X,Y)=0.2936$

Some Information Theory
Channel capacity



- The channel is a model of the transmission link.
- Transmit X and receive Y. How much information can the receiver get from the transmitter?
- The *channel capacity* is defined as

$$C = \max_{p(x)} I(X;Y)$$

AWGN
Additive White Gaussian Noise channel

- Let *X* be band-limited in bandwidth *W*
- $Y = X + N$, where $N \sim N\left(0, \sqrt{N_0 / 2}\right)$
- The capacity is

$$C = W \log_2 \left(1 + \frac{P}{N_0 W} \right) \quad \text{[bps]}$$

- where *P* is the power of *X,* i.e. $E[X^2]=P$.
- It is not possible to get higher data rate on this channel!

AWGN Example (VDSL)

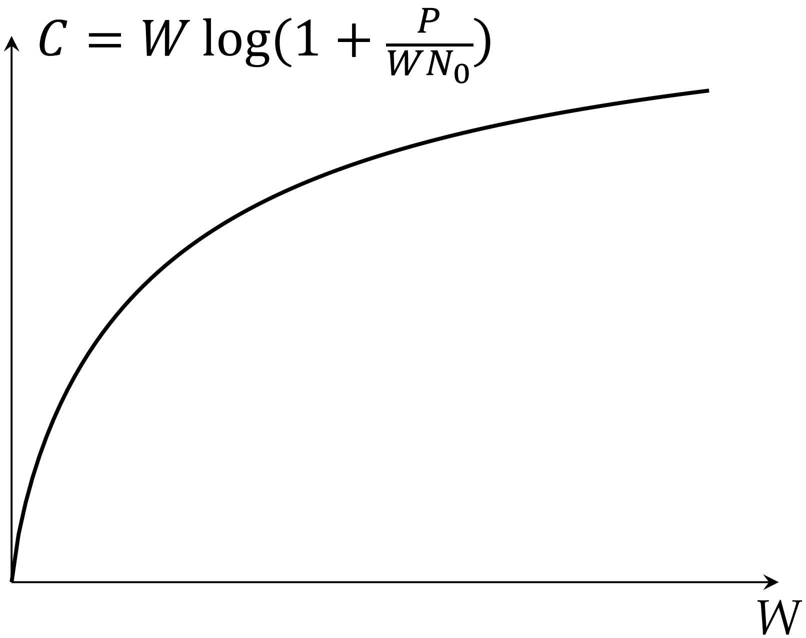
- Consider a channel with

$$W = 17\ MHz$$
$$P_\Delta = -60\ dBm/Hz$$
$$N_0 = -145\ dBm/Hz$$

- Power $P = 10^{-60/10} \cdot 17 \cdot 10^6$ mW
- Noise $N_0 = 10^{-145/10}$ mW/Hz
- Capacity $C = W \log\left(1 + \frac{P}{N_0 W}\right) = W \log\left(1 + \frac{10^{-60/10}}{10^{-145/10}}\right) = 480$ Mbps

Shannon's fundamental limit

- Plot capacity vs *W*



- Is there a limit?

- Let *W* -> ∞

$$C_\infty = \lim_{W\to\infty} W\log\left(1+\frac{P/N_0}{W}\right)$$

$$= \lim_{W\to\infty} \log\left(1+\frac{P/N_0}{W}\right)^W = \log e^{P/N_0} = \frac{P/N_0}{\ln 2}$$

- With $E_b=PT_b$ and $T_s=kT_b$

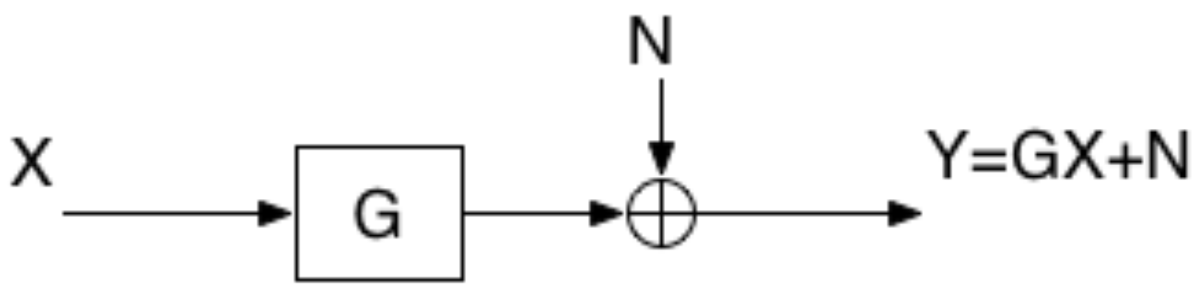
$$\frac{C_\infty}{R_b} = \frac{E_b/N_0}{\ln 2} > 1$$

- Which gives the *fundamental limit*

$$\frac{E_b}{N_0} > \ln 2 = -1.59dB$$

AWGN with attenuation

- Let *X* be bandlimited in bandwidth *W*
- Let *G* be attenuation on channel, *G<1*



- The capacity is

$$C = W \log_2\left(1 + \frac{|G|^2 P}{N_0 W}\right) \quad \text{[in bit/s]}$$