

ETSF15

Physical layer communication

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Physical layer

- Analog vs digital (Previous lecture)
- Transmission media
- Modulation
 - Represent digital data in a continuous world
- Disturbances, Noise and distortion
- Information

Transmission media

Guided media

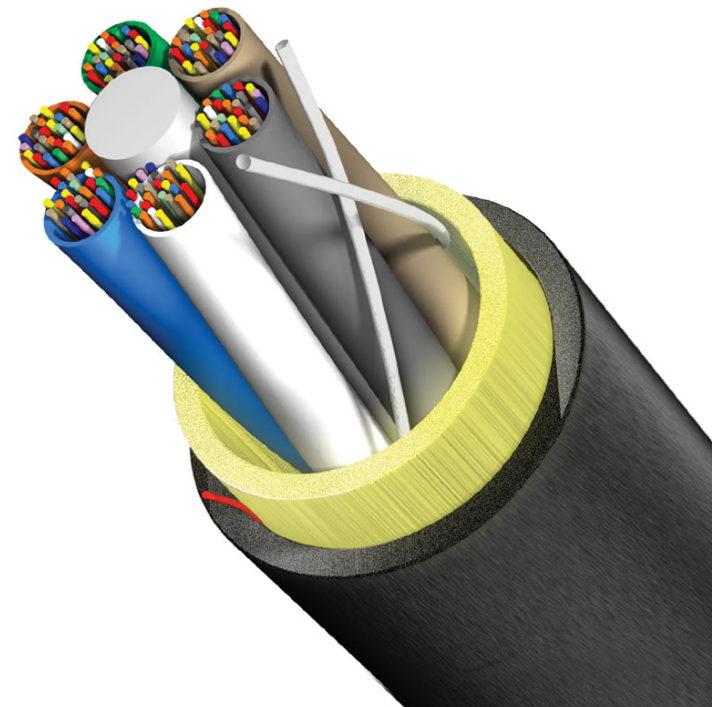
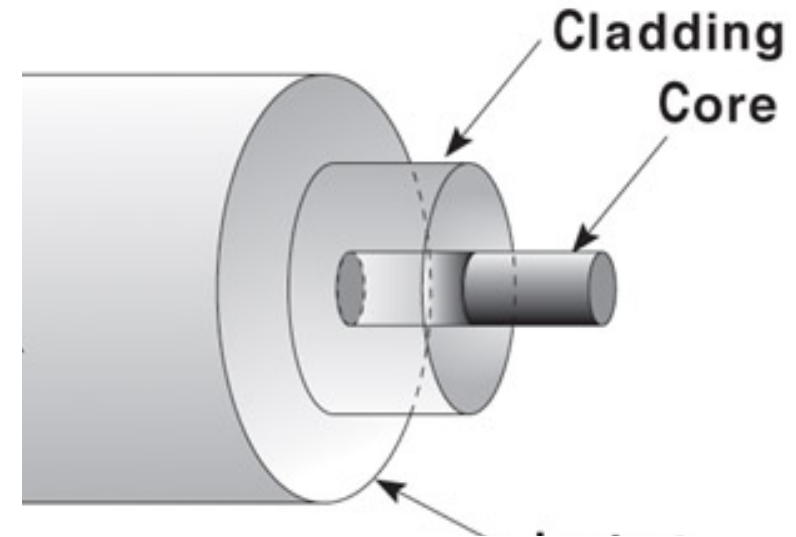
- Fibre optic cable
- Twisted pair copper cables
- Coax cable

Unguided media

- Radio
- Microwave
- Infra red

Fibre optic

- Transmission is done by light in a glass core (very thin)
- Total reflection from core to cladding
- Bundle several fibres in one cable
- Not disturbed by radio signals



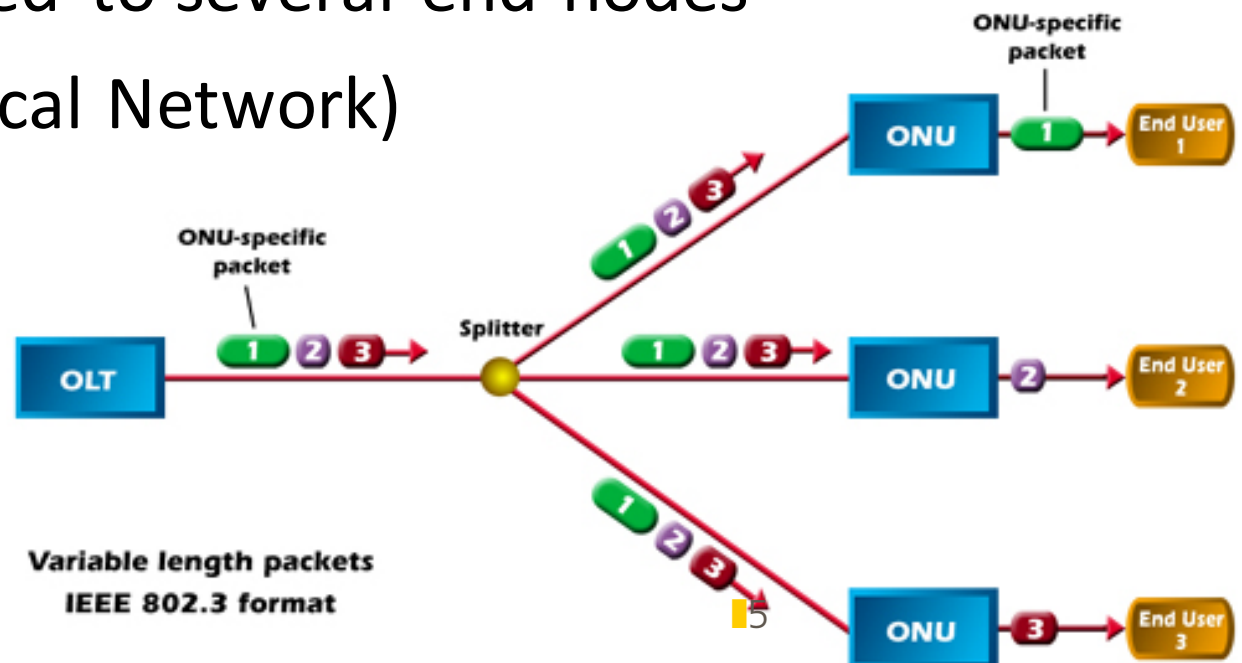
Optical network architecture

Point to point

- Two nodes are connected by one dedicated fibre

Point to multi-point

- One point is connected to several end nodes
 - PON (Passive Optical Network)



Twisted pair copper cables

Two copper lines twisted around each other

- Twisting decreases disturbances (and emission)
- Used for
 - Telephony grid (CAT3)
 - Ethernet (CAT5, CAT6 and sometimes CAT 7)



Coax cable

One conductor surrounded by a shield

- Used for
 - Antenna signals
 - Measurement instrumentations

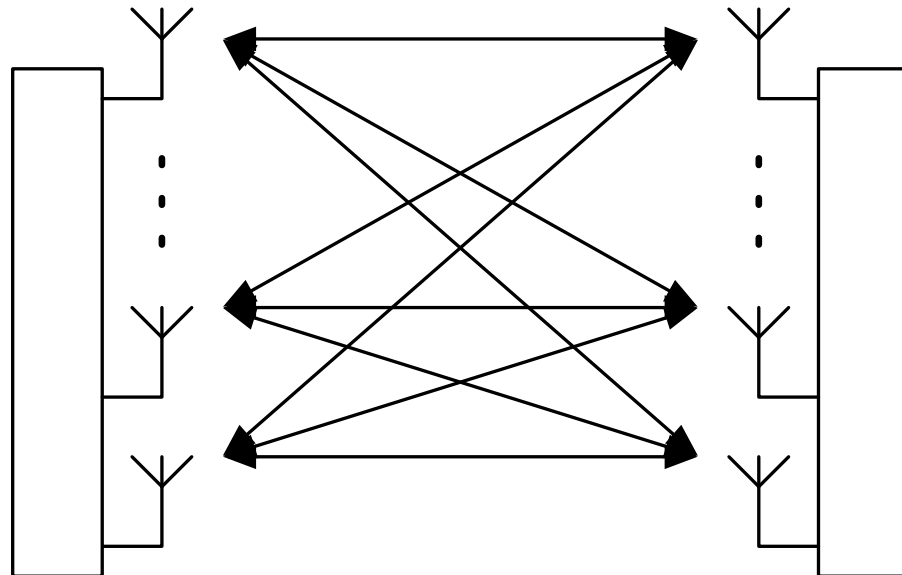


Radio structures

- Single antenna system

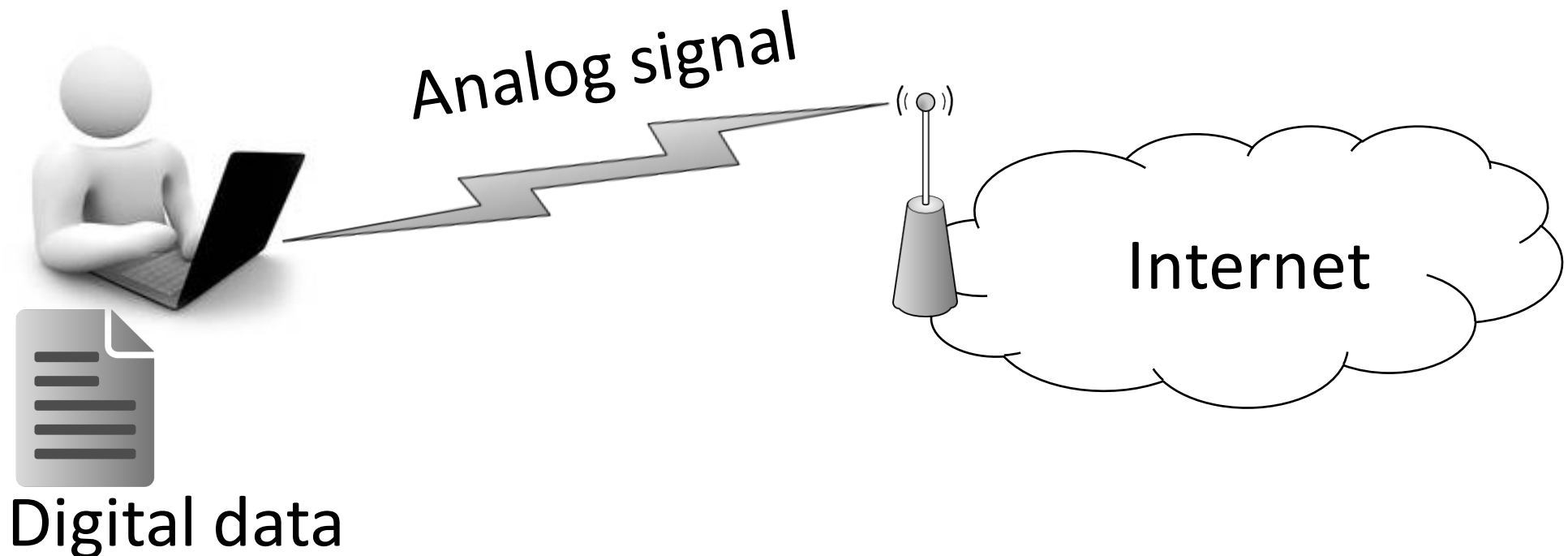


- MIMO (Multiple In Multiple Out)



From bits to signals

- Principles of digital communications



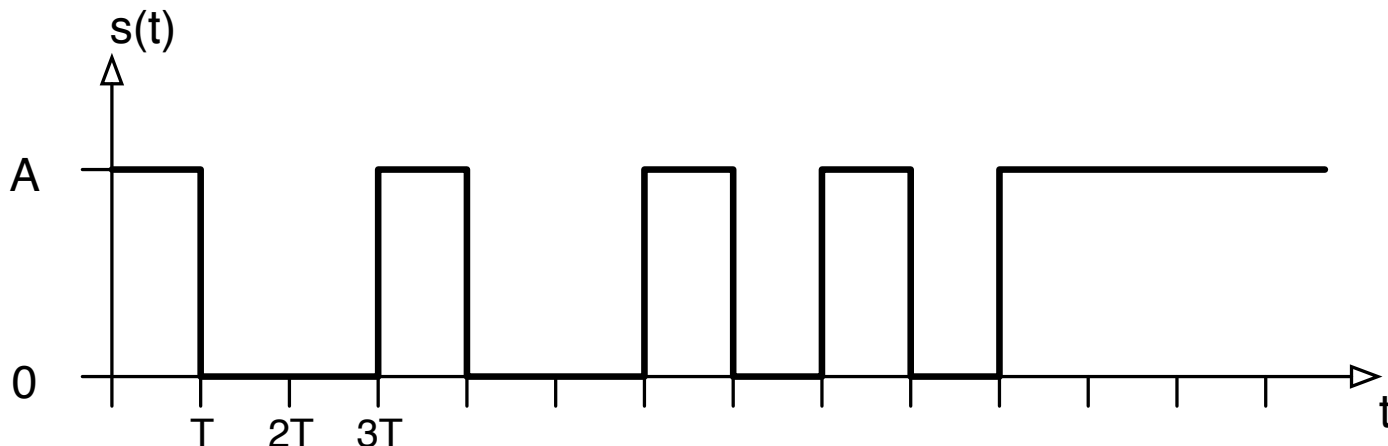
On-off keying

- Send one bit during T_b seconds and use two signal levels, “on” and “off”, for 1 and 0.

$$a(t) = A \cdot x \quad 0 \leq t \leq T_b$$

Ex.

$x=10010010101111100$



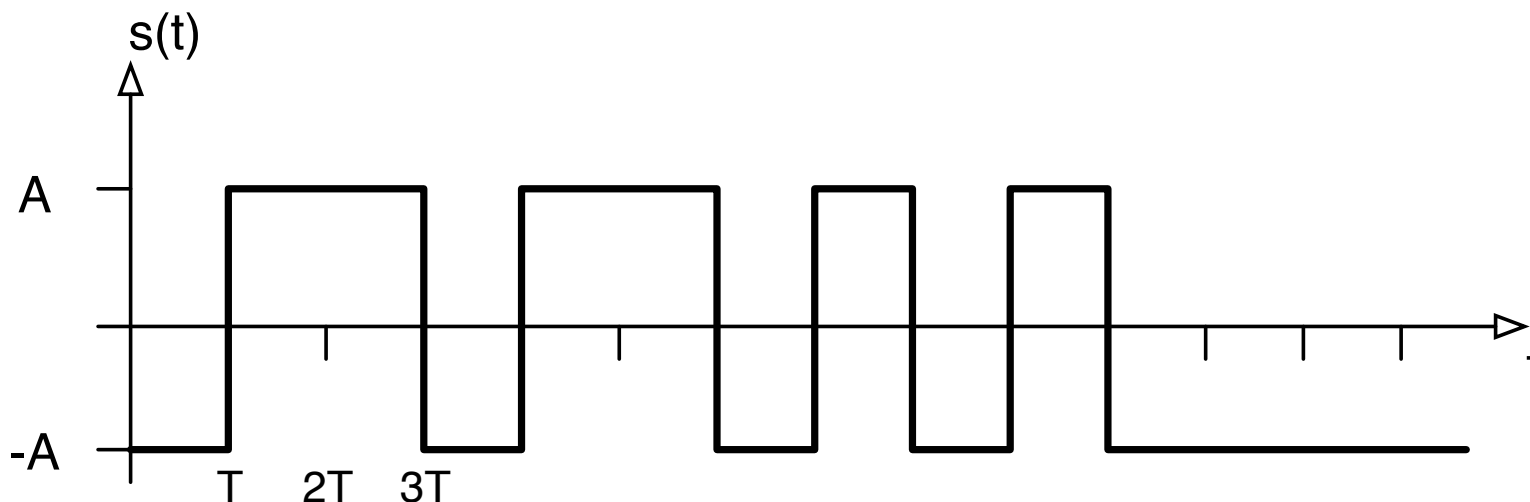
Non-return to zero (NRZ)

- Send one bit during T_b seconds and use two signal levels, $+A$ and $-A$, for 0 and 1.

$$a(t) = A \cdot (-1)^x \quad 0 \leq t \leq T_b$$

Ex.

$x=10010010101111100$



Description of general signal

With the pulse form $g(t) = A, 0 \leq t < T_s$, the signals can be described as

$$s(t) = \sum_n a_n g(t - nT_s)$$

Two signal alternatives

On-off keying

$$a_n = x_n \quad \Rightarrow \quad s_0(t) = 0 \text{ and } s_1(t) = g(t)$$

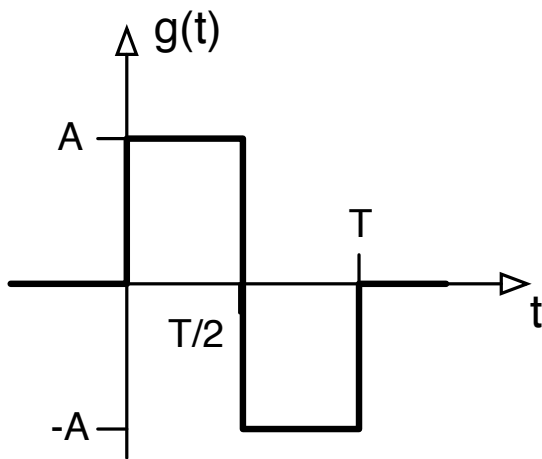
NRZ

$$a_n = (-1)^{x_n} \quad \Rightarrow \quad s_0(t) = g(t) \text{ and } s_1(t) = -g(t)$$

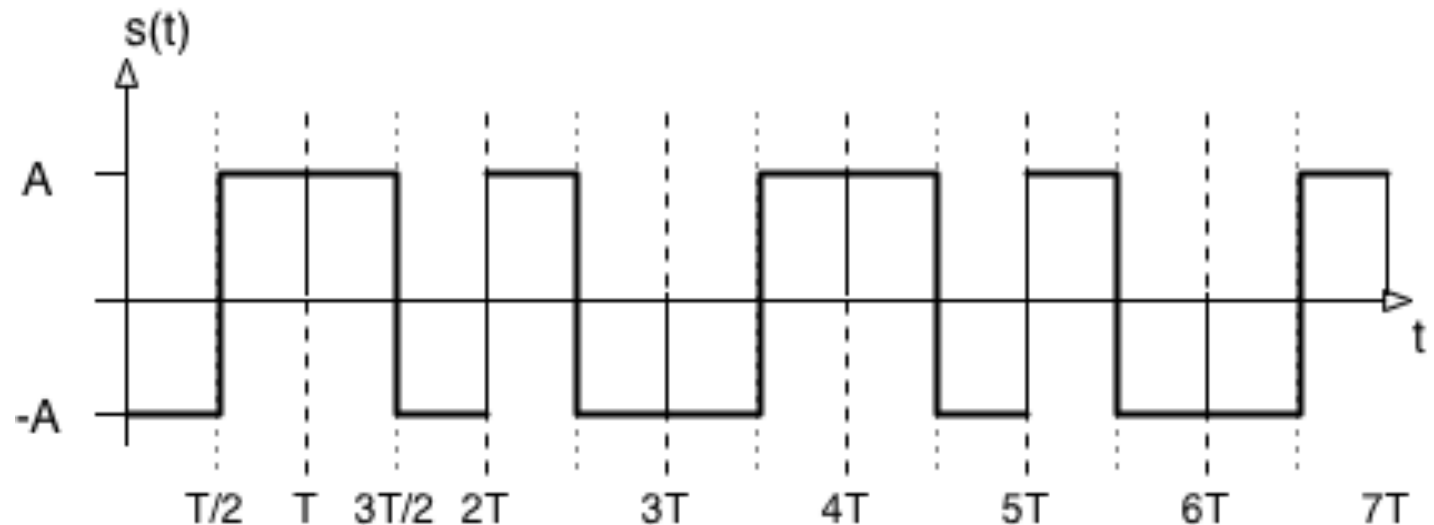
Manchester coding

- To get a zero passing in each signal time, split the pulse shape $g(t)$ in two parts and use +/- as amplitude.

Ex.

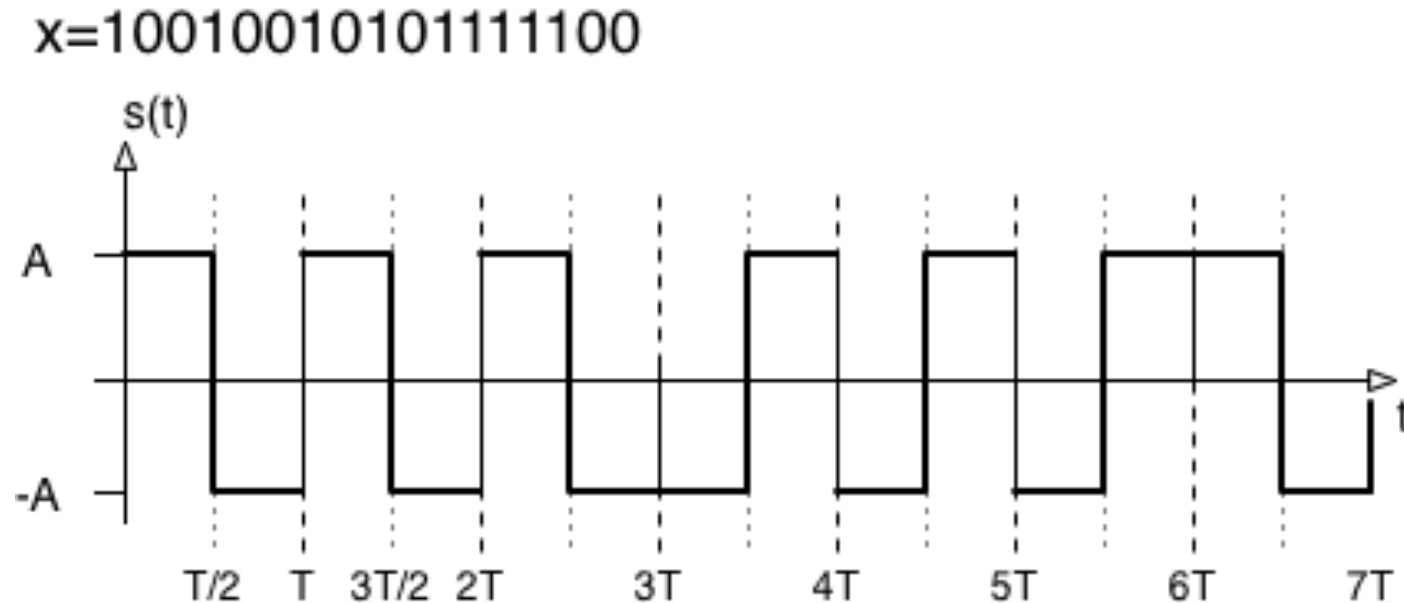


$x=10010010101111100$



Differential Manchester coding

- Use a zero transition at the start to indicate the data.
- For a transmitted 0 the same pulse as previous slot is used, while for a transmitted 1 the inverted pulse is used, i.e. $a_n = a_{n-1}(-1)^{x_n}$

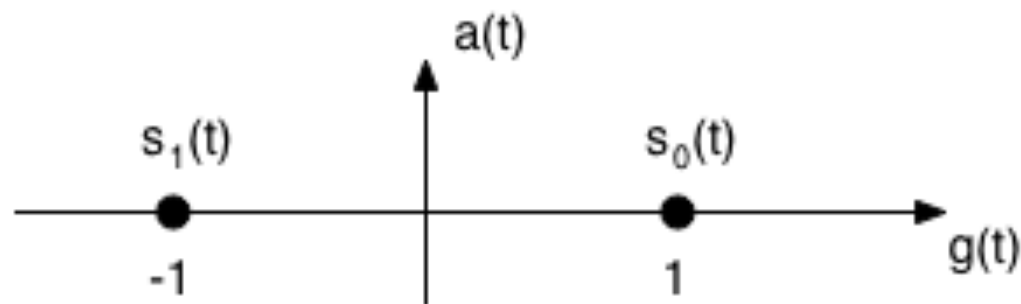


PAM (Pulse Amplitude Modulation)

- NRZ and Manchester are forms of binary PAM
- The data is stored in the amplitude and transmitted with a pulse shape $g(t)$

$$a(t) = a_n \cdot g(t) \quad a_n = (-1)^x$$

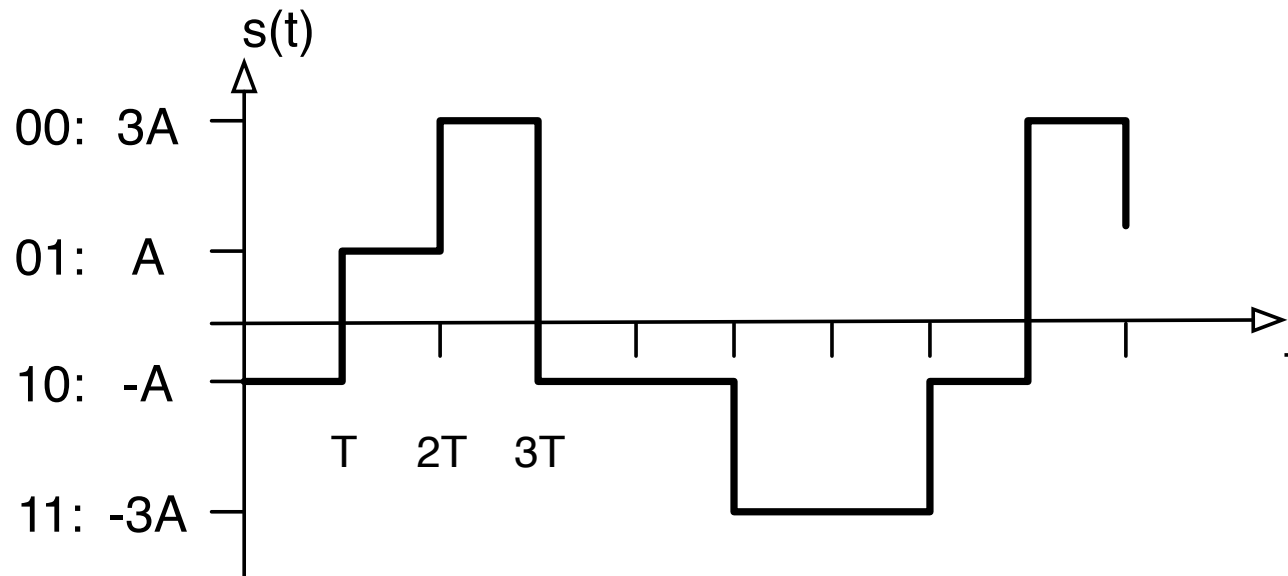
- Graphical representation



M-PAM

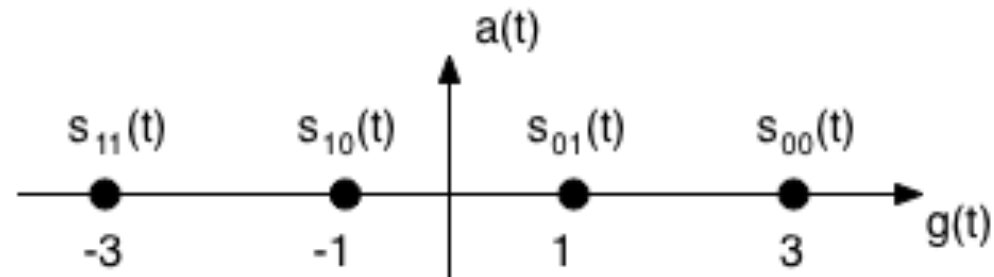
- Use M amplitude levels to represent $k = \log_2(M)$ bits
- Ex. Two bits per signal (4-PAM)

$x = 10\ 01\ 00\ 10\ 10\ 11\ 11\ 10\ 00$

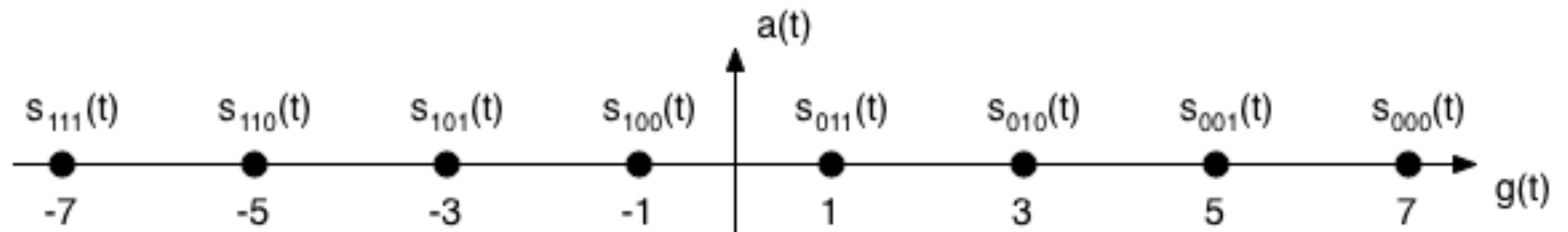


M-PAM

- Ex: 4-PAM

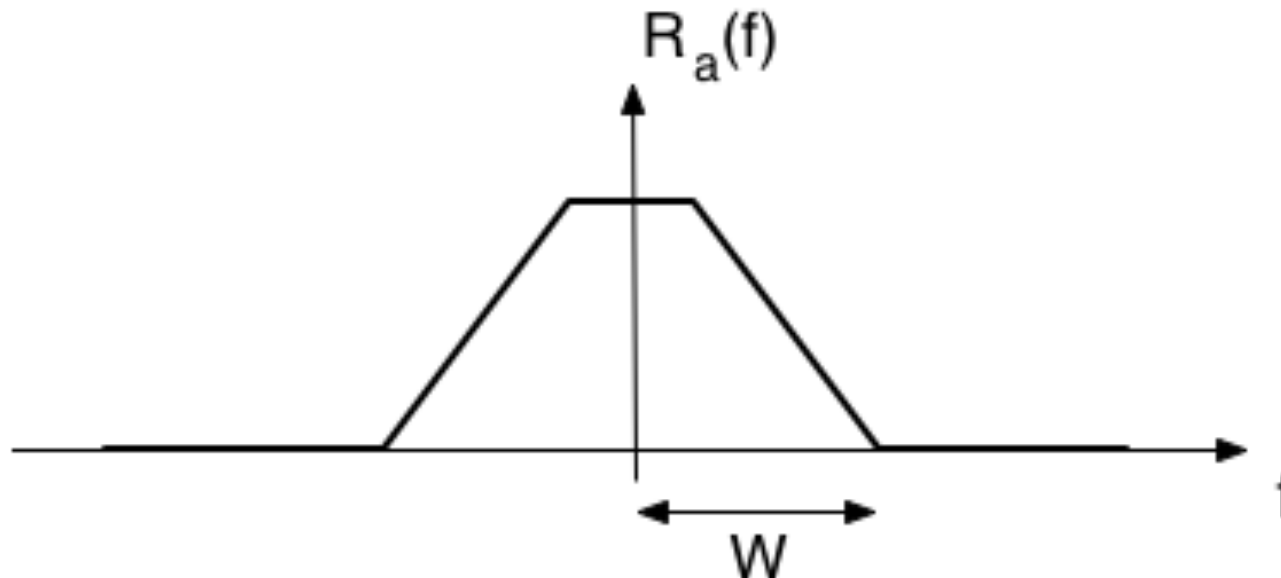


- Ex: 8-PAM



Bandwidth of signal

- The **bandwidth**, W , is the (positive side) frequency band occupied by the signal
- So far only **base-band** signals (centered around $f=0$)



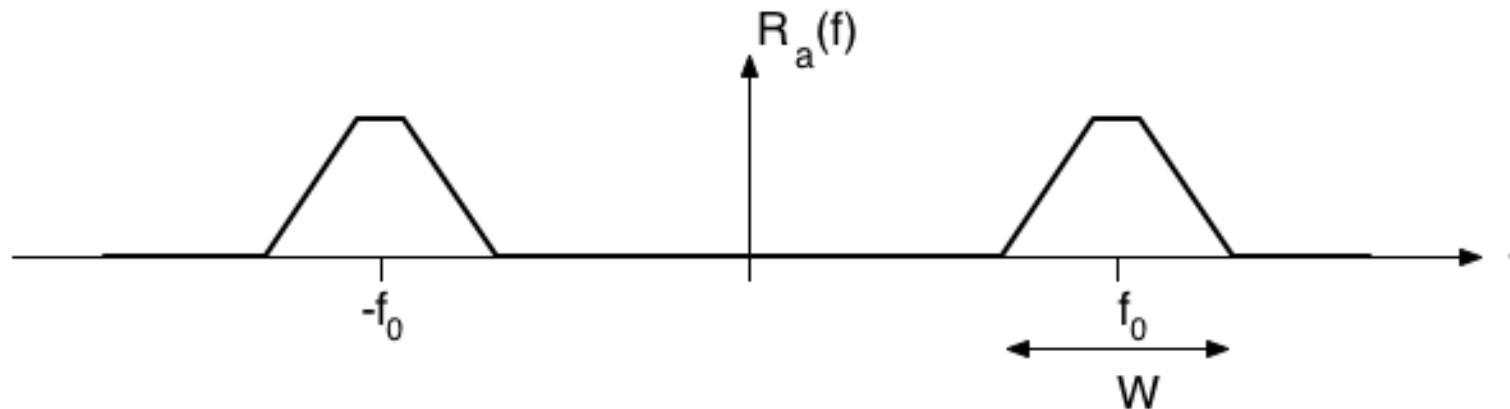
Pass-band signal

- The following multiplication centers the signal around the carrier frequency f_0

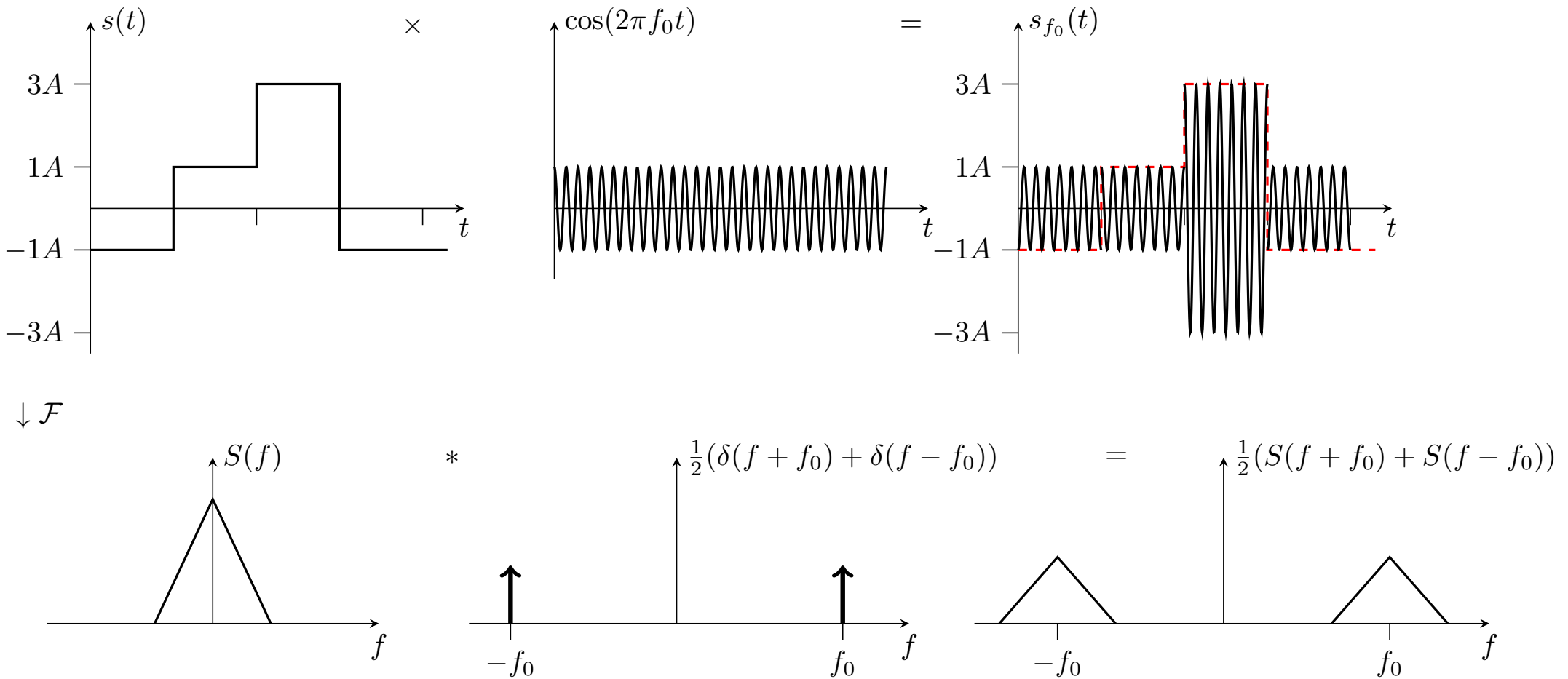
$$s(t) = a(t) \cdot \cos(2\pi f_0 t)$$

where $a(t)$ is a base-band signal

- Frequency modulate the signal to a carrier frequency f_0



Modulation in frequency



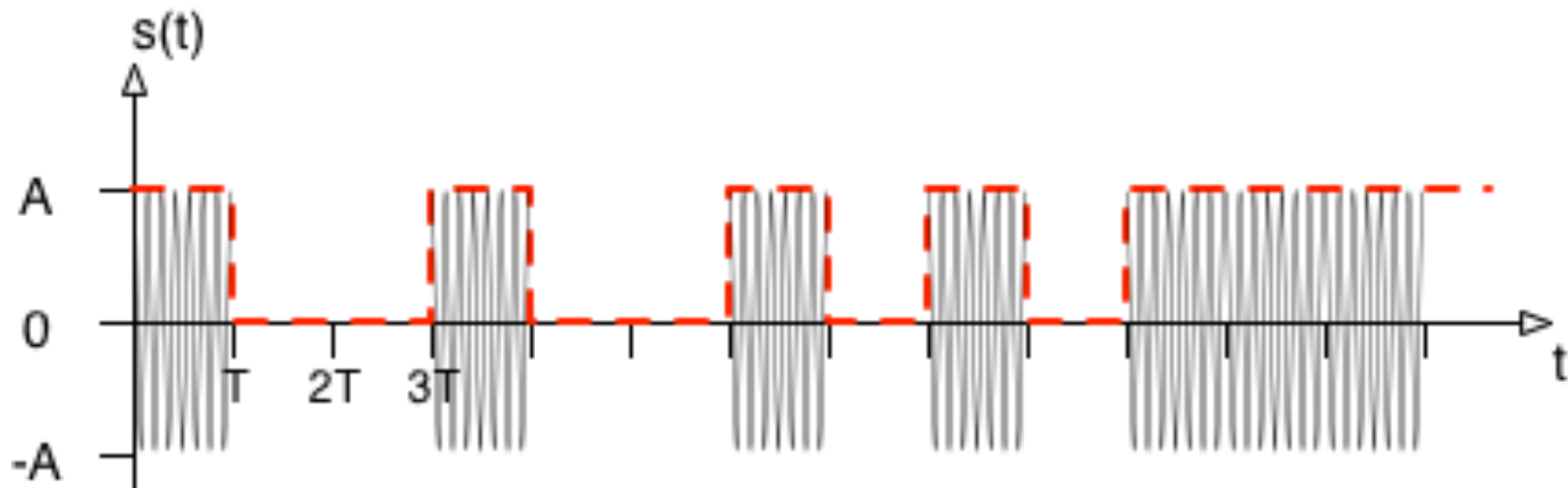
ASK (Amplitude Shift Keying)

- Use on-off keying at frequency f_0 .

$$s(t) = \sum_n x_n g(t - nT) \cos(2\pi f_0 t)$$

- Ex.

$x=10010010101111100$

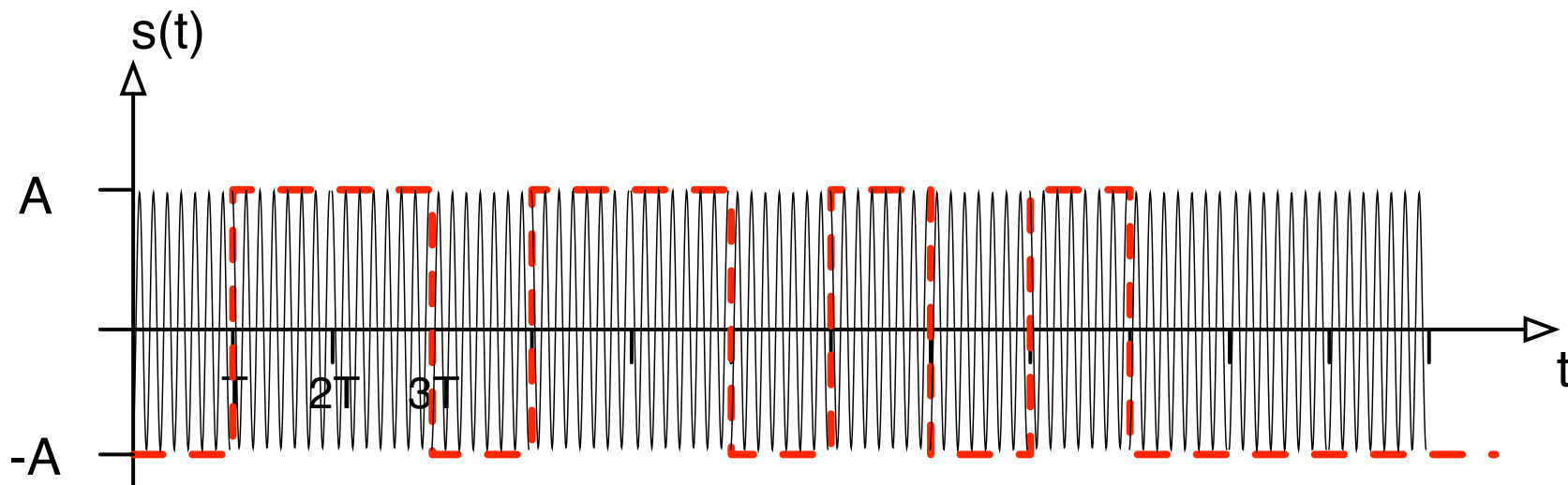


BPSK (Binary Phase Shift Keying)

- Use NRZ at frequency f_0 , but view information in phase

$$s(t) = \sum_n (-1)^{x_n} g(t - nT) \cos(2\pi f_0 t) = \sum_n g(t - nT) \cos(2\pi f_0 t + x_n \pi)$$

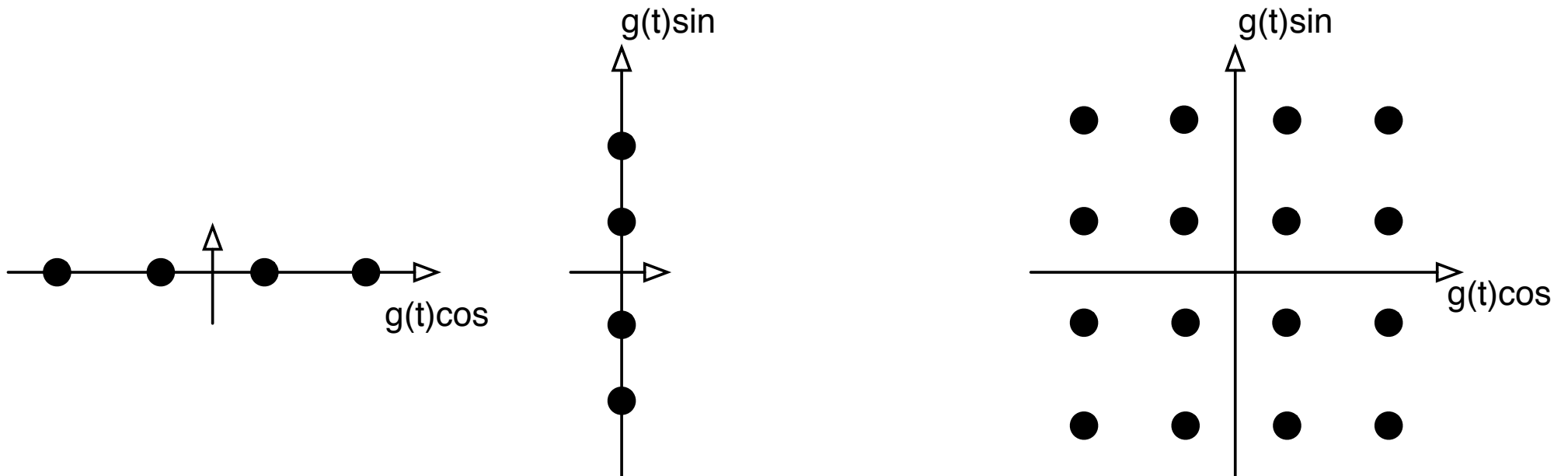
$x=10010010101111100$



M-QAM

(Quadrature Amplitude Modulation)

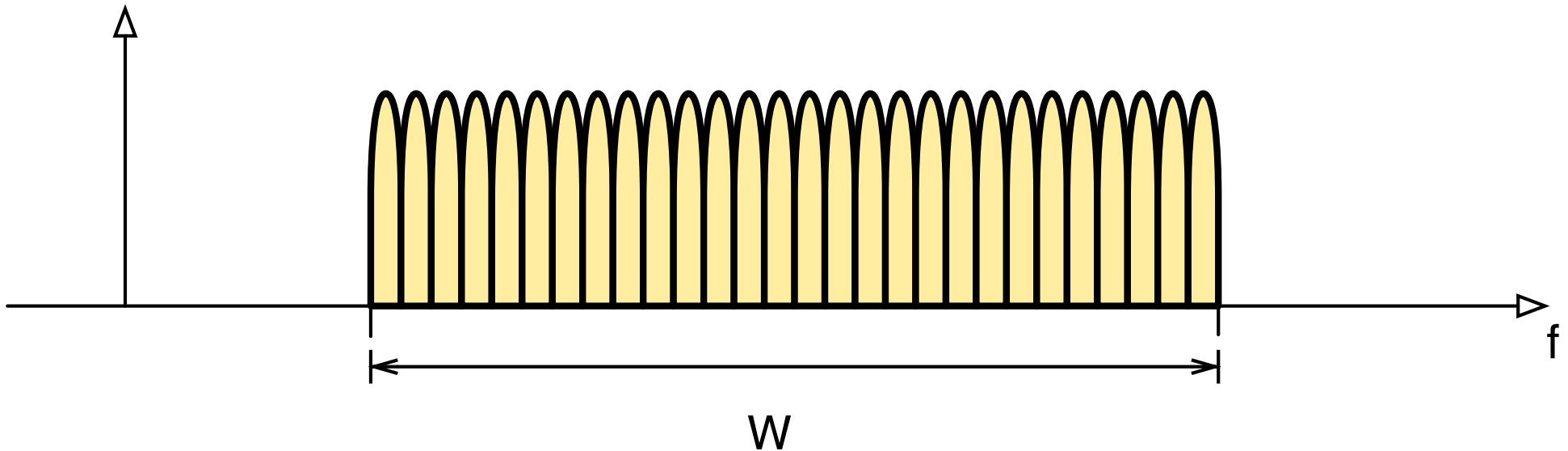
Use that $\cos(2\pi f_0 t)$ and $\sin(2\pi f_0 t)$ are orthogonal (for high f_0) to combine two orthogonal M-PAM constellations



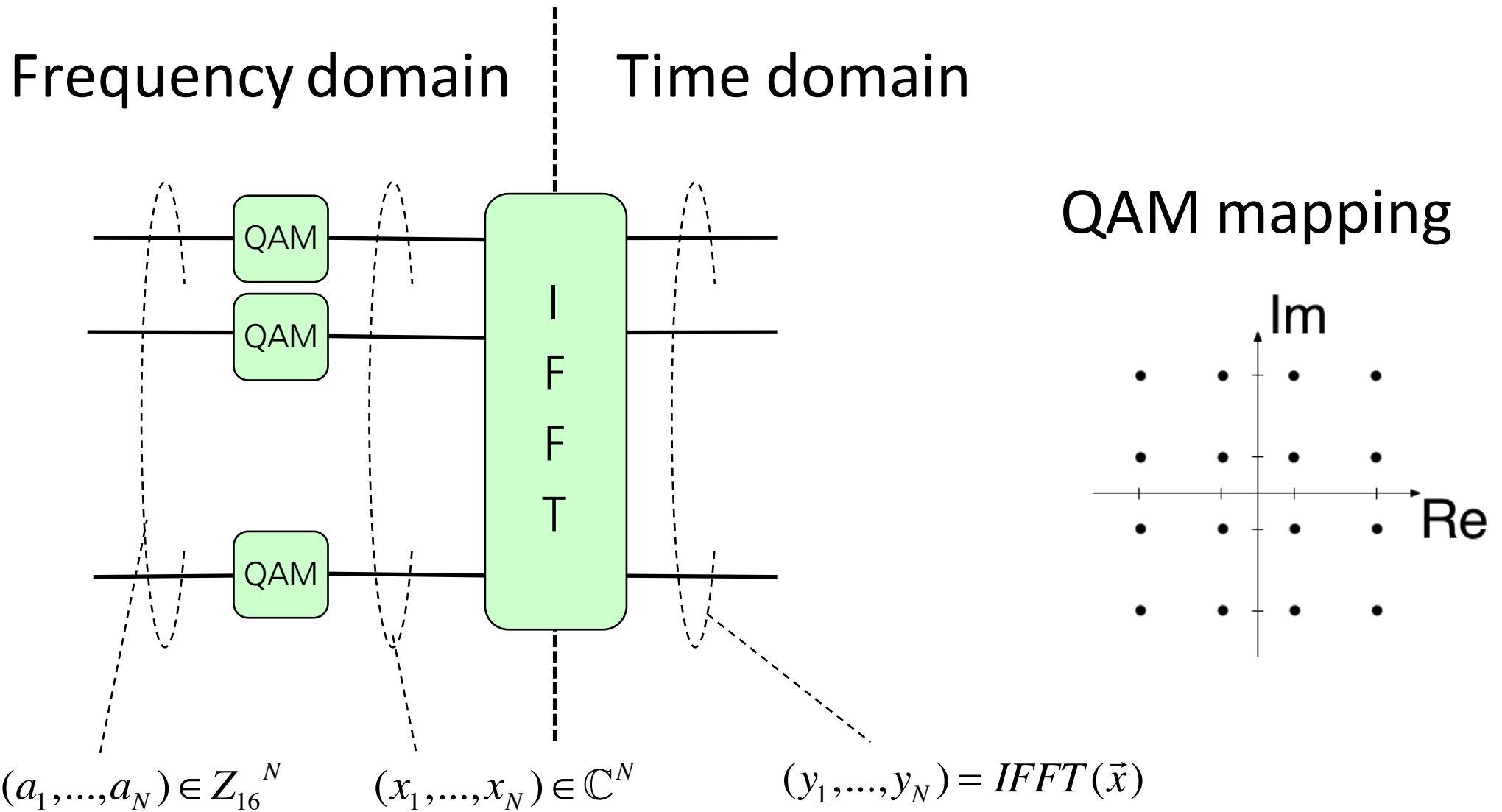
OFDM

Orthogonal Frequency Division Multiplexing

- N QAM signals combined in an orthogonal manner
- Used in e.g. ADSL, VDSL, WiFi, DVB-C&T&H, LTE, etc



Idea of OFDM implementation



Some important parameters

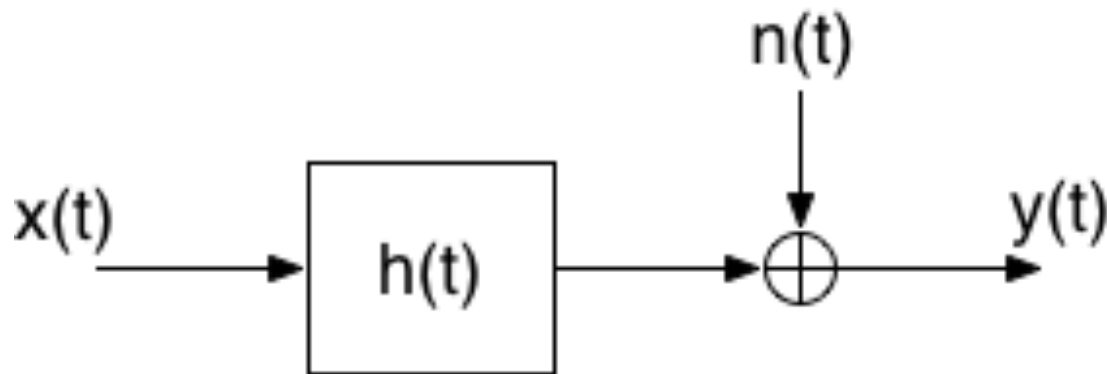
k =bit per symbol

- T_s time per symbol
 - R_s symbol per second
 - E_s energy per symbol
 - SNR, Signal to noise ratio
 - ◆ average signal power relative to noise power
 - W Bandwidth, frequency band occupied by signal
 - Bandwidth utilisation: bits per second per Hz [bps/Hz]
- $T_b = T_s / k$ time per bit
 - $R_b = kR_s$ bit per second [bps]
 - $E_b = E_s / k$ energy per bit

$$\rho = \frac{R_b}{W}$$

Impairments on the communication channel (link)

- Attenuation
- Multipath propagation (fading)
- Noise



$$y(t) = x(t) * h(t) + n(t)$$

Noise disturbances

- Thermal noise (Johnson-Nyquist)
 - Generated by current in a conductor
 - -174 dBm/Hz ($=3.98 \cdot 10^{-18}$ mW/Hz)
- Impulse noise (Often user generated, e.g. electrical switches)
- Intermodulation noise (From other systems)
- Cross-talk (Users in the same system)
- Background noise (Misc disturbances)

https://en.wikipedia.org/wiki/Johnson-Nyquist_noise

Some Information Theory

Entropy

- Discrete case: X discrete random variable

$$H(X) = E[-\log_2 p(X)] = -\sum_x p(x) \log_2 p(x)$$

Entropy is uncertainty of outcome (for discrete case)

- Continuous case: X continuous random variable

$$H(X) = E[-\log_2 f(X)] = -\int_R f(x) \log_2 f(x) dx$$

Example Entropy

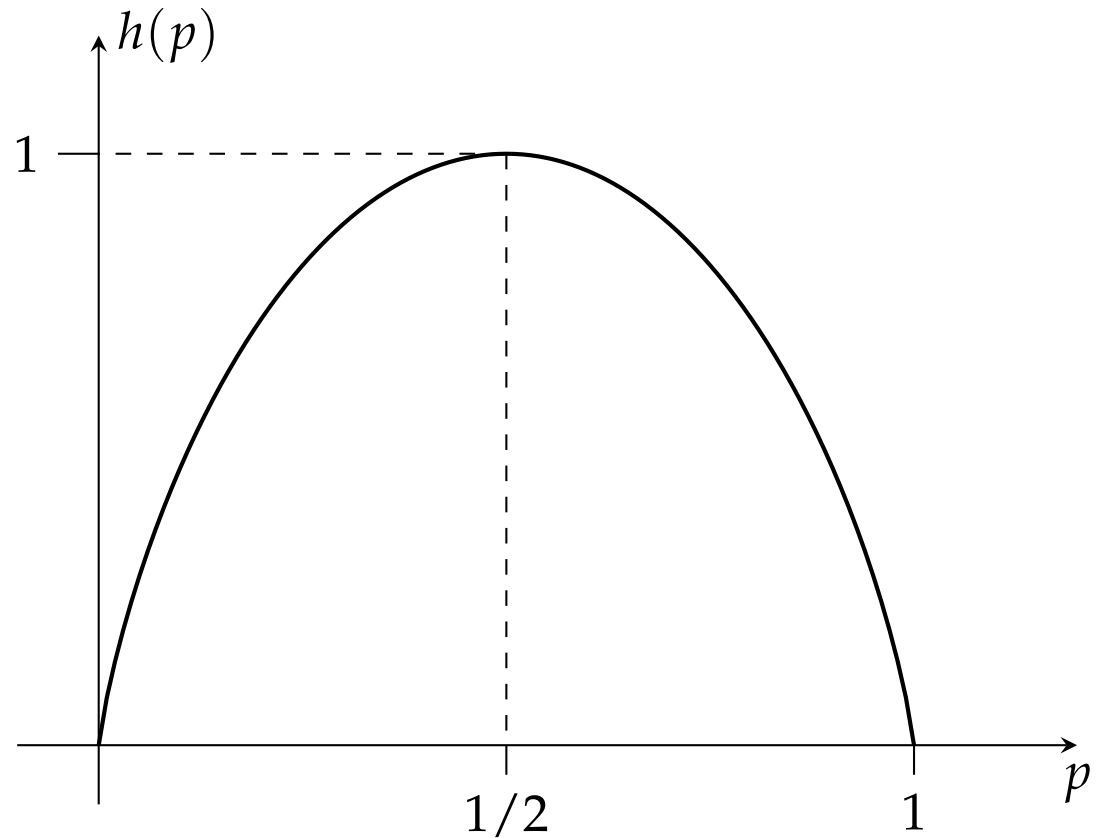
Let X be a binary random variable with

$$P(X=0)=p$$

$$P(X=1)=1-p.$$

The binary entropy function is

$$h(p) = -p \log_2 p - (1-p) \log_2 (1-p)$$



Compression

The entropy sets a limit on the compression ratio

- Consider a source for X with N symbols and the distribution $P(N)$. In average a symbol must be represented by at least $H(P)$ bits.
- Well known compression algorithms are *zip*, *gz*, *png*, *Huffman*
- Lossy compression e.g. *jpeg* and *mpeg*

Some more Information Theory

Mutual information

- Let X and Y be two random variables
- The information about X by observing Y is given by

$$I(X;Y) = E \left[\log_2 \frac{P(X,Y)}{P(X)P(Y)} \right]$$

- This gives

$$I(X;Y) = H(X) + H(Y) - H(X,Y)$$

Example Mutual Information

The random variables X and Y has the joint distribution

$P(X,Y)$	$Y=0$	$Y=1$
$X=0$	0	$3/4$
$X=1$	$1/8$	$1/8$

That gives

$$P(X=0) = 3/4 \quad \text{and} \quad P(X=1) = 1/4$$

$$P(Y=0) = 1/8 \quad \text{and} \quad P(Y=1) = 7/8$$

Entropies: $H(X) = h(\frac{1}{4}) = 0.8114$

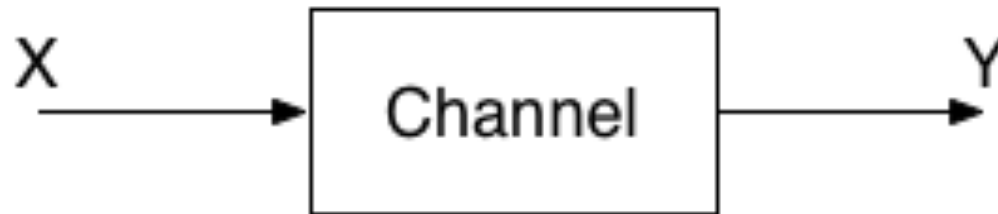
$$H(Y) = h(\frac{1}{8}) = 0.5436$$

$$H(X,Y) = -\frac{3}{4} \log \frac{3}{4} - \frac{1}{8} \log \frac{1}{8} - \frac{1}{8} \log \frac{1}{8} = 1.0613$$

Information: $I(X;Y) = H(X) + H(Y) - H(X,Y) = 0.2936$

Some Information Theory

Channel capacity



- The channel is a model of the transmission link.
- Transmit X and receive Y . How much information can the receiver get from the transmitter?
- The *channel capacity* is defined as

$$C = \max_{p(x)} I(X;Y)$$

AWGN

Additive White Gaussian Noise channel

- Let X be bandlimited in bandwidth W
- $Y = X + N$, where $N \sim N\left(0, \sqrt{N_0 / 2}\right)$
- The capacity is

$$C = W \log_2 \left(1 + \frac{P}{N_0 W} \right) \quad [\text{bps}]$$

- where P is the power of X , i.e. $E[X^2]=P$.
- It is not possible to get higher data rate on this channel!

AWGN Example (VDSL)

- Consider a channel with

$$W = 17 \text{ MHz}$$

$$P_{\Delta} = -60 \text{ dBm/Hz}$$

$$N_0 = -145 \text{ dBm/Hz}$$

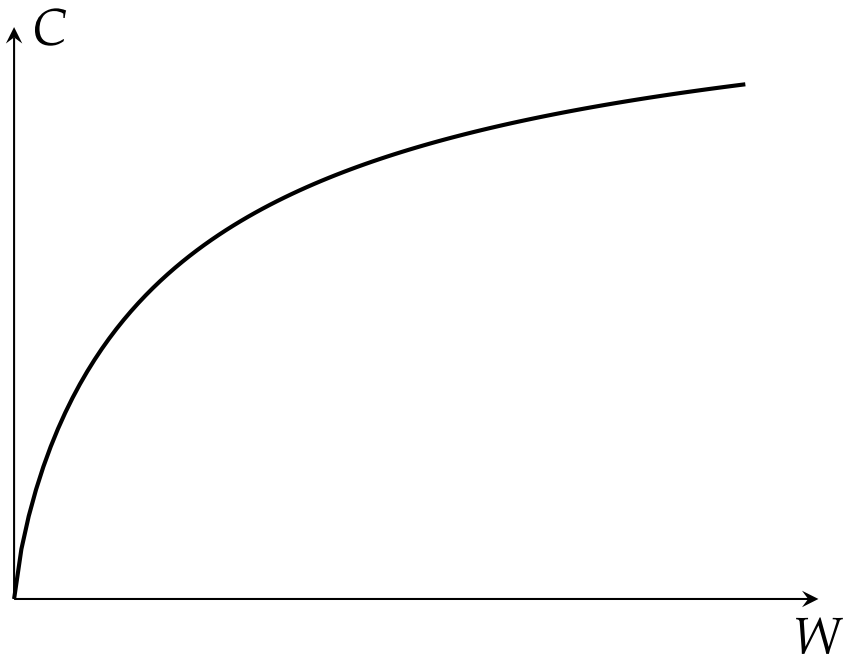
- Power $P = 10^{-60/10} \cdot 17 \cdot 10^6 \text{ mW}$

- Noise $N_0 = 10^{-145/10} \text{ mW/Hz}$

- Capacity $C = W \log\left(1 + \frac{P}{N_0 W}\right) = W \log\left(1 + \frac{10^{-60/10}}{10^{-145/10}}\right) = 480 \text{ Mbps}$

Shannon's fundamental limit

- Plot capacity vs W



- Is there a limit?

- Let $W \rightarrow \infty$

$$C_{\infty} = \lim_{W \rightarrow \infty} W \log \left(1 + \frac{P/N_0}{W} \right)$$

$$= \lim_{W \rightarrow \infty} \log \left(1 + \frac{P/N_0}{W} \right)^W = \log e^{P/N_0} = \frac{P/N_0}{\ln 2}$$

- With $E_b = PT_b$ and $T_s = kT_b$

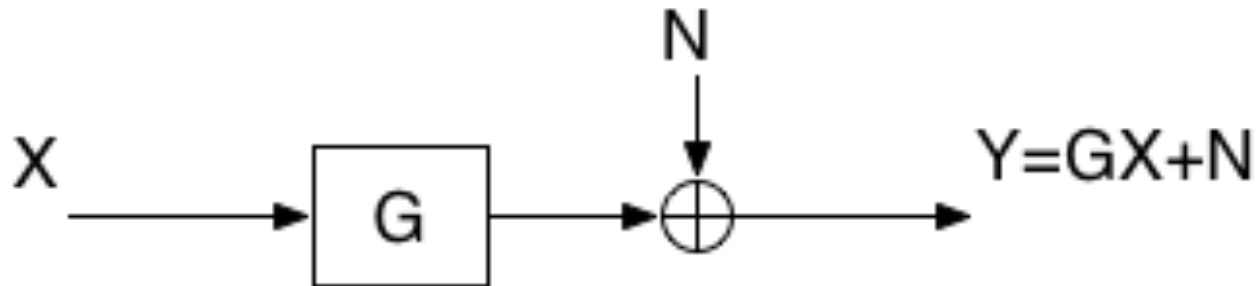
$$\frac{C_{\infty}}{R_b} = \frac{E_b / N_0}{\ln 2} > 1$$

- Which gives the *fundamental limit*

$$\frac{E_b}{N_0} > \ln 2 = -1.59 \text{ dB}$$

AWGN with attenuation

- Let X be bandlimited in bandwidth W
- Let G be attenuation on channel, $G < 1$



- The capacity is

$$C = W \log_2 \left(1 + \frac{|G|^2 P}{N_0 W} \right) \quad [\text{in bit/s}]$$