ETSF15
Physical layer

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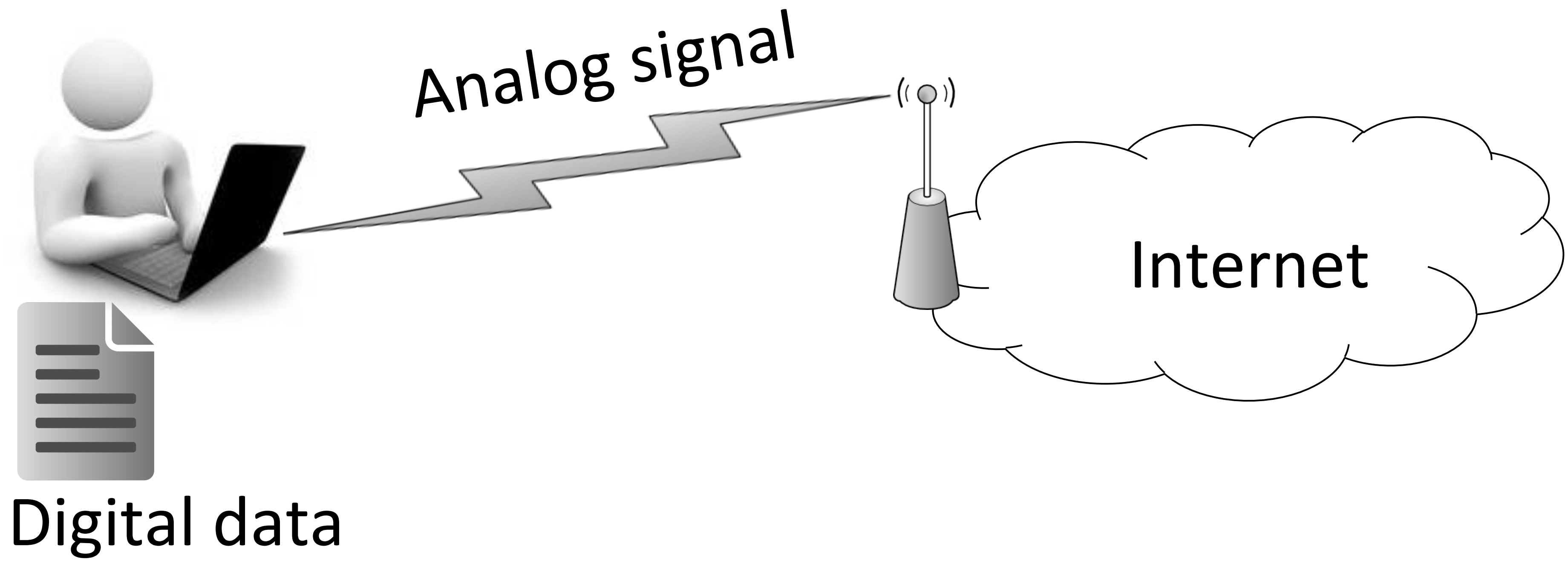
Physical layer

- Analog vs digital
  - Sampling, quantisation
- Modulation
  - Represent digital data in a continuous world
- Disturbances
  - Noise and distortion
- Digital data processing
  - Information

From bits to signals

- Principles of digital communications



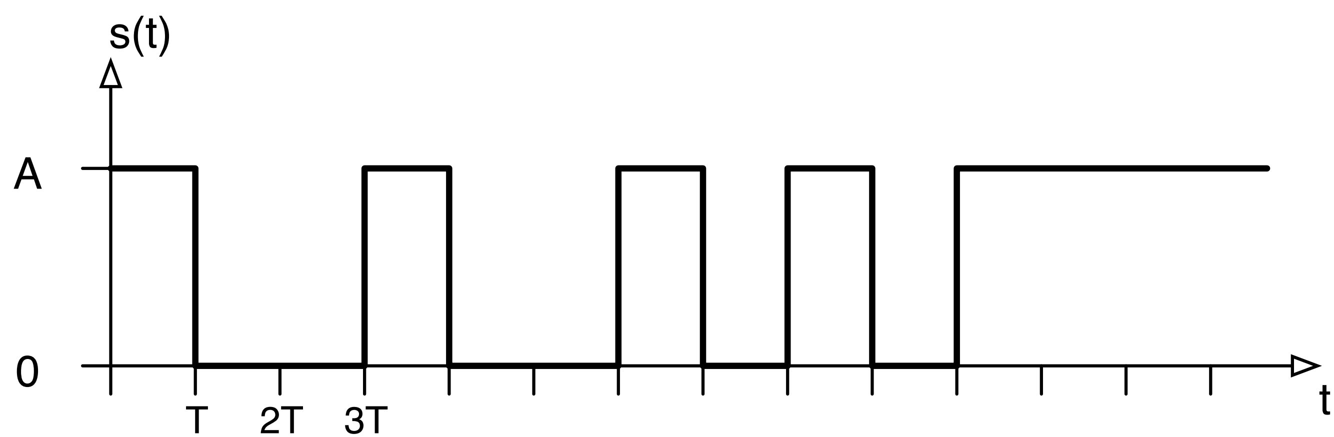
On-Off keying

- Send one bit during T_b seconds and use two signal levels, “on” and “off”, for 1 and 0.

$$a(t) = A \cdot x \qquad 0 \leq t \leq T_b$$

Ex.

x=1001001010111100



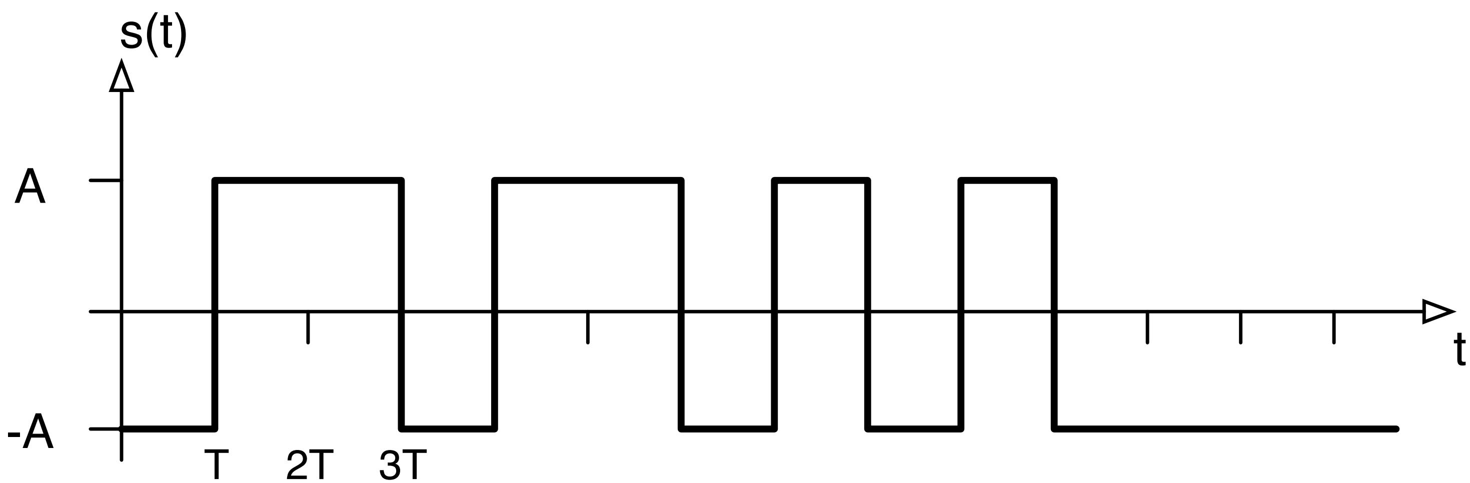
Non-return to zero (NRZ)

- Send one bit during T_b seconds and use two signal levels, *+A* and *-A*, for 0 and 1.

$$a(t) = A \cdot (-1)^x \quad 0 \le t \le T_b$$

Ex.

x=1001001010111100



Mathematical description

- With $g(t)=A$, $0<t<T$, the signals can be described as

$$s(t)=\sum_n a_n g(t-nT)$$

- On-off

$$a_n = x_n$$

- NRZ

$$a_n = (-1)^{x_n}$$

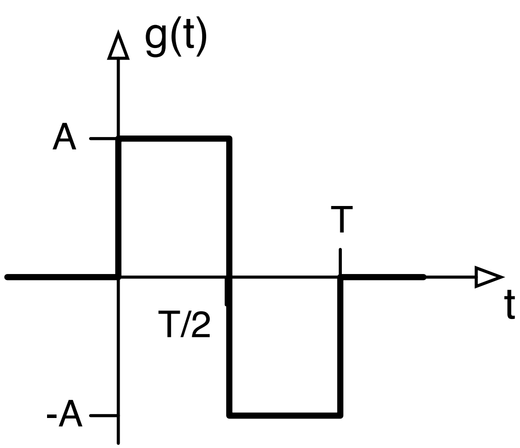
Two signal alternatives

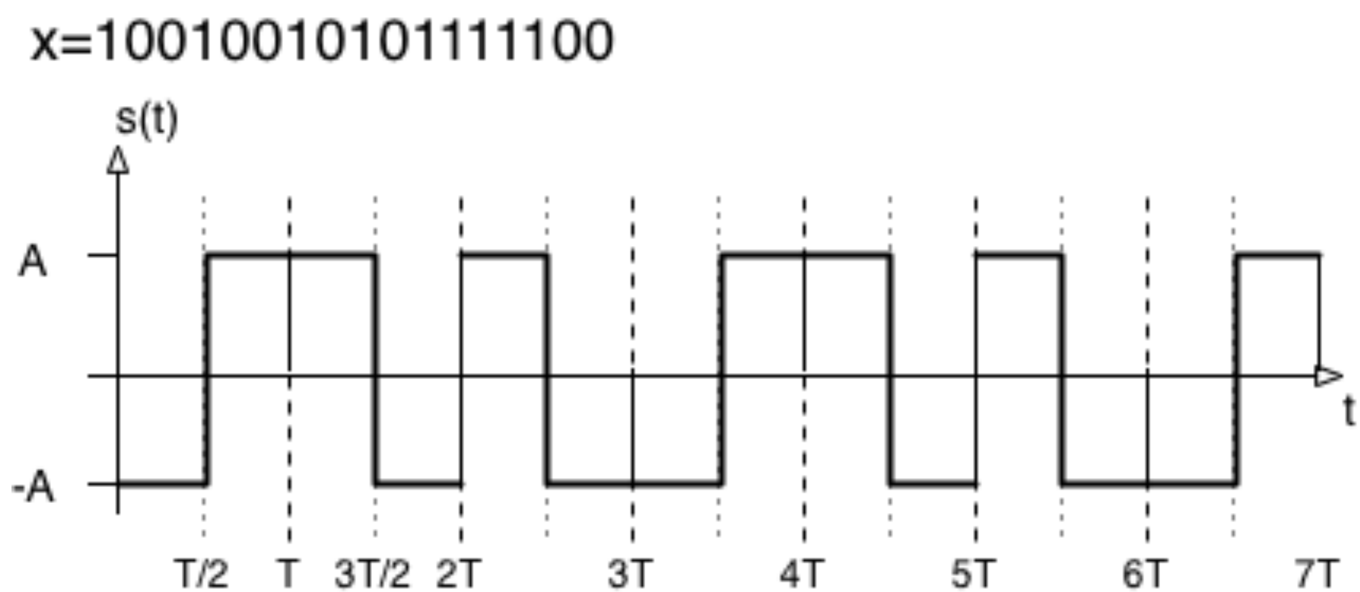
- $s_0(t)=0$ and $s_1(t)=g(t)$
- $s_0(t)=g(t)$ and $s_1(t)=-g(t)$

Manchester coding

- To get a zero passing in each signal time, split the pulse shape *g(t)* in two parts and use +/- as amplitude.

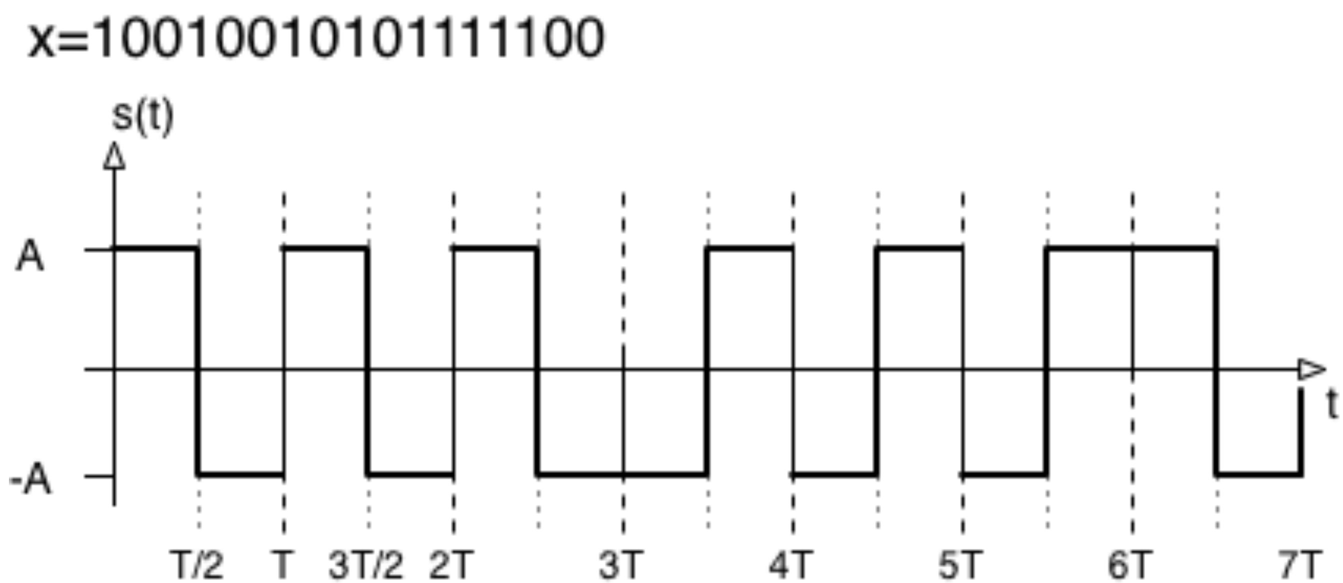
Ex.





Differential Manchester coding

- Use a zero transition at the start to indicate the data.
- For a transmitted 0 the same pulse as previous slot is used, while for a transmitted 1 the inverted pulse is used.

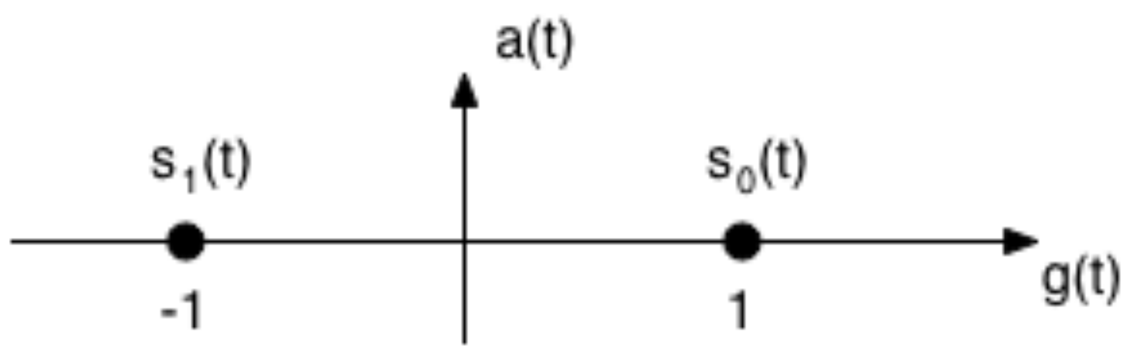


PAM (Pulse Amplitude Modulation)

- NRZ and Manchester are forms of binary PAM
- The data is stored in the amplitude and transmitted with a pulse shape g(t)

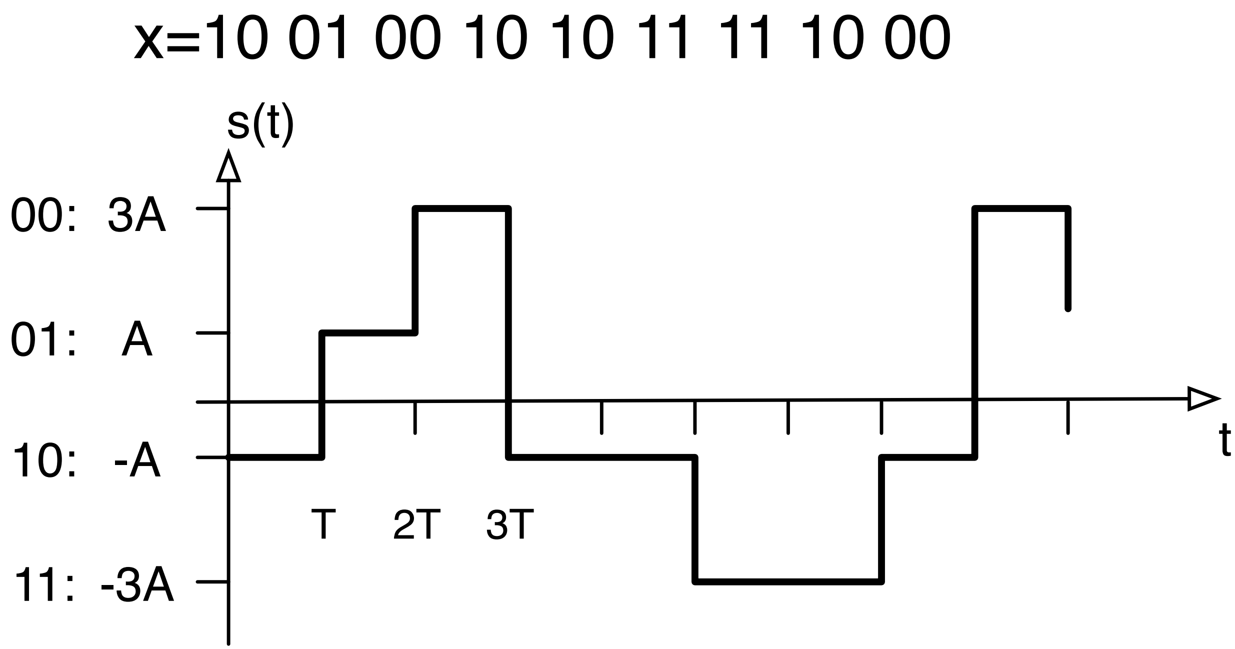
$$a(t) = a_n \cdot g(t) \qquad a_n = (-1)^x$$

- Graphical representation



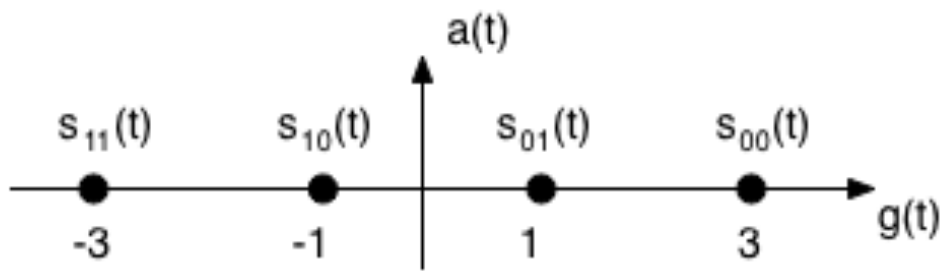
M-PAM

- Use $M=2^k$ amplitude levels to represent k bits
- Ex. Two bits per signal (4-PAM)

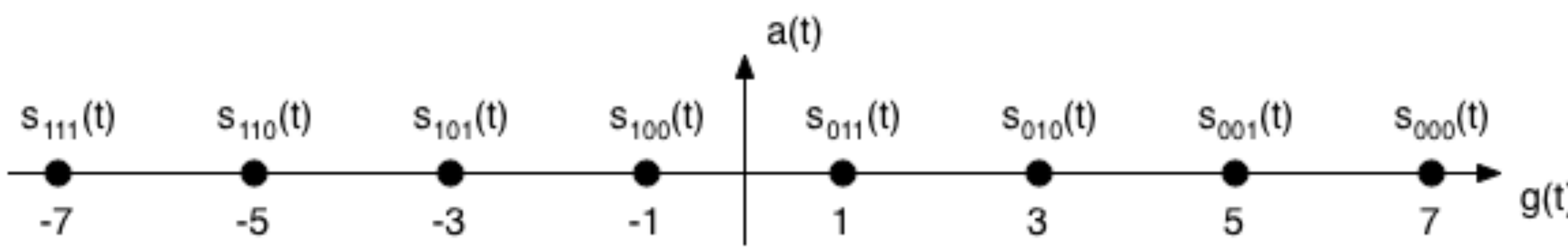


M-PAM

- Ex: 4-PAM

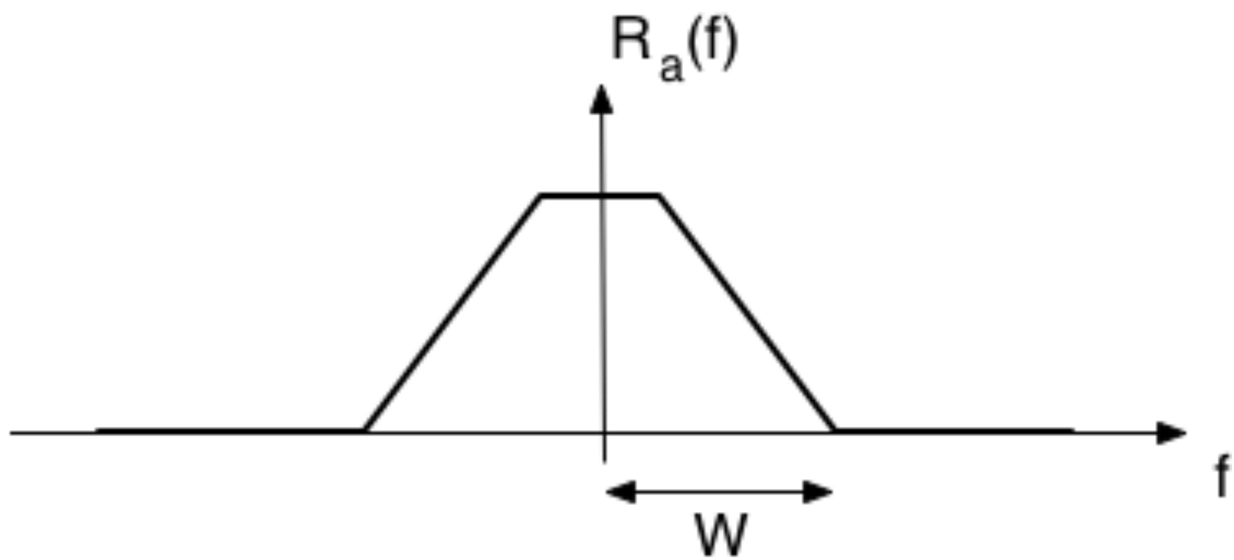


- Ex: 8-PAM



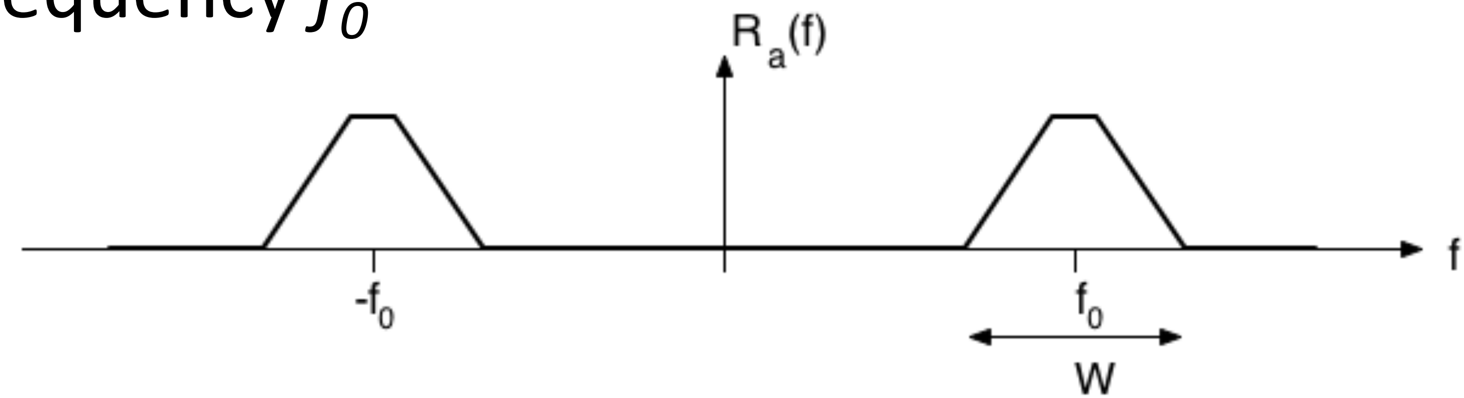
Bandwidth of signal

- The **bandwidth**, *W*, is the (positive side) frequency band occupied by the signal
- So far only **base-band** signals (centered around *f=0*)



Pass-band signal

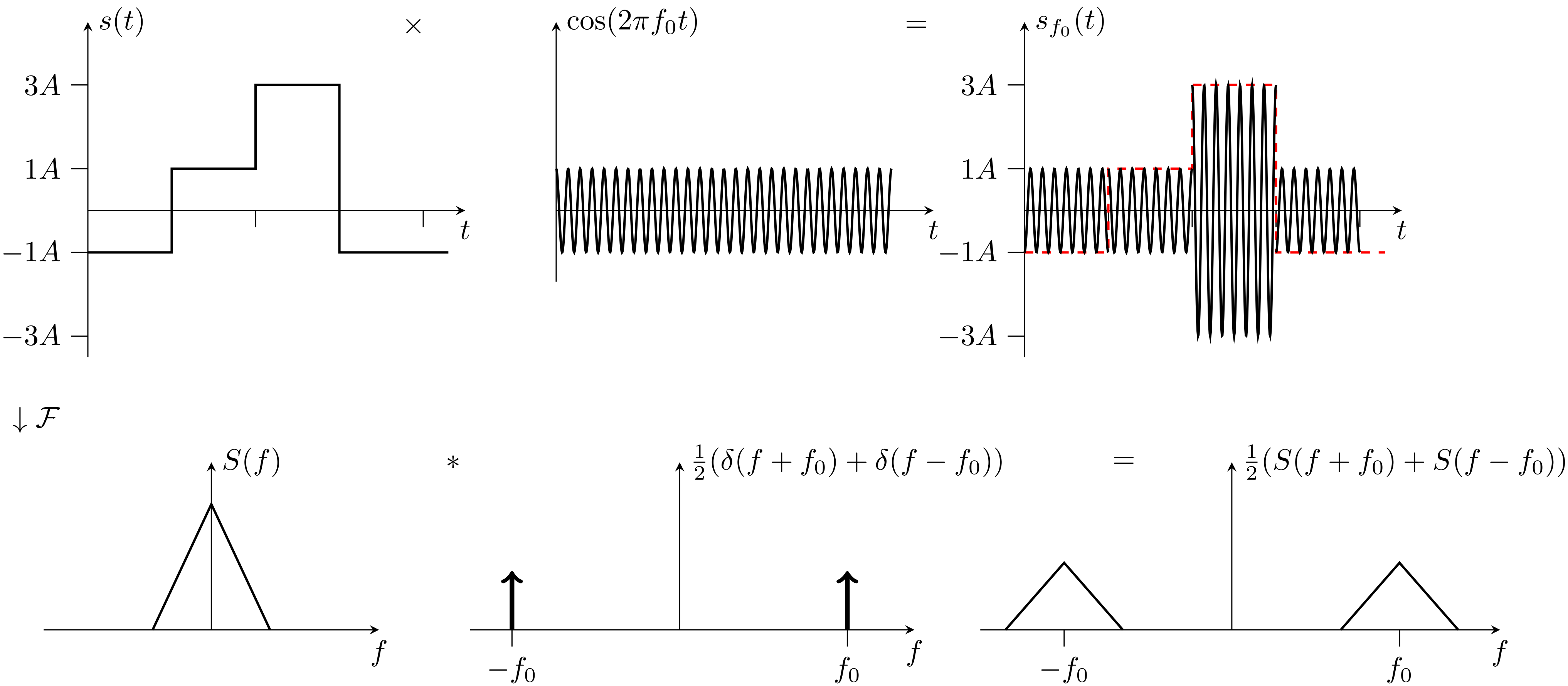
- Frequency modulate the signal to a carrier frequency f_0



- The following multiplication centers the signal around f_0

$$s(t) = a(t) \cdot \cos(2\pi f_0 t)$$

Modulation in frequency



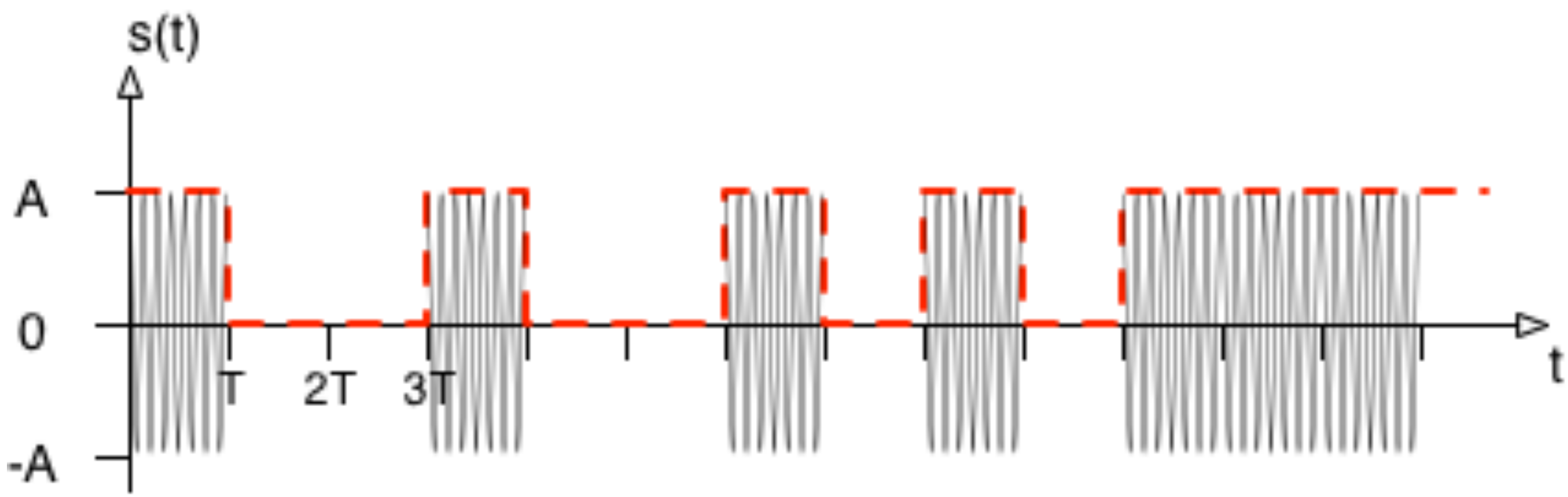
Modulated On-Off keying

- Use on-off keying at frequency f_0.

$$s(t) = \sum_n x_n g(t - nT)\cos(2\pi f_0 t)$$

- Ex.

x=1001001010111100

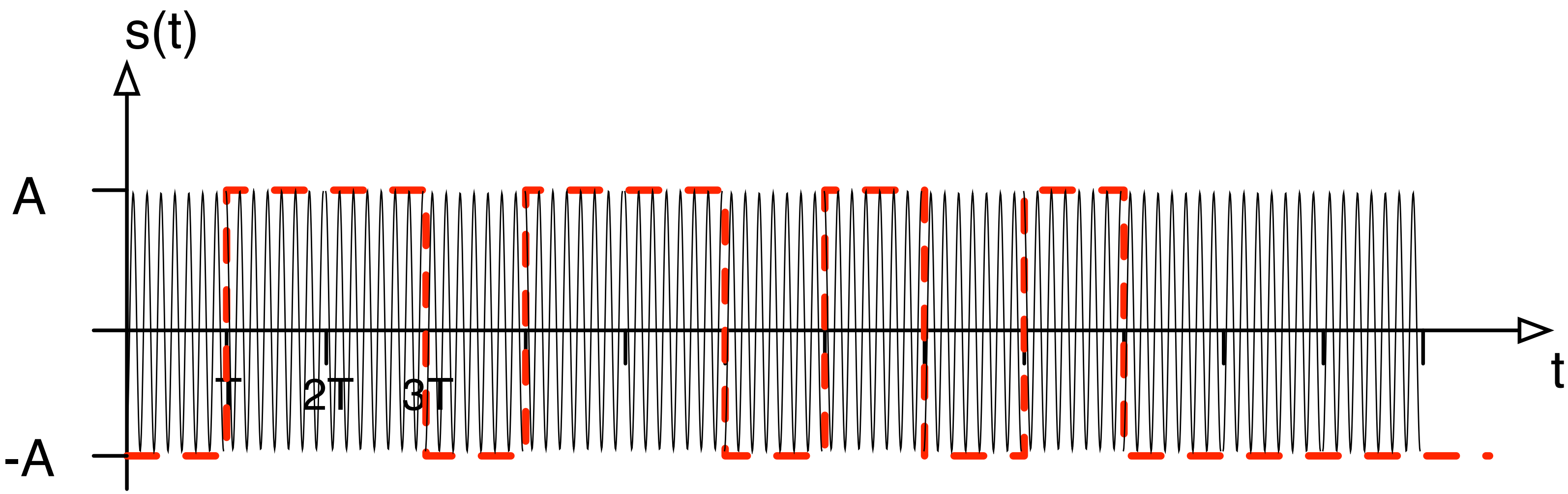


BPSK (Binary Phase Shift Keying)

- Use NRZ at frequency f_0, but view information in phase

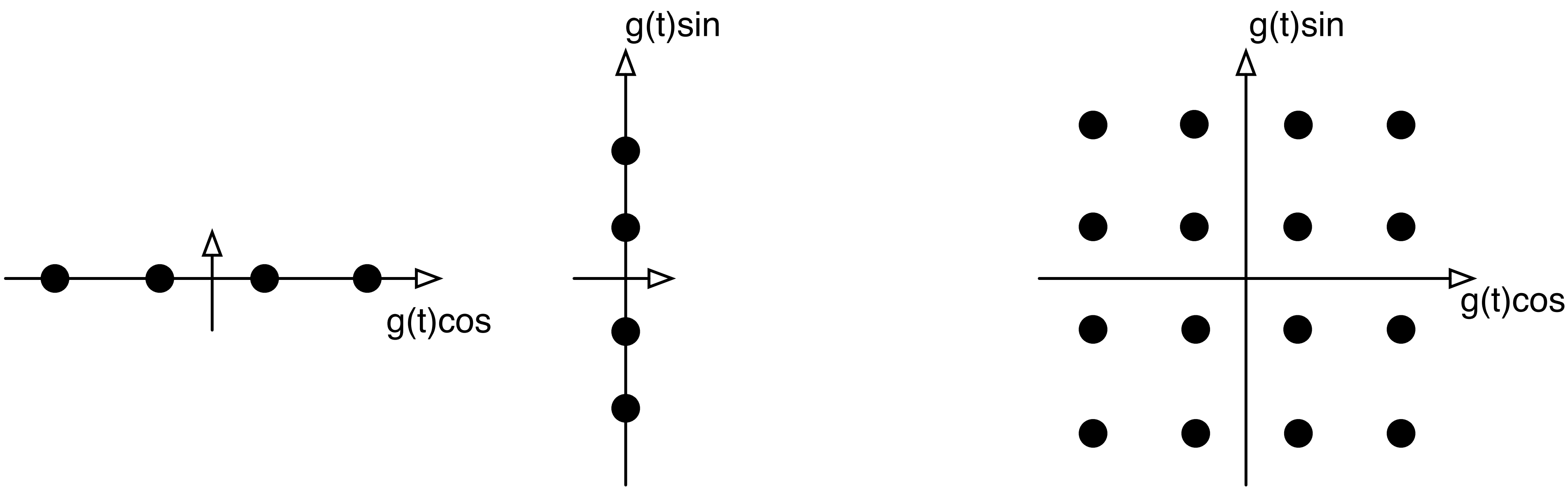
$$s(t) = \sum_n (-1)^{x_n} g(t - nT)\cos(2\pi f_0 t) = \sum_n g(t - nT)\cos(2\pi f_0 t + x_n \pi)$$

x=100100101011111100



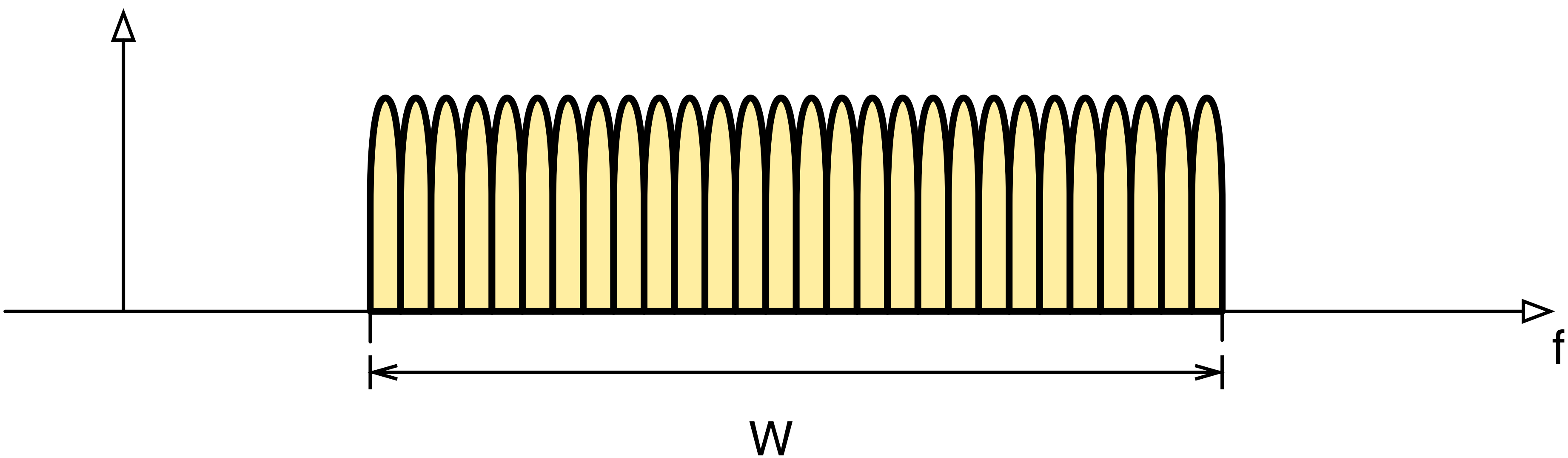
M-QAM (Quadrature amplitude Modulation)

Use that *cos(2πf*$_0$*t)* and *sin(2πf*$_0$*t)* are orthogonal (for high f_0) to combine two orthogonal PAM constellations



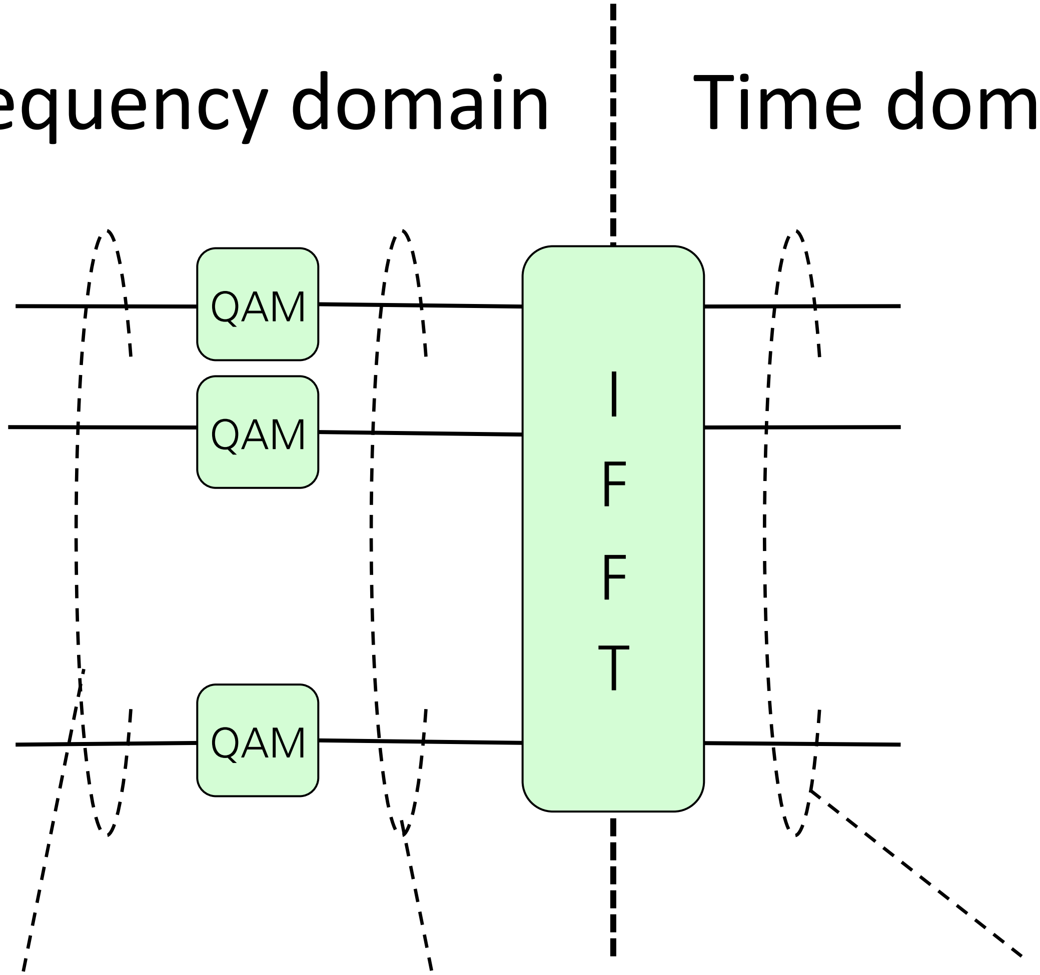
OFDM
Orthogonal Frequency Division Multiplexing

- N QAM signals combined in an orthogonal manner
- Used in e.g. ADSL, VDSL, WiFi, DVB-C&T&H, LTE, etc



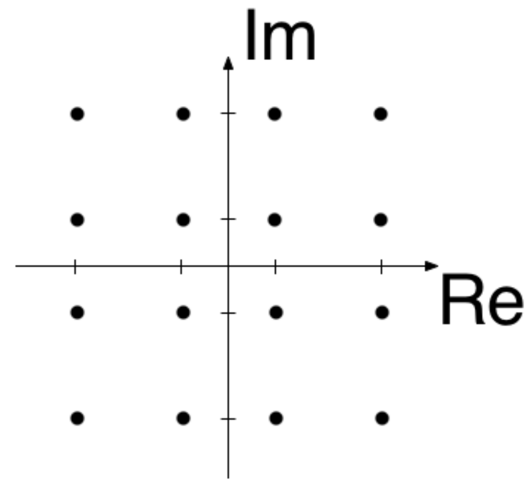
Idea of OFDM implementation

Frequency domain | Time domain



$(a_1,\ldots,a_N)\in Z_{16}{}^N$ $(x_1,\ldots,x_N)\in\mathbb{C}^N$ $(y_1,\ldots,y_N)=IFFT(\vec{x})$

QAM mapping



Some important parameters

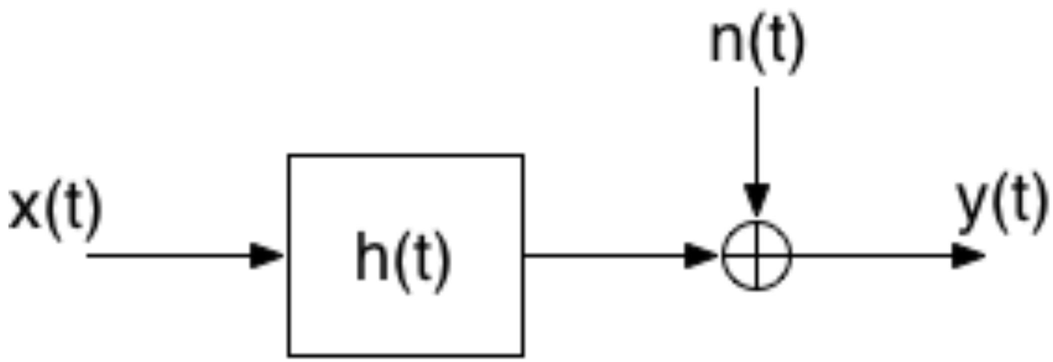
- T_s time per symbol
- R_s symbol per second
- E_s energy per symbol
- $T_b=T_s/k$ time per bit
- $R_b=kR_s$ bit per second [bps]
- $E_b=E_s/k$ energy per bit

- SNR, Signal to noise ratio: ratio of signal energy and noise energy
- W Bandwidth, frequency band occupied by signal
- Bandwidth utilisation: bits per second per Hz [bps/Hz]

$$\rho = \frac{R_b}{W}$$

Impairments on the communication channel (link)

- Attenuation
- Multipath propagation (fading)
- Noise



$$y(t) = x(t) * h(t) + n(t)$$

Noise disturbances

- Thermal noise (Johnson-Nyquist)
  - Generated by current in a conductor
  - -174 dBm/Hz (=$3.98 \cdot 10^{-18}$ mW/Hz)
- Impulse noise (Often user generated, e.g. electrical switches)
- Intermodulation noise (From other systems)
- Cross-talk (Users in the same system)
- Background noise (Misc disturbances)

https://en.wikipedia.org/wiki/Johnson-Nyquist_noise

Some Information Theory

Entropy

- Discrete case: *X* discrete random variable

$$H(X) = E[-\log_2 p(X)] = -\sum_x p(x)\log_2 p(x)$$

  Entropy is uncertainty of outcome (for discrete case)

- Continuous case: *X* continuous random variable

$$H(X) = E[-\log_2 f(X)] = -\int_R f(x)\log_2 f(x)dx$$

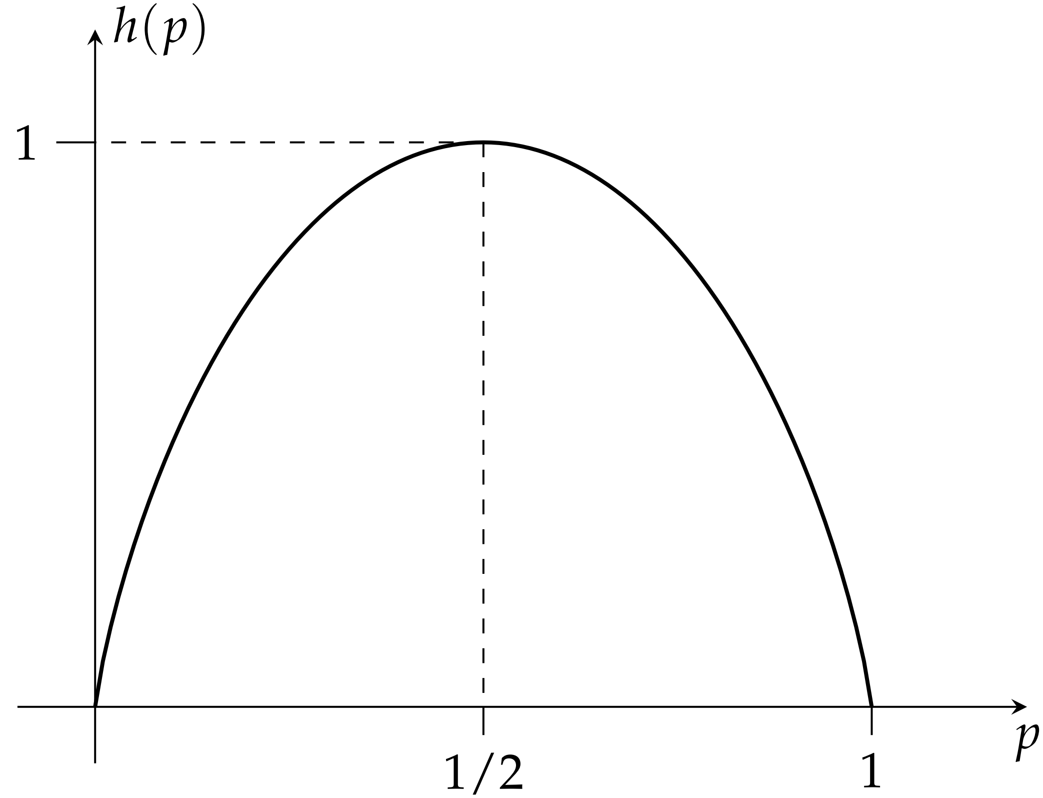
Example Entropy

Let *X* be a binary random variable with

P(X=0)=p

P(X=1)=1-p.

The binary entropy function is



$$h(p) = -p\log_2 p - (1-p)\log_2(1-p)$$

Compression

The entropy sets a limit on the compression ratio

- Consider a source for *X* with *N* different symbols and the distribution *P(X)*. In average a symbol must be represented by *H(X)* bits.

- Well known compression algorithms are zip, gz, png, Huffman
- Lossy compression e.g. jpeg and mpeg

Huffman coding

Given a random variable $X \in \{x_1, x_2, \ldots, x_N\}$ with probabilities $P(X = x_i) = p_i$

Algorithm:

- INIT: List all symbols as nodes
- REPEAT:
  - Merge the two least probable nodes, i and j, in a binary tree, and list as one node with probability p_i+p_j
  - If only one node left STOP
- Label the branches of the constructed tree with 0 and 1

The obtained compression code is optimal for i.i.d. sequences.

Optimal means minimal expected length per symbol, over all codes

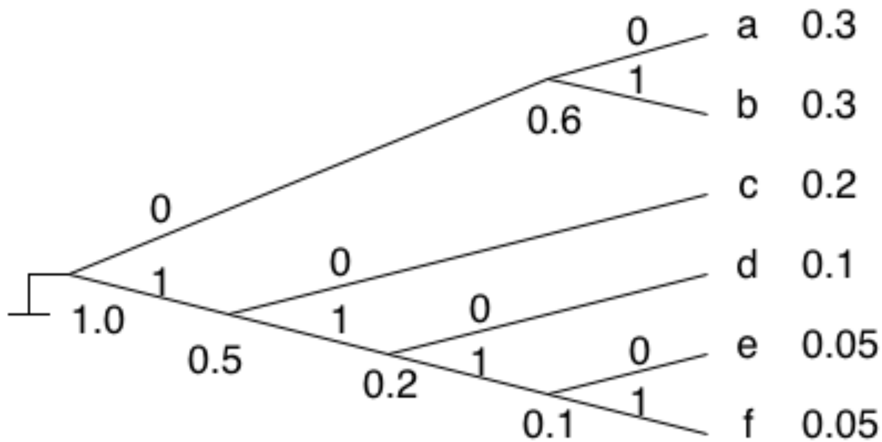
Huffman example

$X \in \{a,b,c,d,e,f\}$

Probabilities

X	P(X)
a	0.3
b	0.3
c	0.2
d	0.1
e	0.05
f	0.05

Construct tree



Code book

X	Y	L
a	00	2
b	01	2
c	10	2
d	110	3
e	1110	4
f	1111	4

Average codeword length $E[L] = 0.3 \cdot 2 + \cdots + 0.05 \cdot 4 = 2.3$ bit/symb

Entropy $H(X) = -0.3 \cdot \log 0.3 - \cdots - 0.05 \cdot \log 0.05 = 2.27$ bit

Some Information Theory
Mutual information

- Let *X* and *Y* be two random variables
- The information about *X* by observing *Y* is given by

$$I(X;Y) = E\left[\log_2 \frac{P(X,Y)}{P(X)P(Y)}\right]$$

- This gives

$$I(X;Y) = H(X) + H(Y) - H(X,Y)$$

Example Mutual Information

The random variables *X* and *Y* has the joint distribution

P(X,Y)	Y=0	Y=1
X=0	0	3/4
X=1	1/8	1/8

That gives

$$P(X=0)=3/4 \quad \text{and} \quad P(X=1)=1/4$$

$$P(Y=0)=1/8 \quad \text{and} \quad P(Y=1)=7/8$$

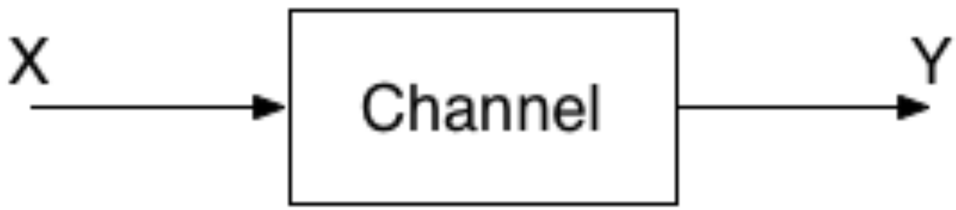
Entropies: $H(X)=h(\tfrac{1}{4})=0.8114$ bit

$H(Y)=h(\tfrac{1}{8})=0.5436$ bit

$H(X,Y)=-\tfrac{3}{4}\log\tfrac{3}{4}-\tfrac{1}{8}\log\tfrac{1}{8}-\tfrac{1}{8}\log\tfrac{1}{8}=1.0613$ bit

Information: $I(X;Y)=H(X)+H(Y)-H(X,Y)=0.2936$ bit

Some Information Theory
Channel capacity



- The channel is a model of the transmission link.
- Transmit *X* and receive *Y*. How much information can the receiver get from the transmitter?
- The *channel capacity* is defined as

$$C = \max_{p(x)} I(X;Y)$$

AWGN
Additive White Gaussian Noise channel

- Let X be bandlimited in bandwidth W
- $Y = X + N$, where $N \sim N\left(0, \sqrt{N_0 / 2}\right)$
- The capacity is

$$C = W \log_2\left(1 + \frac{P}{N_0 W}\right) \quad \text{[in bit/s]}$$

- where P is the power of X, i.e. $E[X^2]=P$.
- It is not possible to get higher data rate on this channel!

AWGN Example (VDSL)

- Consider a channel with

$$W = 17 \cdot 10^6\, Hz$$

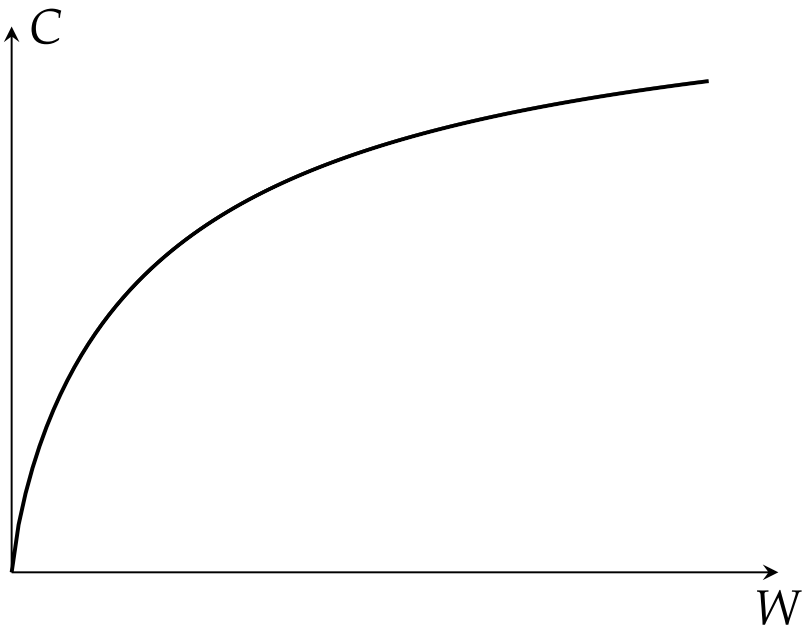
$$P_\Delta = -60\, dBm / Hz$$

$$N_0 = 145\, dBm / Hz$$

- Power $P = 10^{-60/10} \cdot 17 \cdot 10^6$ mW
- Noise $N_0 = 10^{-145/10}$ mW/Hz
- Capacity $C = W \log\left(1 + \frac{P}{N_0 W}\right) = W \log\left(1 + \frac{10^{-60/10}}{10^{-145/10}}\right) = 480$ Mbps

Shannon's fundamental limit

- Plot capacity vs W



- Is there a limit?

- Let *W* -> ∞

$$C_\infty = \lim_{W \to \infty} W \log\left(1 + \frac{P/N_0}{W}\right)$$

$$= \lim_{W \to \infty} \log\left(1 + \frac{P/N_0}{W}\right)^W = \log e^{P/N_0} = \frac{P/N_0}{\ln 2}$$

- With $E_b = PT_b$ and $R_b = 1/T_b$

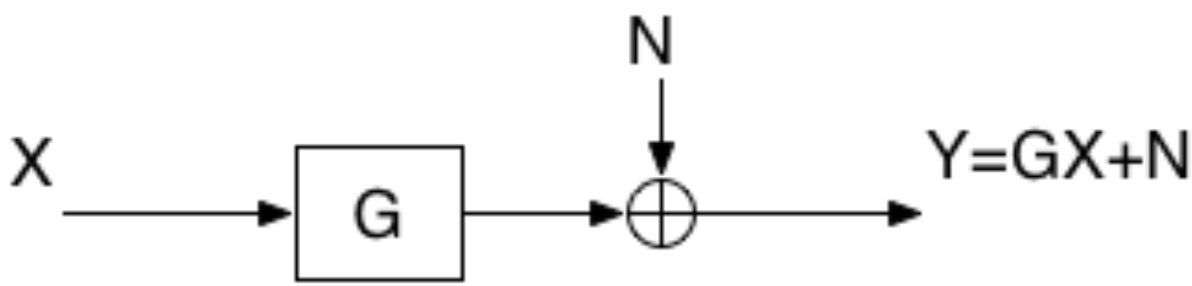
$$\frac{C_\infty}{R_b} = \frac{E_b/N_0}{\ln 2} > 1$$

- Which gives the *fundamental limit*

$$\frac{E_b}{N_0} > \ln 2 = -1.59 dB$$

AWGN with attenuation

- Let *X* be bandlimited in bandwidth *W*
- Let *G* be attenuation on channel, *G<1*



- The capacity is

$$C = W \log_2 \left(1 + \frac{|G|^2 P}{N_0 W} \right) \quad \text{[in bit/s]}$$