ETSF15
Analog/Digital

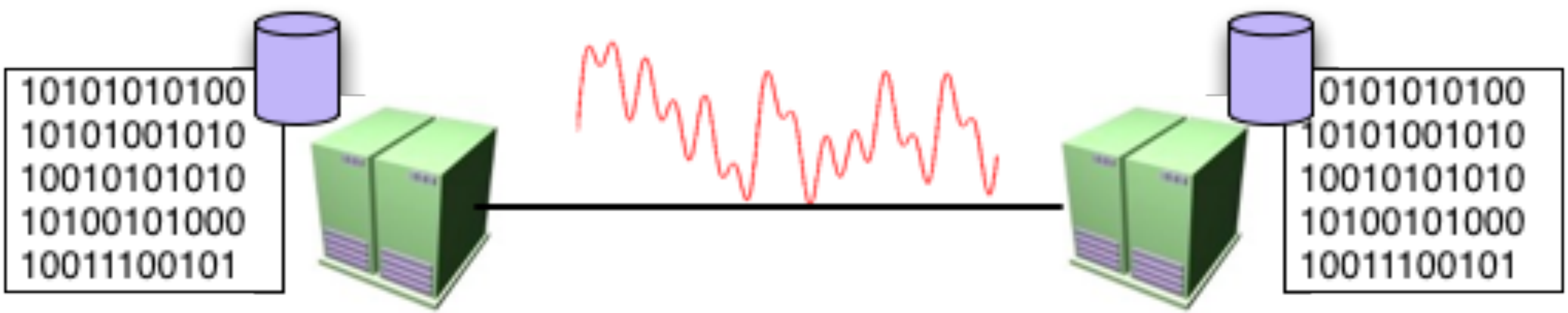
Stafan Höst

Physical layer

- Analog vs digital
  - Sampling, quantisation
- Modulation
  - Represent digital data in a continuous world
- Disturbances
  - Noise and distortion
- Digital data processing
  - Information

Data vs Signal vs Information

- **Data**: Static representation of information
- **Signal**: Dynamic representation of information
- **Information**: Information content in data or signal



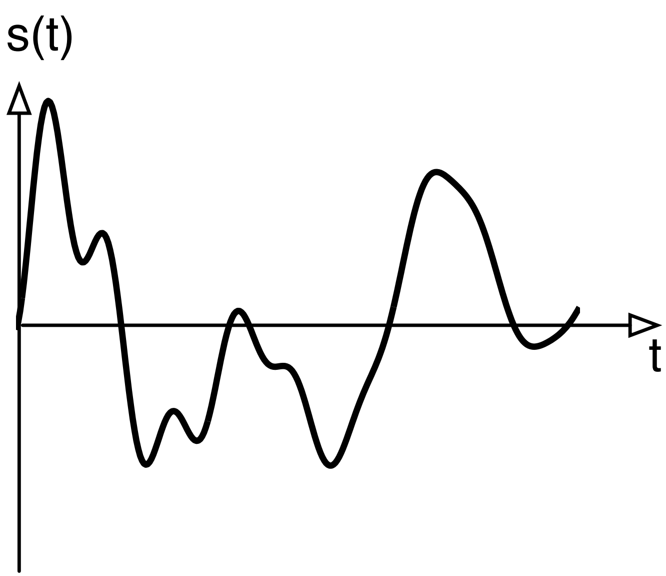
Analog vs digital

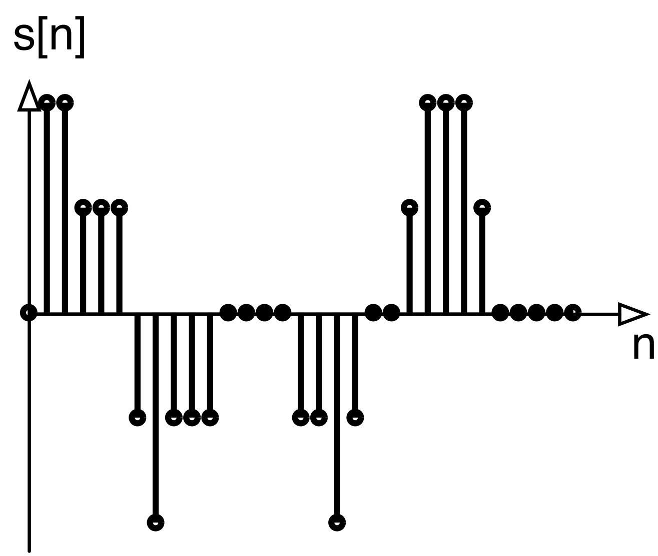
Analog

- Continuous time and amplitude signal
- Electrical/optical domain

Digital

- Discrete time and amplitude
- Binary representation





Digitalization of analog signals

Performed in three steps:

1. Sampling: Discretization in time
2. Quantization: Discretization in amplitude
3. Encoding: Binary representation of amplitude levels

In practice:

- ADC: Analog to Digital Converter
- DAC: Digital to Analog Converter

Sampling

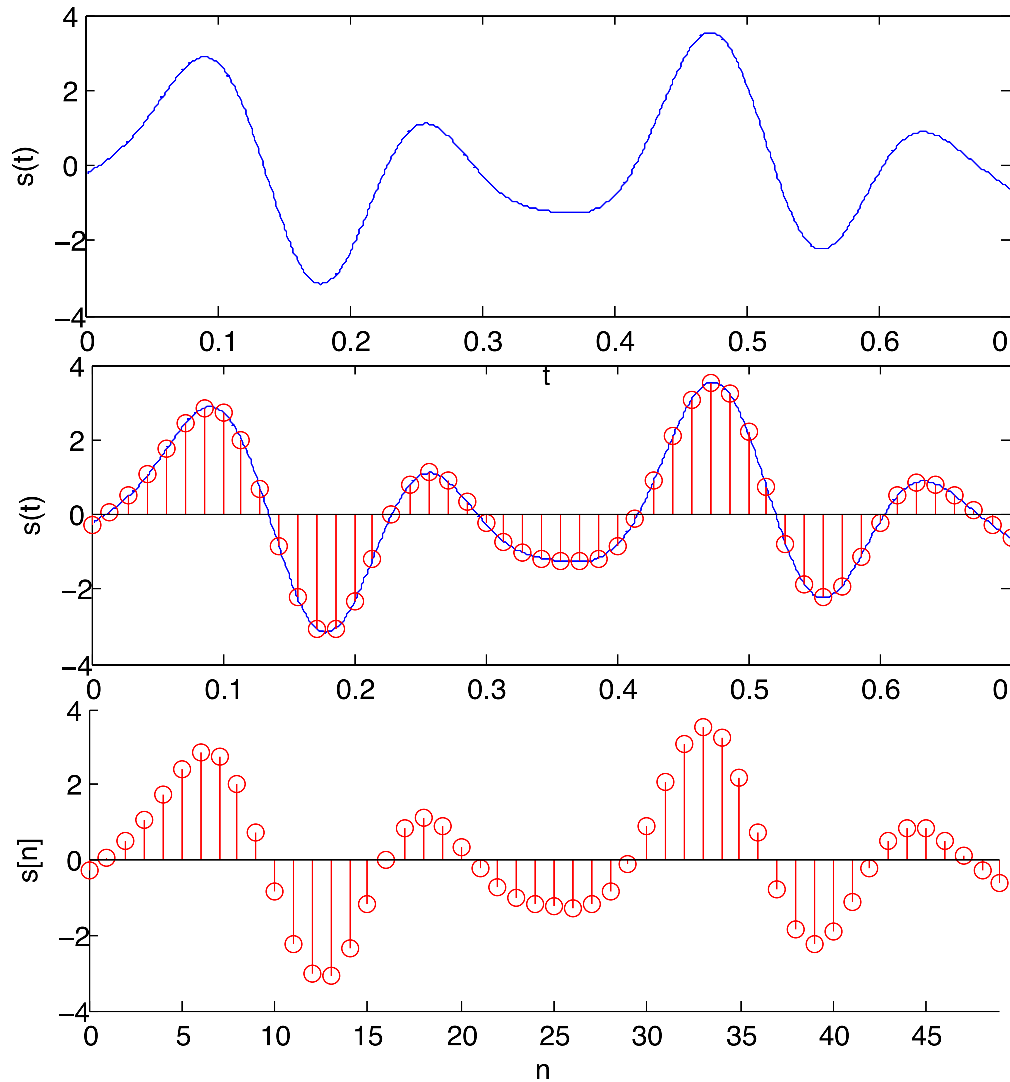
- The process of discretizing time of a continuous signal.

$$s[n] = s(nT_s)$$

- Sampling time: T_s
- Sampling frequnecy:

$$F_s = 1/T_s$$

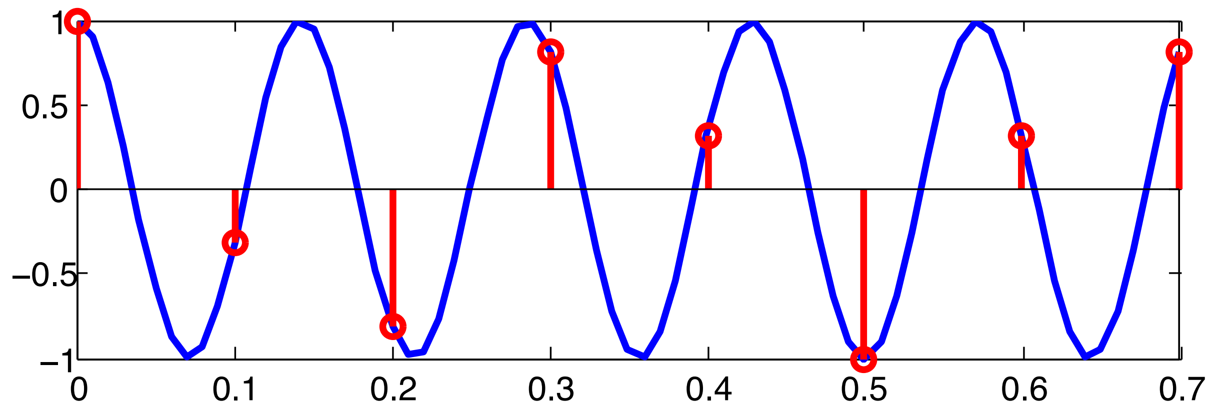
- Loose iformation about time



Aliasing

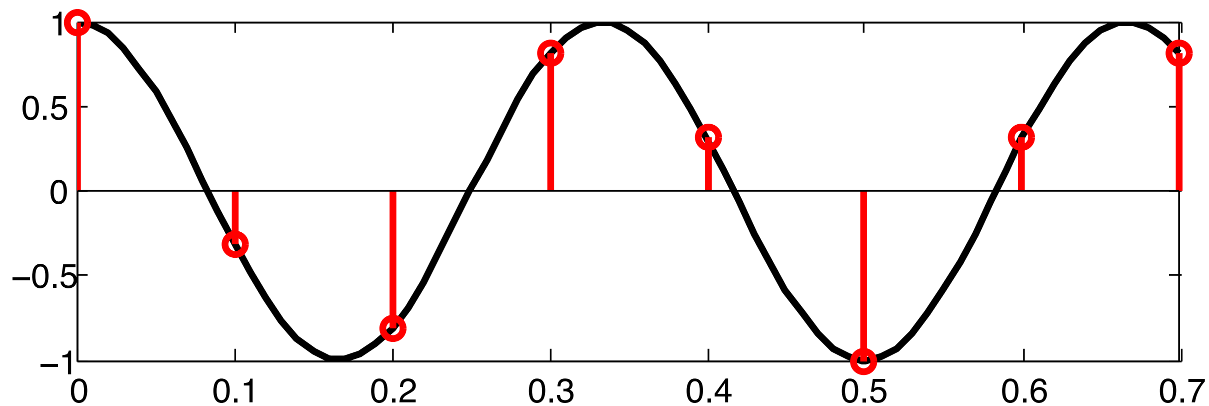
$y(t)=\cos(2\pi 7t)$

$F_s=10$ Hz



Reconstruction to lowest possible frequency

$y(t)=\cos(2\pi 3t)$



Shannon-Nyquist Sampling Theorem

- If s(t) is a band limited signal with highest frequency component Fmax, then s(t) is uniquely determined by the samples s[n] = s(nT) if and only if

$$F_s = \frac{1}{T} \geq 2F_{\max}$$

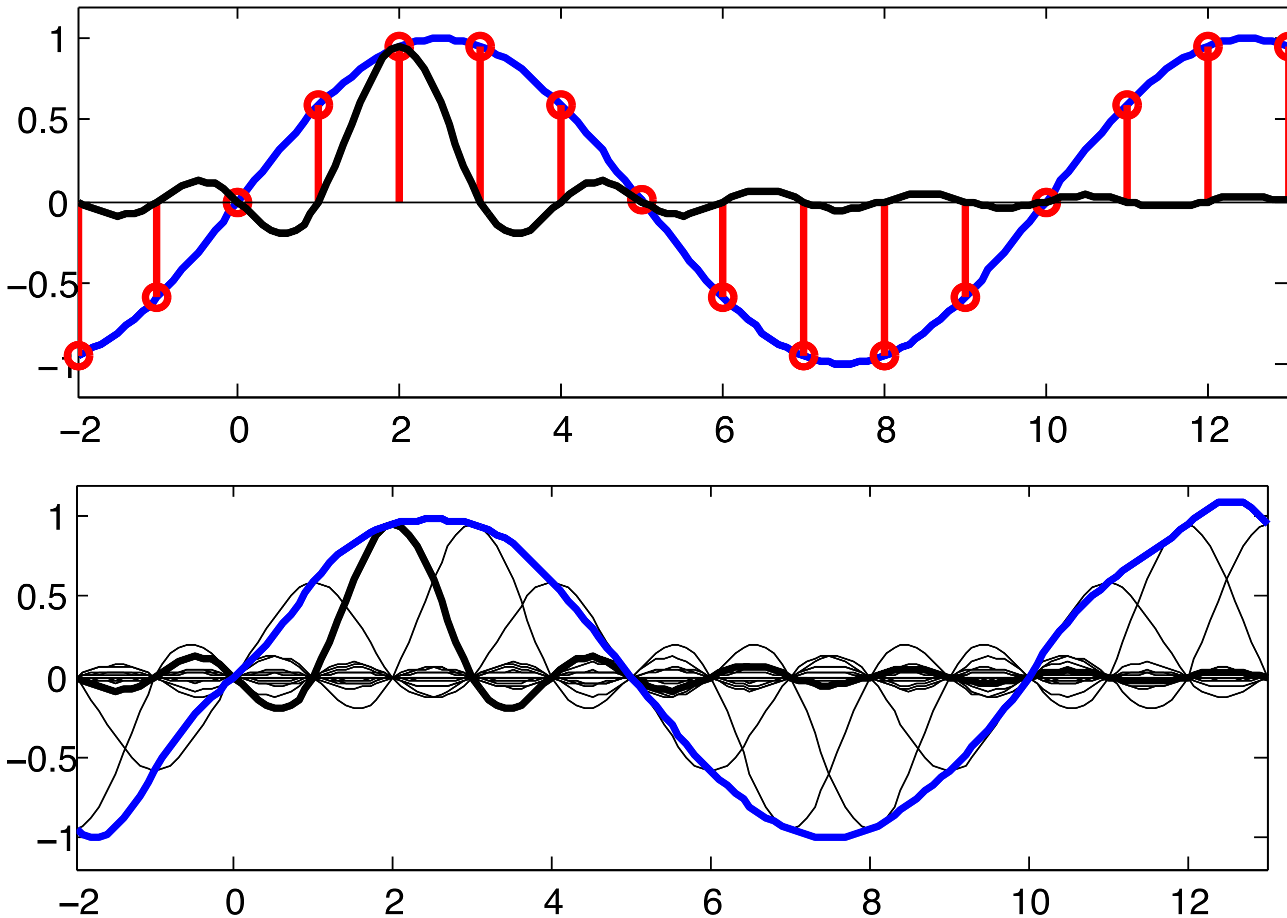
- The signal can be (perfectly) reconstructed with

$$s(t) = \sum_{n=-\infty}^{\infty} s[n] \sin c\left(\frac{t - nT_s}{T_s} \right)$$

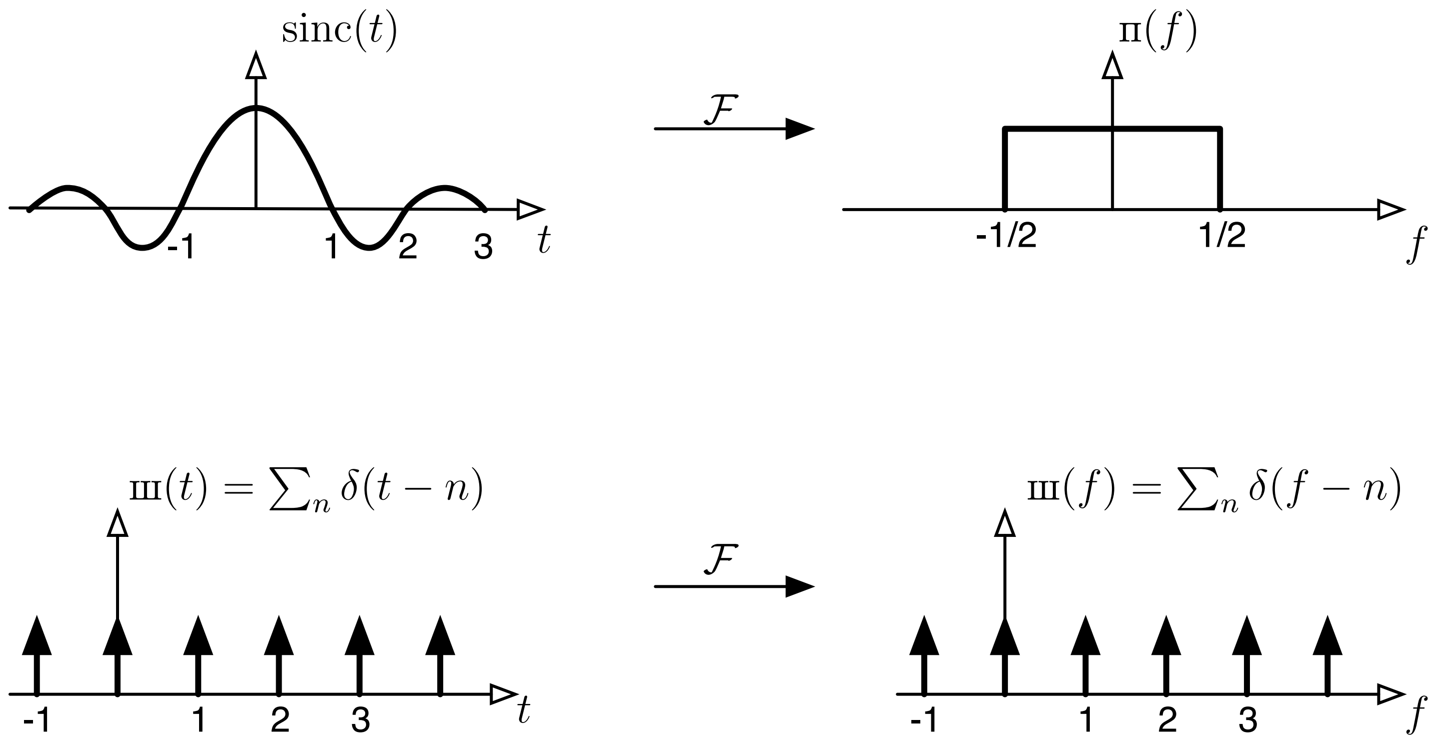
- Fs/2 is the Nyquist frequency and 2Fmax the Nyquist rate

Reconstruction Example

$y(t)=\sin(2/7\pi t)$, $F_s=1Hz$

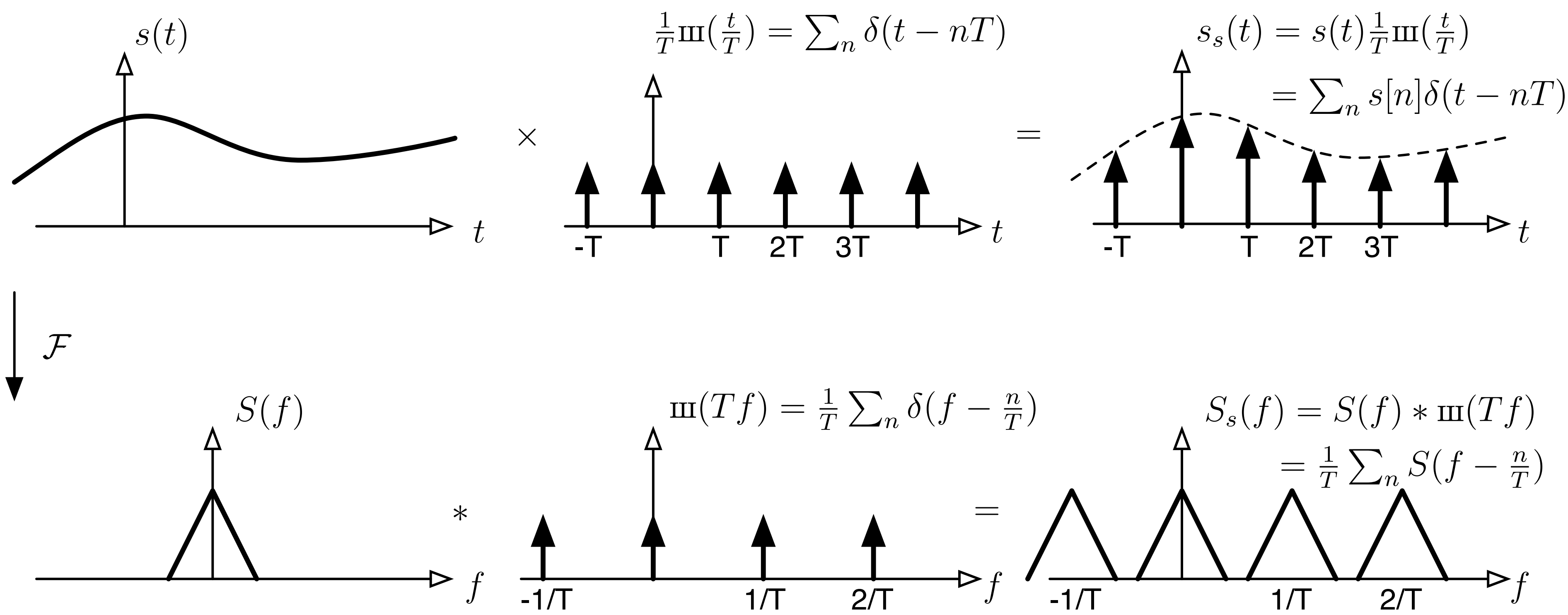


Sampling theorem proof
Two important transforms

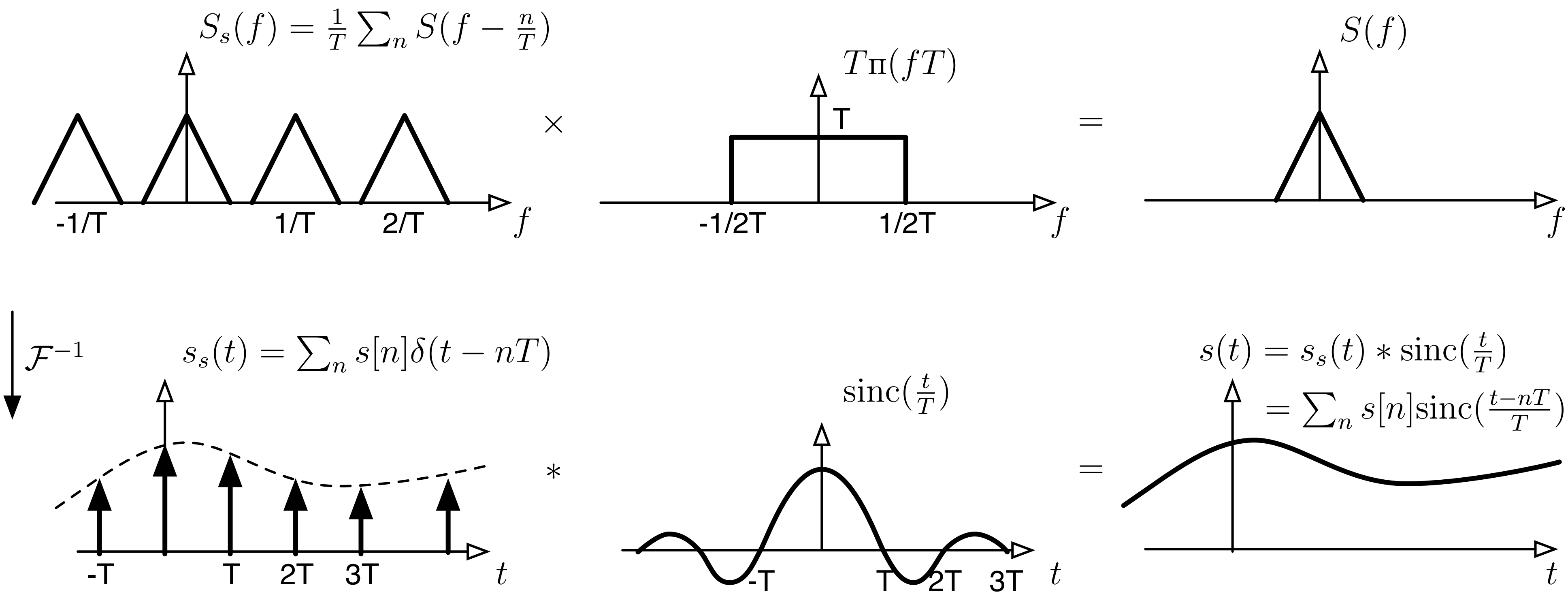


Sampling theorem proof

Mathematical description of sampling

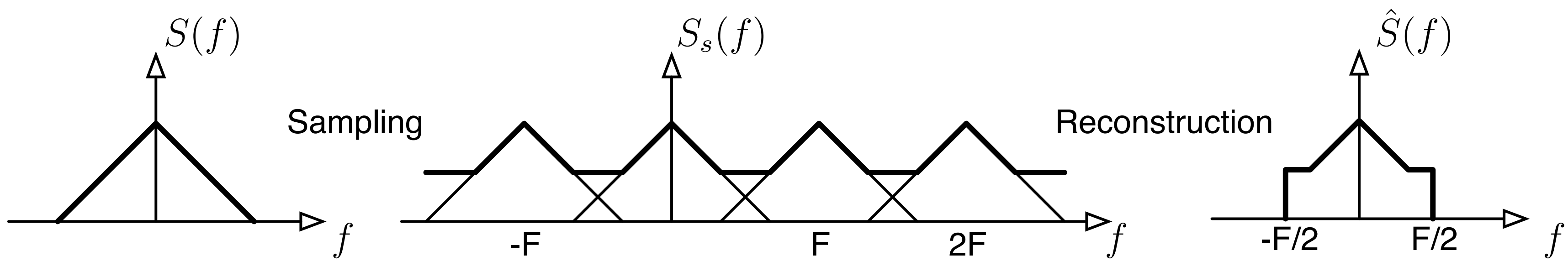


Sampling theorem proof
Reconstruction



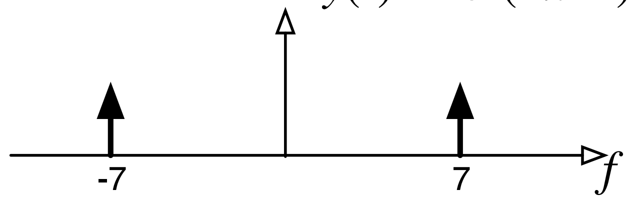
Sampling theorem
Aliasing

- Let $F_s < 2F_{max}$

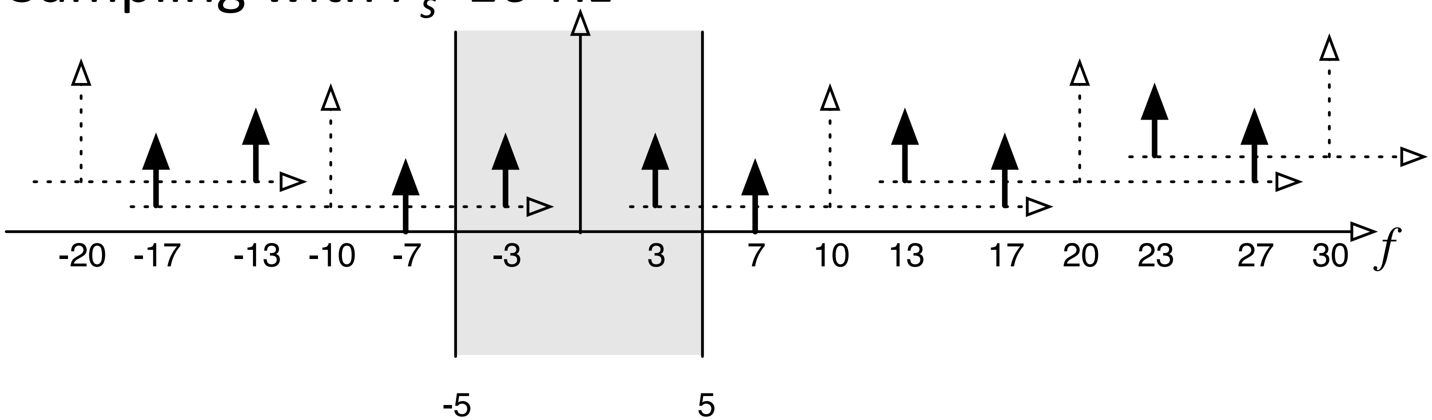


Example

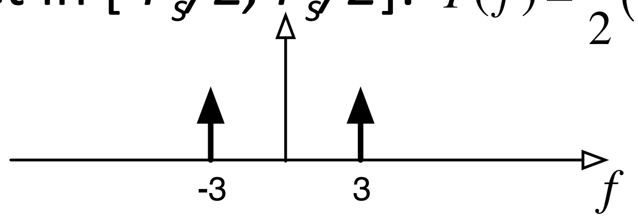
$$y(t) = \cos(2\pi 7t) \rightarrow Y(f) = \frac{1}{2}\left(\delta(f+7) + \delta(f-7)\right)$$



Sampling with $F_s=10$ Hz

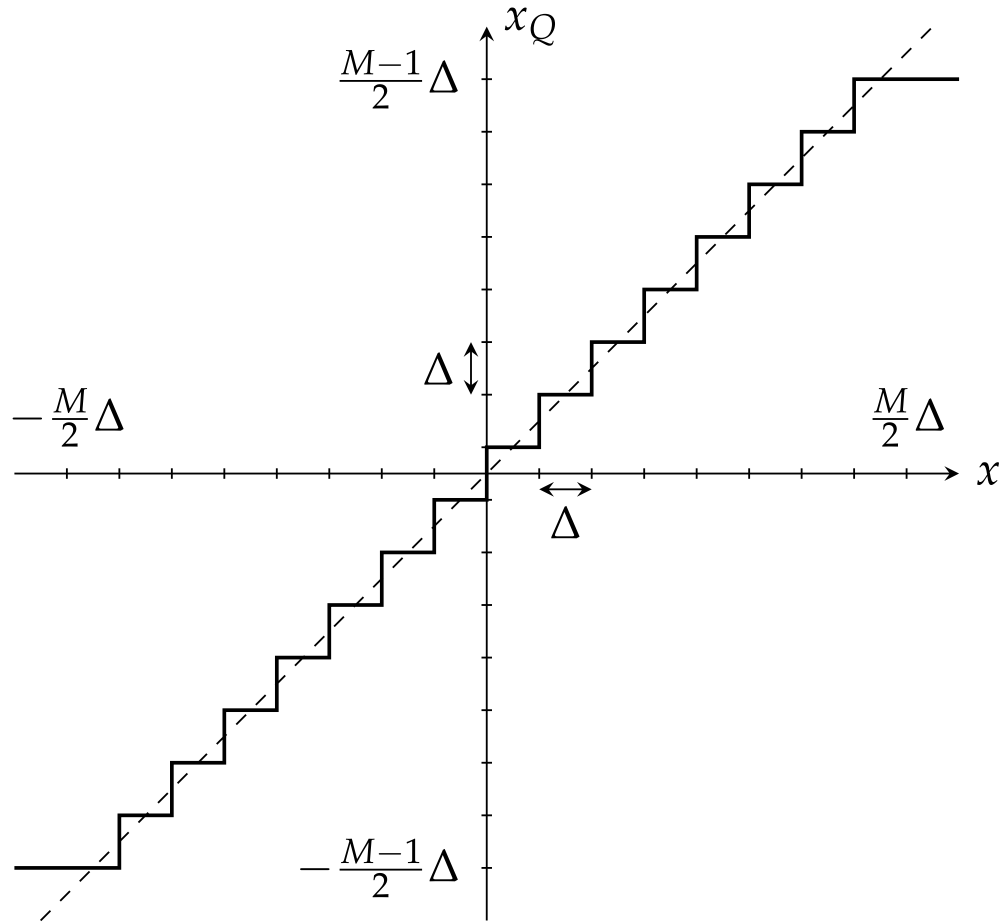


Reconstruct in $[-F_s/2, F_s/2]$: $\hat{Y}(f) = \frac{1}{2}\left(\delta(f+3) + \delta(f-3)\right) \rightarrow \hat{y}(t) = \cos(2\pi 3t)$



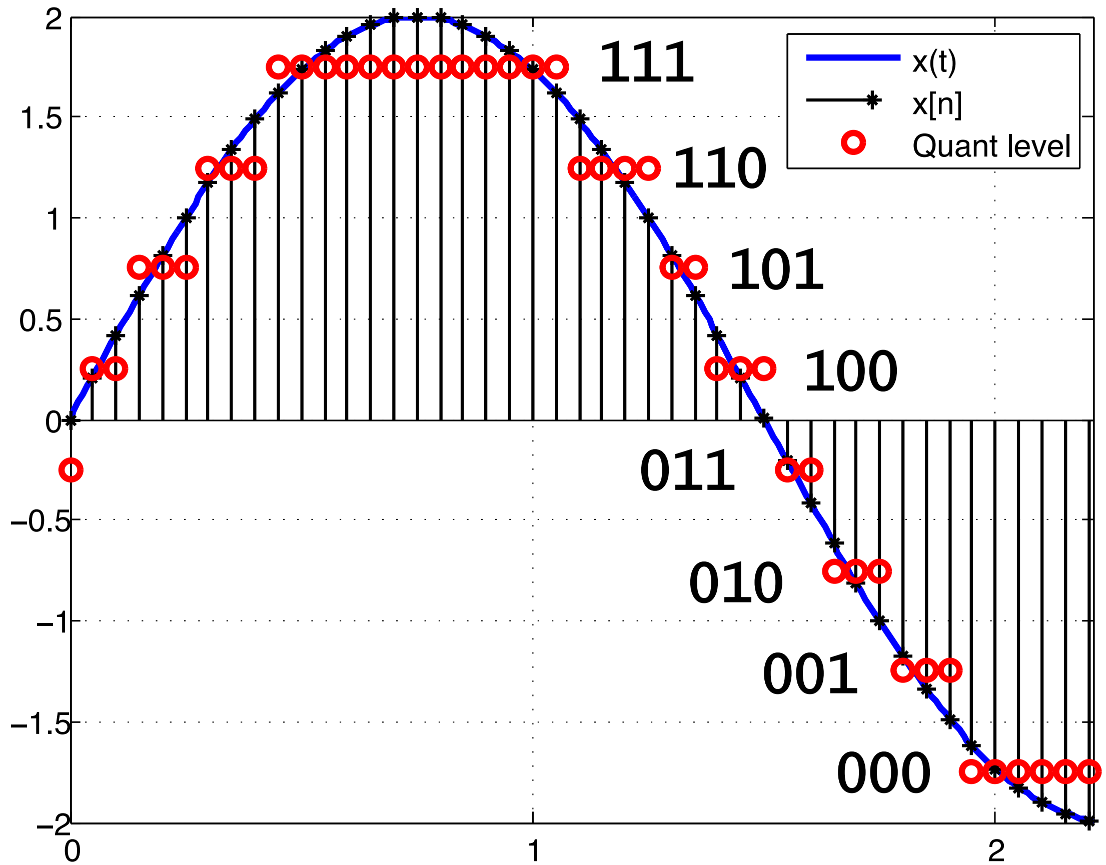
Quantization

- Linear Quantization for k bits
- $M=2^k$ equidistant levels
- Represent sample with k bits



Encoding

- Representation of quantized samples in bits



x=011100100101101101111111111…

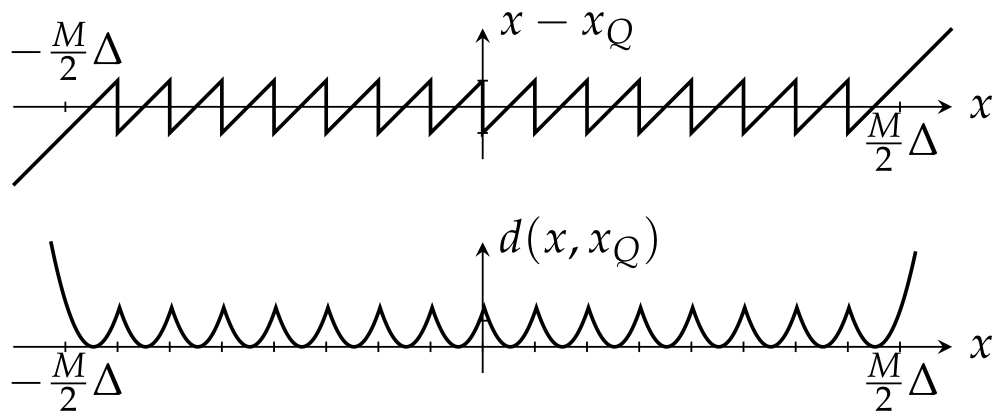
Quantisation distortion

- Distortion (noise):

$$d(x, x_Q) = (x - x_Q)^2$$

- Average distortion for uniform input:

$$E\left[\left(X - X_Q\right)^2\right] = \int_{-\Delta/2}^{\Delta/2} x^2 \frac{1}{\Delta} dx = \frac{\Delta^2}{12}$$



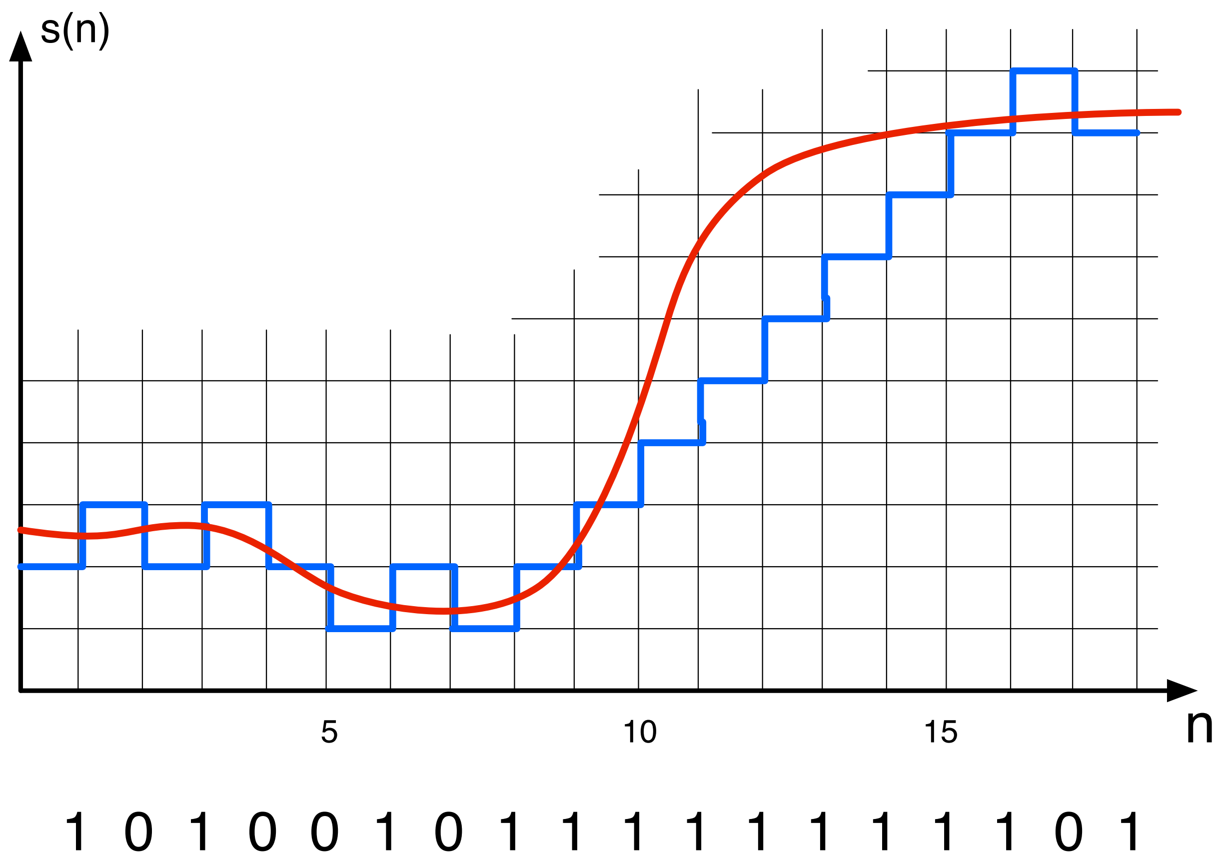
- Signal to quantisation noise ratio

$$SQNR = \frac{E\left[X^2\right]}{E\left[\left(X - X_Q\right)^2\right]} = \frac{(M\Delta)^2/12}{\Delta^2/12} = M^2 = 2^{2k}$$

$$SQNR_{dB} = 10\log 2^{2k} = k \cdot 6dB$$

Delta modulation

- Represent change in amplitude with 1 bit
  - 1: +1
  - 0: −1
- Must be higher sampling rate



Examples

Telephony

F_{max}= 4 kHz

F_s= 8 kHz (samples per sec)

8 bit/sample => 64 kb/s

CD

F_{max}= 20 kHz

F_s= 44.1 kHz (samples per sec)

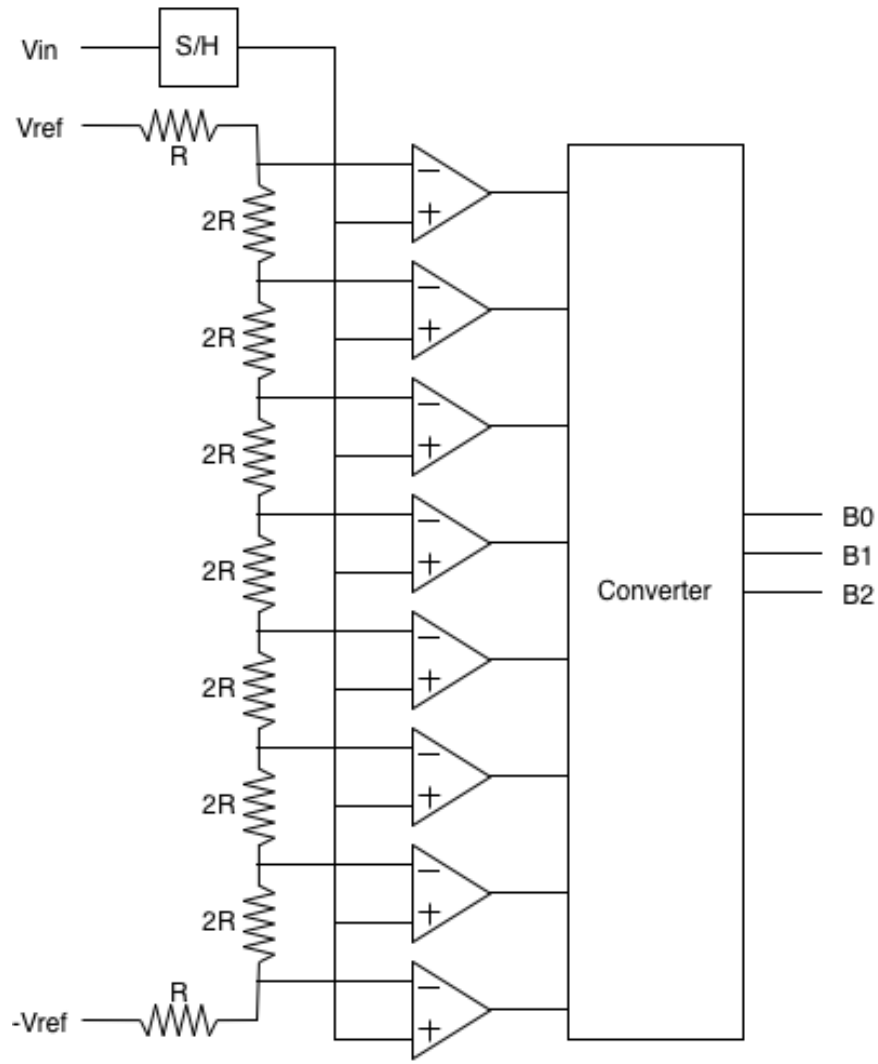
16 bit/sample => 705.6 kb/s

2 channels (stereo)
=> 1.4 Mb/s

ADC
Analog to Digital Converter

- Sample and hold circuit freeze the analog value during conversion
- ADC methods
  - Direct conversion (flash ADC)
  - Integrating
  - Wilkinson
  - Sigma-delta
  - Etc (see e.g.
    https://en.wikipedia.org/wiki/Analog-to-digital_converter)

ADC Direct conversion example (3 bit)



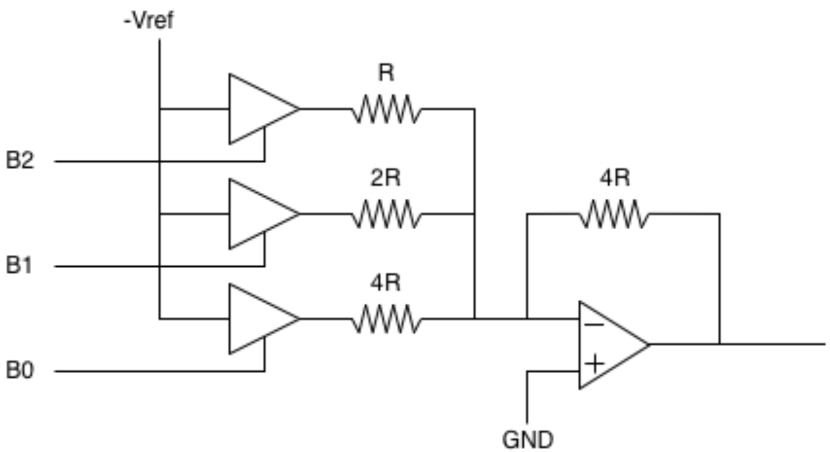
DAC
Digital to Analaog Coverter

- Pulse width modultor
- R-2R ladder
- Interpolating
- Binary weighting

See more on https://en.wikipedia.org/wiki/Digital-to-analog_converter

DAC Example

Weighted resistor



R-2R ladder