Queuing Systems

Computer Exercise 2

Loss Systems and Queuing Networks

Instructions

To be able to understand the computer exercise and to complete it within the given time limits, you have to prepare yourself properly by completing all home assignments. If you have any questions regarding the home assignments, you are welcome to ask your tutor. However, when you arrive to the computer exercise, all assignments **must** have been completed. Further, make sure to **be in time** for the exercise.

To be given access to the computer exercise, you have to

- Complete all home assignments
- Pass a small test covering these assignments and basic queuing theory

In this computer exercise, queuing theory will be used to analyze delay and loss systems both theoretically and through Matlab simulations. To be able to run the simulation programs, some Matlab-files need to be downloaded from the course home page before the start of the computer exercise and copied to your home directory.

In the simulations we use a seed for the randomization, this is used to always draw the random values in the same order. Now we can compare the simulations with each other in a simple way, because we know that the behavior of the system we simulated was the same in all simulations.

During the computer exercise, several different tasks have to be solved. For every task, there are a number of questions that have to be answered. Some of the questions are marked (**) and these shall be discussed with the tutor before proceeding with the computer exercise. However, you are supposed to do some thinking yourself before talking to the tutor. If the tutor happens to be occupied elsewhere, it is allowed to continue for a short while until he/she becomes available.

1. Home Assignments

Part A: Loss Systems
Loss systems can for example be found in wireless cellular networks such as GSM. We assume that a cell in a GSM network has access to m radio channels. We also assume that the call length is exponentially distributed. With a mean value of $\frac{1}{\mu}$. The number of subscribers in the cell is M (limited amount of subscribers). And the call arrival rate of every single inactive subscriber is assumed to be β.

1.Draw the state transition diagram.

2.We assume that the steady state probabilities, p_k, of the system are known. Express the time congestion from these probabilities.

3. Express the call congestion from the steady state probabilities, p_k.

4. Which of the congestion measures (call or time congestion) are more important? Why?

If the number of subscribers is large in comparison to the number of radio channels ($\frac{M}{m} > 10$), our model can be altered.
5. Draw the state transition diagram of the new model.

Part B: Queuing Networks

In the theory, we have used a theorem saying that for an M/M/m-system with infinite queue, the departure process is exponentially distributed with the same mean value as the arrival process. We are now going to validate this statement through simulations.

1. A Poissonian traffic arrives to an M/M/m-system, why is the traffic out from the system also Poissonian?

2. Explain in the same way that two links with Poisson traffics can be merged into just one link with Poisson traffic.

For the queuing network in Figure 1 make the following calculations.

3. For the different nodes in the queuing network, calculate the mean number of customers in the different M/M/1-systems. $\lambda_1 = 7.5\ s^{-1}$, $\lambda_2 = 10\ s^{-1}$, $\mu_1 = 10\ s^{-1}$, $\mu_2 = 14\ s^{-1}$, $\mu_3 = 22\ s^{-1}$, $\mu_4 = 9\ s^{-1}$, $\mu_5 = 11\ s^{-1}$ and $\alpha = 0.4$

4. For the different nodes in the queuing network, calculate the mean number of customers in the queues.

5. Calculate the total number of customers in the queuing system.

6. Calculate the mean response time for the customers, for the different nodes, with using Little's theorem.

7. Replace system 5 with an M/M/1*loss system (a system without queue), with the same mean service time. Calculate the mean response time in the system for a customer that get completely served, i.e., not get blocked.

8. Make modifications in the matlab-file MM1kosystem.m so you can measure the number of customers in the queues and the service time in the different systems. *Hints: You will get 10 vectors and you can name them N1q,..., N5q and T1s, ..., T5s. The number of customers in the queue is the number of customers in the system –1, if there is at least one customer in the system and zero if the system is empty.*

9. Simulate your queuing network (simulation time 1000s) with the values given in home assignment number 3 and you should get the following results, N1q= 2.2732, N3q= 3.4947, T3s= 0.0457 s and T4s= 0.1120 s, to validate your program.

1. M/M/m Loss system with limited amount of subscribers:

A ´MatLab program called Loss calculates the time and call congestion for an M/M/m Loss system with no queue and M customers. Every customer arrives to the system with a certain rate, meaning that the total arrival rate is calculated through multiplication by the number of customers and the customer arrival rate.

1. Plot the diagram over the call and time congestion for Mmax=20.

2. Determine which graph shows the time congestion and which graph shows the call congestion. Describe your decision.

3. How many radio channels exist in the system? Describe how you found it out.

4. What happens to the curves when the number of customers becomes very large (e.g., Mmax=50) (**)

2. M/M/m Loss system, large amount of subscribers

CSN has one call center in Lund and one in Vaxjo, both with 10 operators, dimensioned as loss systems. The arrival rate of each call center during busy hours is 2 calls per minute and the mean service time is 5 minutes. We assume that there are no repeated calls since the callers are aware of the fact that it is useless to try again.

1. Calculate the call congestion, offered traffic ρ, carried traffic ρ_c, and lost (blocked) traffic ρ_b for any of the two call centers. Hint: use the function pkMMmloss.

2. How many operators will in total be needed to keep the call congestion in each of the two call centers during the busy hours below 5%? How large is the server utilization (Carried traffic per operator)?

3.How large will the percentual increase for the call congestion during the busy hour be if the offered traffic is increased by 10%? Use the number of operators obtained in the previous part.

CSN has discovered that their customer service could be made more efficient if an arriving can be directed to an available operator in any of the two call centers.

4. How many operators will be needed to hold the call congestion during busy hours below 5% if the arrival streams are merged in the described way? Use the traffic parameters of part 2.2. How large is the server utilization (Carried traffic per operator)?

5. How large will the percentual increase for the call congestion during busy hours be for this merged system if the offered traffic is increased by 10%? Use the number of operators obtained in exercise 2.4.

6. Compare and evaluate the results of the different call center strategies. Describe the basis of your evaluation.

3. Queuing networks with M/M/1-systems

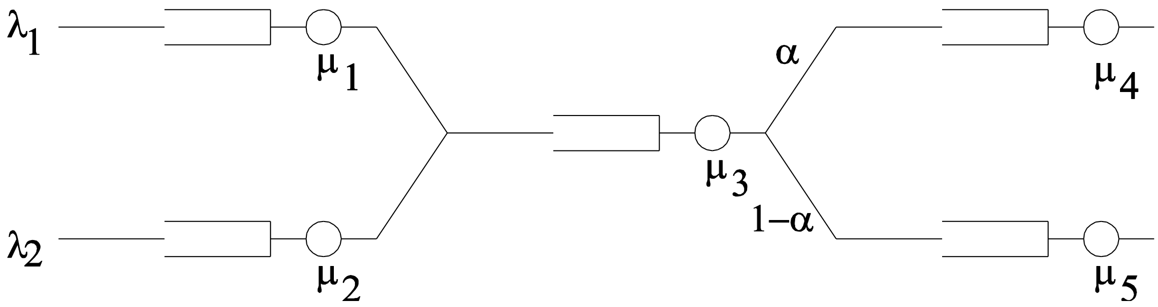

Figure 1: Queuing network

$\lambda_1 = 7.5\ s^{-1}$, $\lambda_2 = 10\ s^{-1}$, $\mu_1 = 10\ s^{-1}$, $\mu_2 = 14\ s^{-1}$, $\mu_3 = 22\ s^{-1}$, $\mu_4 = 9\ s^{-1}$, $\mu_5 = 11\ s^{-1}$, $\alpha = 0.4$

In the rest of the computer exercise use 300 s as simulation time.

1. Simulate your queuing network with the values given and compare with your results made in the home assignment.

2. Verify Little's theorem for each node (N and T), from your simulations. Does it work?

3. Increase the mean number of arrivals with 20% to system 2 in your queuing network. Measure the mean number of customers and the mean time in each system.

4. Why do you get a result like this? Where is the bottleneck?

5. What can you do to make it better? Name at least two different actions. (**)

4. Queuing networks with M/G/1-systems

Here we use the system in Figure 1, with the same mean values, but we change the service time distribution.

1. Change the service time distribution to be deterministic (constant) and simulate the mean response time and mean number of customers in each node. How does the change in service time distribution affect the mean response time for each node? Higher/lower, why?

2. Do the same thing when the service time distribution is a hyper-exponential distribution (use the matlab file h2rnd.m and call it with h2rnd(0.5, μ-2, μ+2) where you of course put in the values of the different systems μ). How does the mean response time differ for each node, when the service time distributions change?

3. Try to verify Little's theorem, when the service time distribution is a hyper-exponential distribution.

4. Which of the system 5 systems are really M/G/1-systems?

5. Compare these results with the results you got from the exponential distribution. Which have the longest mean response time? Why? (**)

5. The black box problem.

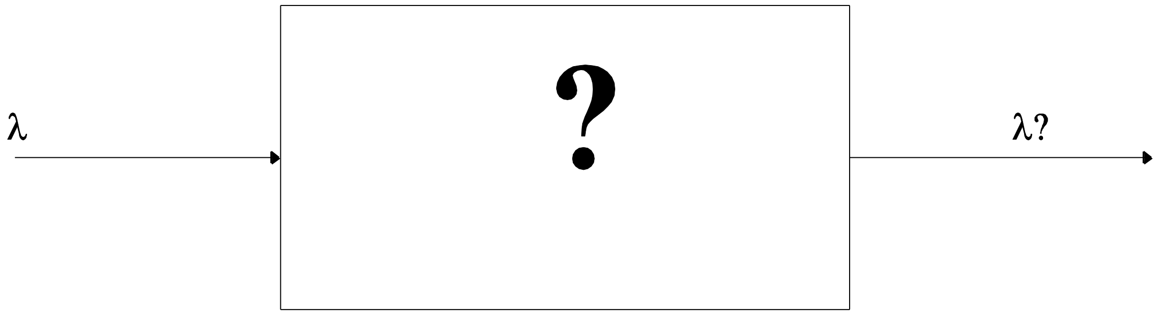

Figure 2: Black box

Simulate a black box (Matlab file blackbox.m) from the file you get certain parameters that can be useful in the following exercises. In the black box we have 5 different systems, the customers arrive at first to system one and depart from the black box from system five. Parameters that might be useful are nbrarrivedtosystem, nbrarrivedX, nbrdepartedX, Ttot, TX, TXs, NX and NXq. Where the X stands for the node number of the system.

1. Given a black box, try to verify Little's theorem, for the whole system, use the parameters given above. Assume that the arrival process is a Poisson process with $\lambda = 7.5\ s^{-1}$.

2. Try to find out if any customers get blocked in the black box.

3. Use the vector Ttot and plot the last 100 $Ttot_n$-values against the corresponding $Ttot_{n-1}$ –values, i.e. write plot(Ttot(404:503),Ttot(403:502),'*') in the command prompt. Describe what have you plotted (define each axix). What can you say about the dependency when the arrival process is a Poisson process with $\lambda = 7.5\ s^{-1}$?

4. What conclusions, about the system in the black box, can you draw from your measurements? (**)

Appendix :

histogram: Plots a histogram over the elements in a vector.
pkMMmloss(rho,m) Calculates the steady state probabilities for an M/M/m Loss system.