LUND UNIVERSITY
Electrical and Information Technology

Communication and Networks

Problems Phy layer

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Problems

1. An analog signal is sampled and reconstructed by an ideal system using the samplung frequency $F_s = 16$ Hz. What is the reconstructed signal if the original is

   (a) $s(t) = \cos(2\pi 10t)$

   (b) $s(t) = \sin(2\pi 10t)$

   (c) $s(t) = 2\sin(2\pi 5t) + \sin(2\pi 11t)$

2. What is the duration of a bit if the signalling rate is

   (a) 100 kb/s

   (b) 2 Mb/s

3. Assume that we want to transmit a bit sequence consisting of 5 zeros followed by 5 ones. Code the sequence with the following line coding schemes:

   (a) On-Off keying
   (b) NRZ
   (c) Manchester
   (d) Differential Manchester

4. Decode the waveforms according to

   (a) Manchester coding
   (b) Differential Manchester

i)


ii)


iii)


5. An analog information source is sampled with a rate of 44100 samples/s, and each sample is represented by 16 bits. The average transmitted signal power equals 10^{-2} $[V^2]$. Calculate the average transmitted energy per bit.

6. Assume the communication channel consists of an attenuation of α and addition of noise. Then, if $s(t)$ is transmitted the received signal is $r(t) = \alpha s(t) + w(t)$, where $|\alpha| \leq 1$ and $w(t)$ is the noise. If the communication distance is long the attenuation can be too large, i.e. the value $|\alpha|$ can be too small. Then the received information

carrying signal $z(t) = \alpha s(t)$ can be too weak compared with the additive noise $w(t)$, resulting in too many bit errors. Determine an expression of the average received energy per information bit, denoted $E_b^{(rec)}$, that contains α, the bit rate R_b and the average transmitted signal power P.

7. An analog (real baseband) speech signal s(t) is modulated up to $f_0 = 20$ kHz using

$$s_m(t) = s(t) \cdot \cos(2\pi f_0 t)$$

   (a) To down convert the signal the same modulation by cosinus can be used followed by an ideal low-pass filter. What would you set the breaking frequency to in the LP filter and what is the amplitude of the resulting signal?

   (b) The signal can also be down converted using sampling and alisasing in the reconstruction. What would you set the sampling frequency to?

8. Given an OFDM based channel were Each OFDM symbol consists of 512 carriers. The number of bits for each carrier is, of course, dependent on the SNR for that carrier. Let us assume that the bit-loading is such that the 200 first carriers can be loaded with 15 bits each, the next 150 carriers can be loaded with 11 bits and the remaining carriers can be loaded with 4 bits each. What is the channels bit rate if the OFDM symbol rate is 8000 symbols per second?

9. Consider a 16-QAM constellation. It is stated in the text that this can be viewed as two orthogonal 4-PAM constellations. This is in general true, but it also has to do with the mapping between bits and signal alternatives.

   (a) Show how two different 4-PAM signals can be modulated with orthogonal signals to form a 16-QAM.

   (b) Give two mappings between bits and signals; one where the constellation can be split into two 4-PAM, and one where it cannot be split.

10. A DVD video is compressed using MPEG-2 and in this problem the compression ratio will be estimated. The video is recorded at 25 frames per second and the maximum resolution is 720×576 pixels per frame. A pixel consists of three colours quantised using 8 bits each. A normal movie of about 2 hours takes about 8 GB on the disk. Defining the compression ratio as uncompressed size divided by the compressed size, what is this for a DVD video? If there would be no compression how long time can you record on a DVD?

11. In LTE (the 4G mobile system) a frame represents a transmission time of 10 ms. As described in the Figure 1 a frame consists of ten sub-frames, and each sub-frame consists of two slots. A slot is built from a sequence of seven OFDM frames, where the name comes from the modulation scheme OFDM used in LTE. An OFDM frame consists of a complex sample vector of length *FFTsize* appended with a cyclic prefix, *CP*. The CP of the first OFDM symbol is of length CP_a and for the six remaining the length is CP_b.

    An LTE link can be set up with six different bandwidths $W \in \{1.4, 3, 5, 10, 15, 20\}$ MHz. In the table below the bandwidths and their corresponding FFTsizes and CP lengths, CP_a and CP_b, are listed. The lengths are given in number of samples. Since it is a radio signal the samples are complex valued.

    Internally in the base station, when passing the radio signal from the baseband unit to the radio unit, a protocol called CPRI is often used. Each real sample is then quantised and represented by 15 bits.

    For each of the bandwidths:

    (a) Determine the symbol time T_S and symbol rate R_S for frames, sub-frames, slots and OFDM frames. (Since the OFDM frames are not equal length, give an average value).

    (b) Find the sample frequency F_s, bit time T_b and the bit rate R_b required between the baseband unit and the radio unit.

    (c) The OFDM modulation is based on FFT (Fast Fourier Transform) where a vector of N samples in the frequency domain is transformed to N samples in the time domain. The size of the vectors is the value FFTsize in the description. Each sample before the transform (in frquency domain) represents at most six bits of data (neglecting some synchronisation and management data). There is also a guard band in the frame not used for transmission, meaning only a part of the samples is used for data. In the table the value Data Samples is the number of samples per FFT frame used to store data. Derive the expansion of bit rate, i.e. the ratio between the efficient bit rate for data and the bit rate for the samples between the baseband unit and the radio unit.

W [MHz]	1.4	3	5	10	15	20
FFTsize	128	256	512	1024	1536	2048
CP_a	10	20	40	80	120	160
CP_b	9	18	36	72	108	144
Data Samples	72	180	300	600	900	1200

12. Consider an urn with ten balls, three black and seven white. Draw first one ball and call this X. Then, without replacement, draw a second ball, Y.

    (a) What is the uncertainty (entropy) of X?

    (b) What is the uncertainty (entropy) of Y?

    (c) How much information about the colour of the second ball do you get from the first ball?

13. Consider an OFDM channel with five subchannels, each occupying a bandwidth of 10 kHz. Due to regulations the allowed power level in the utilised band is -60 dBm/Hz. Each of the subchannels have separate attenuation and noise levels according to Figure 2. Notice that the attenuation $|G_i|^2$ is given in dB and the noice N_0, i in dBm/Hz, where i is the subchannel. Derive the total capacity for the channel.

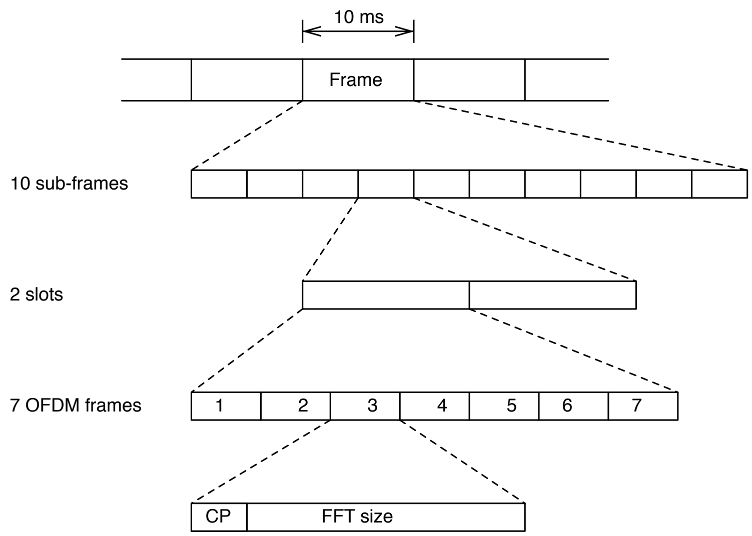


Figure 1: Frame structure in the LTE mobile system.

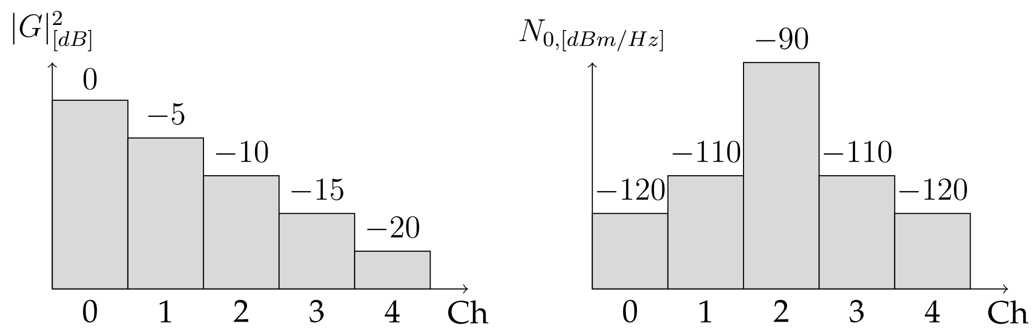


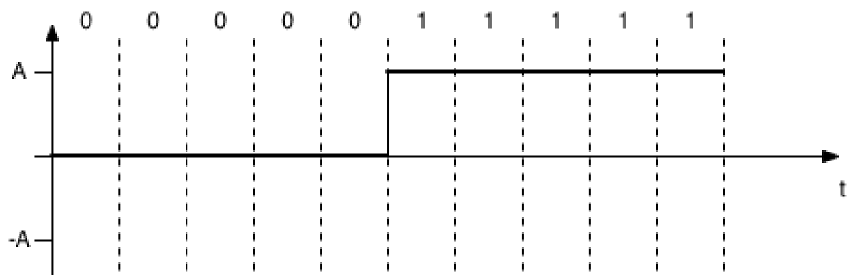
Figure 2: Attenuation and noise in the five subchannels.

Solutions

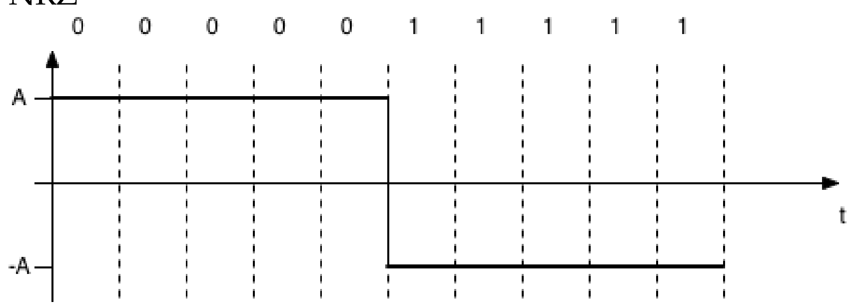
1. (a) $s(t) = \cos(2\pi 6t)$
   (b) $s(t) = -\sin(2\pi 6t)$
   (c) $s(t) = sin(2\pi 5t)$

2. (a) $T_b = 1/R_b = 10\mu s$
   (b) $T_b = 0.5\mu s$

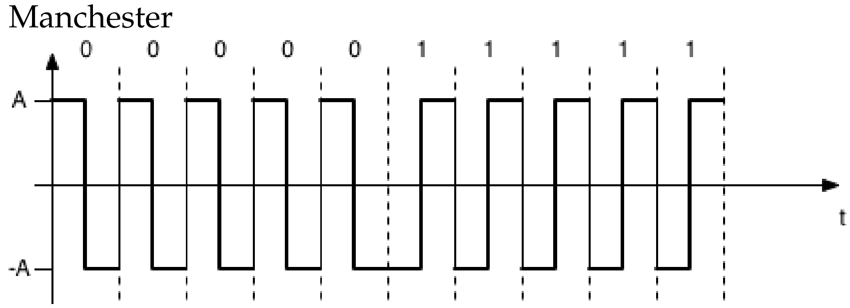
3. (a) On-Off



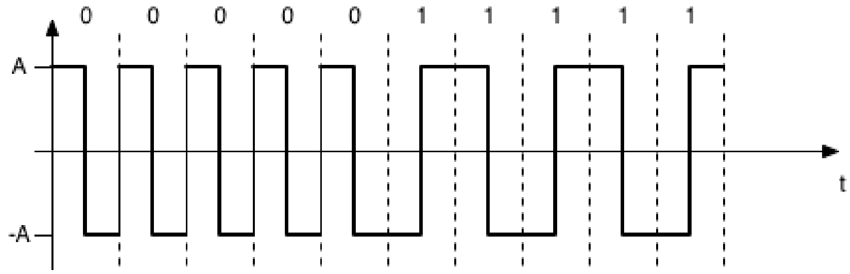
   (b) NRZ



   (c) Manchester



(d) Differential Manchester



4. (a) i) 1110011010 ii) 11001100001 iii) 00011100
   (b) i) 001010111 ii) 0101010001 iii) 0010010

5. $E_b = \dfrac{P}{R_b} = \dfrac{10^{-2}}{44100 \cdot 16} = 1.42 \cdot 10^{-8}\ [V^2 s]$

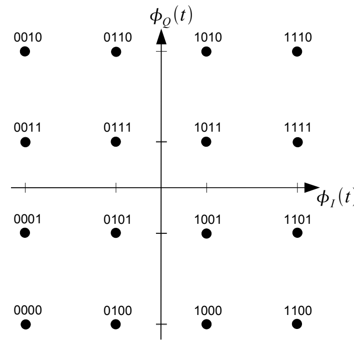
6. $E_b^{(rec)} = \alpha^2 P / R_b$

7. (a) The breaking frequency can e.g. be set to f_0, and the amplitude becomes half the amplitude of the original signal. It should be larger than the bandwidth of the signal but lower than the repetition at $2f_0$
   (b) Set $F_s = f_0$ and filter with a LP filter (reconstruction filter) breaking at $F_s/2$.

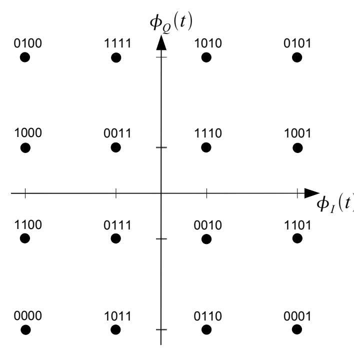
8.

$$R_b = (200 \times 15 + 150 \times 11 + (512 - 200 - 150) \times 4) \times 8000 = 42.4 Mb/s$$

9. In a 16-QAM constellation there are four bits for each signal alternative. Use the first two as a mapping to a 4-PAM constellation in horizontal direction, and the last two as a mapping to a 4-PAM constellation in vertical direction. Then we get the mapping according to the figure below



Of course, it is easy to find a mapping that is not possible to split in two dimensions, for example the following.



10. The bit rate from DVD (in average):

$$R_{b,DVD} = \frac{8 \cdot 10^9 \cdot 8}{2 \cdot 60 \cdot 60} = 8.9 Mb/s$$

Bit rate from uncompressed source:

$$R_{b,uncomp} = 25 \cdot 720 \cdot 576 \cdot 3 \cdot 8 = 249 Mb/s$$

The compression ratio is $249/8.9 = 28$. Without compression the time on a DVD would be $2 \cdot 60/28 = 4.28$ minutes. (Alternative derivation $\frac{8 \cdot 10^3 \cdot 8}{249*60} = 4.28$ minutes).

11. (a) For all bandwidths:

T_s [ms]				R_s [kHz]			
frame	sub frame	slot	OFDM	frame	sub frame	slot	OFDM
10	1	0.5	0.0714	0.1	1	2	14

(b) With $2 \times 15 = 30$ bit/complex sample we get the following table. This is only the one-way direction and for one LTE antenna the total bit rate must be doubled.

W [MHz]	1.4	3	5	10	15	20
FFTsize	128	256	512	1024	1536	2048
CPa	10	20	40	80	120	160
CPb	9	18	36	72	108	144
Sampl/slot (7FFTsize + 6CPb + CPb)	960	1920	3840	7680	11520	15360
F_s [MHz, i.e. S/s] ($S/slot \times 2k$)	1.92	3.84	7.68	15.36	23.04	30.72
R_b [Mb/s] ($F_s \times 30$)	57.6	115.2	230.4	460.8	691.2	921.6
T_b [ns] ($1/R_b$)	17.3	8.68	4.34	2.17	1.45	1.09

(c) With 14 000 OFDM frames per second and 6 b/sample we get the following table.

W [MHz]	1.4	3	5	10	15	20
R_b [Mb/s] data ($DS \times 6 \times 14k$)	6.05	15.1	25.2	50.4	75.6	100.8
Expansion	9.5	7.6	9.1	9.1	9.1	9.1

(In many network architecture discussions today, the BBU is centralized in the network while the RU is located further out. Then it is assumed that CPRI is used for transmitting the radio signal over the network, meaning an extensive increase in traffic since each RU serves several LTE antennas. It should also be considered that the next generation LTE, LTE-A, uses a bandwidth of 100 MHz, containing five 20 MHz LTE channels.)

12. (a) The distribution of the first ball is

X	b	w
$p(x)$	$3/10$	$7/10$

which gives the entropy $H(X) = h(\frac{3}{10}) = 0.8813$ bit.

(b) The probability $P(Y)$ is unconditional of X, and as such there is no knowledge about the outcome of it. In that case the order of the drawn balls are irrelevant and Y gets the same distribution as X. And the same entropy.

(c) To get the joint probability of the pair (X, Y) we can go via the conditional probability $P(Y|X)$ to get $P(X, Y) = P(X)P(Y|X)$ as the tables below.

$P(Y\|X)$		Y: b	Y: w
X	b	$2/9$	$7/9$
	w	$3/9$	$6/9$

$P(XY) = P(X)P(Y\|X)$		Y: b	Y: w
X	b	$\frac{3}{10}\frac{2}{9}$	$\frac{3}{10}\frac{7}{9}$
	w	$\frac{7}{10}\frac{3}{9}$	$\frac{7}{10}\frac{6}{9}$

hence $H(XY) = H(\frac{6}{90}, \frac{21}{90}, \frac{21}{90}, \frac{42}{90}) = 1.7534$ bit. And the mutual information is

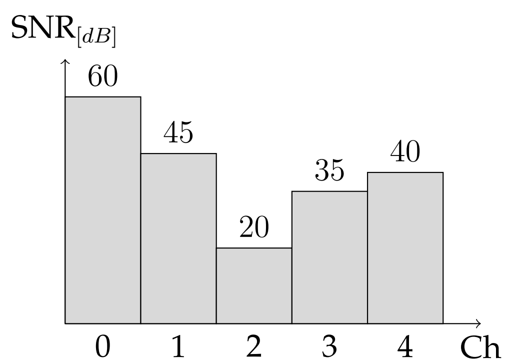
$$I(X;Y) = H(X) + H(Y) - H(XY) = 0.0092 \text{ bit}$$

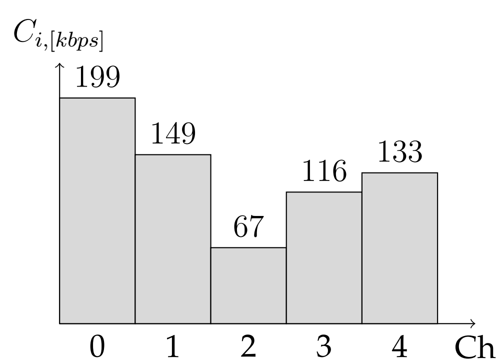
(Which is not much at all)

13. The allowed power level $P_\Delta = -60$ dBm/Hz $= 10^{-60/10}$ mW/Hz gives the total power in a sub-channel as $P = P_\Delta W$ mW, where $W = 10$ kHz is the bandwidthm. A useful measure of the SNR is (in linear scale, i.e. not dB)

$$\text{SNR}_i = \frac{P|G_i|^2}{N_{0,i}W} = \frac{P_\Delta W|G_i|^2}{N_{0,i}W} = \frac{P_\Delta|G_i|^2}{N_{0,i}}$$

In dB scale this gives $\text{SNR}_i = P_\Delta + |G_i|^2 - N_{0,i}$ and is shown below,





The capacity per sub-channel is derived as $C_i = W\log(1 + \text{SNR})$, shown above. The total capaicty is the sum, $C = \sum_i C_i = 199 + 149 + 67 + 116 + 133 = 665$ kbps.