Lecture no: 5

Digital modulation

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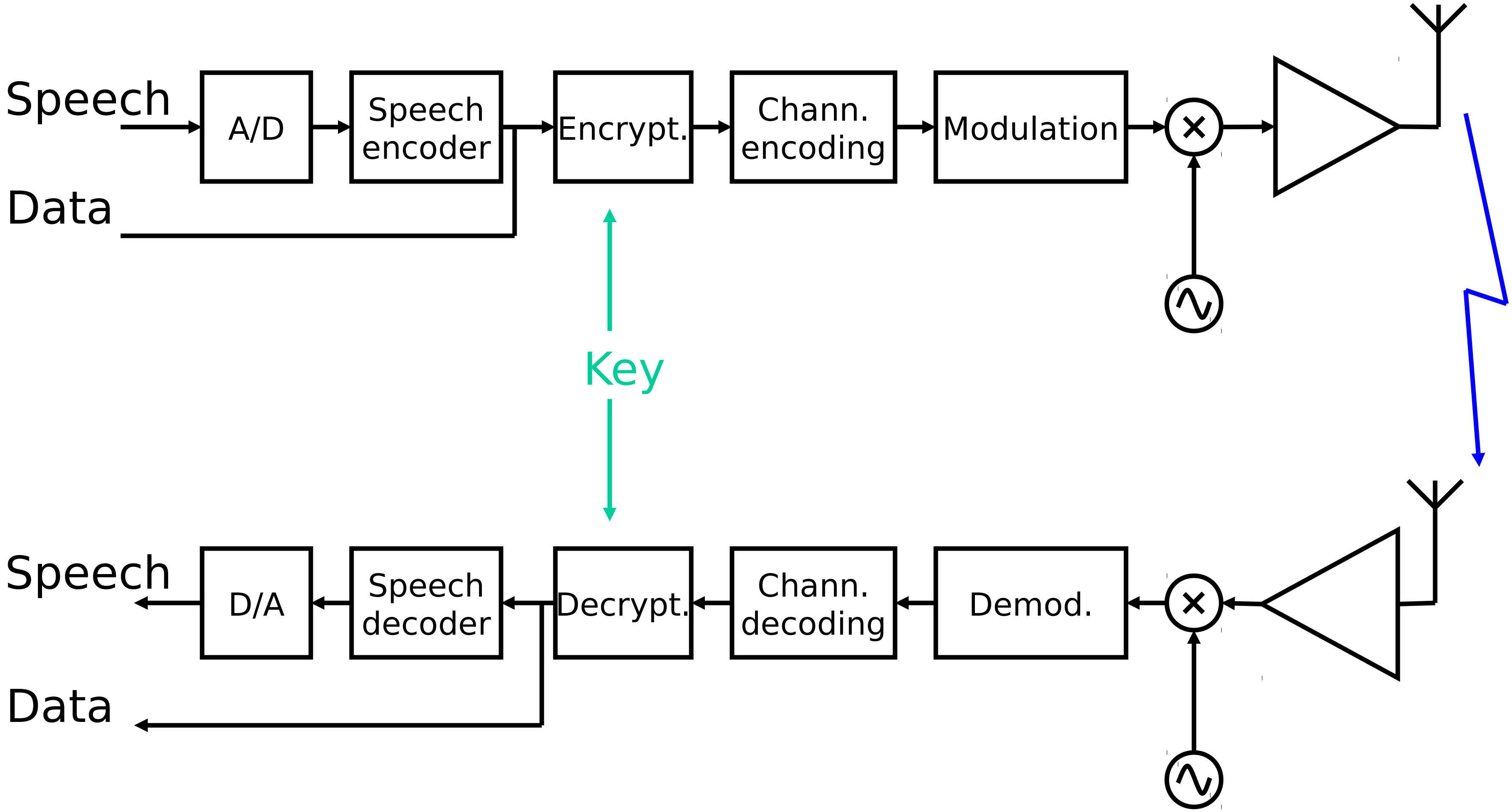
Contents

- Brief overview of a wireless communication link
- Radio signals and complex notation (again)
- Modulation basics
- Important modulation formats

STRUCTURE OF A WIRELESS COMMUNICATION LINK

A simple structure



(Read Chapter 10 for more details)

RADIO SIGNALS AND COMPLEX NOTATION (from Lecture 3)

Simple model of a radio signal

- A transmitted radio signal can be written

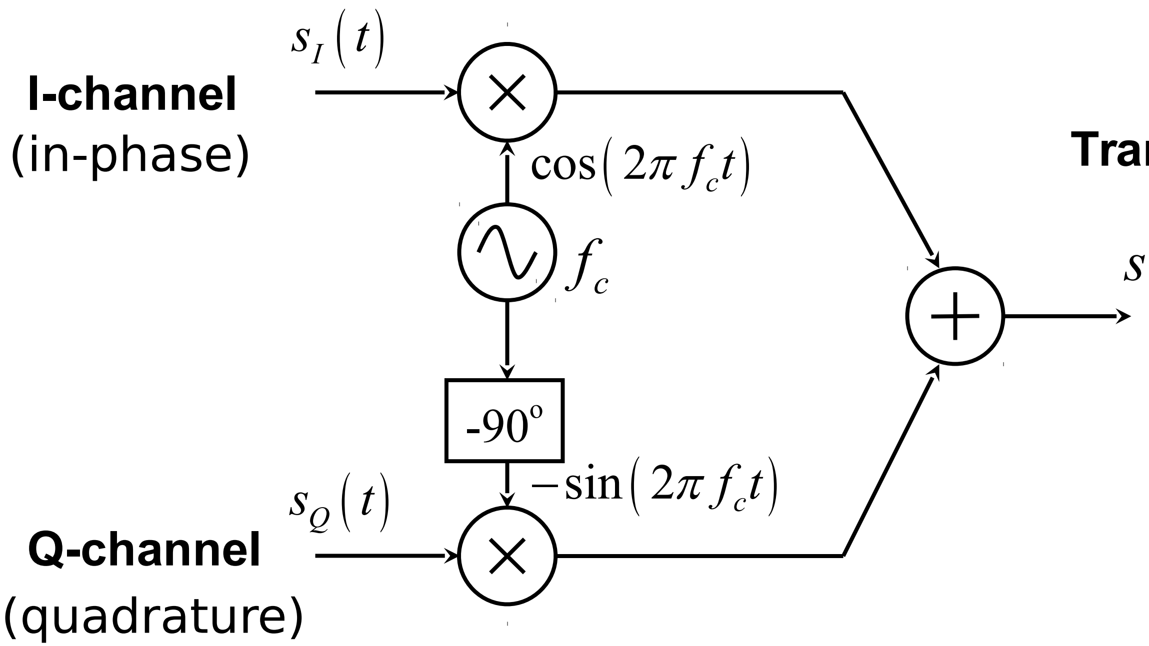
$$s(t) = A\cos(2\pi f t + \phi)$$

Amplitude — Frequency — Phase

- By letting the transmitted information change the amplitude, the frequency, or the phase, we get the tree basic types of digital modulation techniques
  - **ASK** (Amplitude Shift Keying)
  - **FSK** (Frequency Shift Keying)
  - **PSK** (Phase Shift Keying)

Constant amplitude (FSK, PSK)

The IQ modulator



$$s(t) = s_I(t)\cos(2\pi f_c t) - s_Q(t)\sin(2\pi f_c t)$$

Take a step into the complex domain:

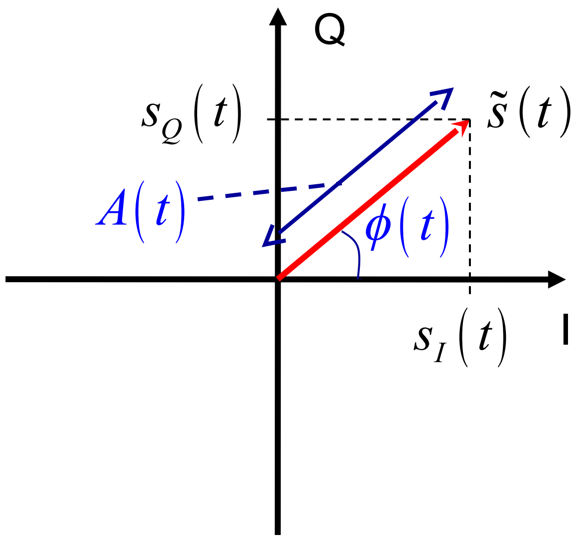
Complex envelope $\tilde{s}(t) = s_I(t) + j\,s_Q(t)$

Carrier factor $e^{j2\pi f_c t}$

$$\Rightarrow \quad s(t) = \mathrm{Re}\left\{\tilde{s}(t)\,e^{j2\pi f_c t}\right\}$$

Interpreting the complex notation

Complex envelope (phasor)



Polar coordinates:

$$\tilde{s}(t) = s_I(t) + j\, s_Q(t) = A(t)\, e^{j\phi(t)}$$

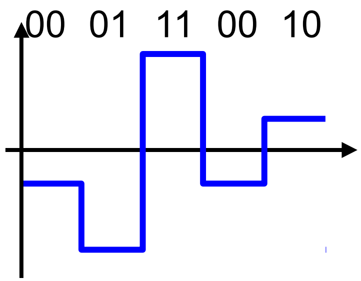
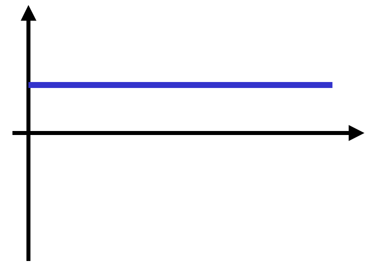
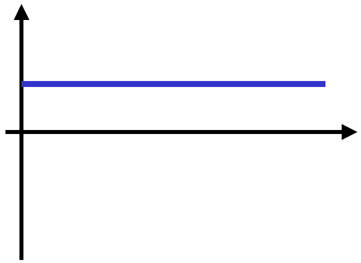
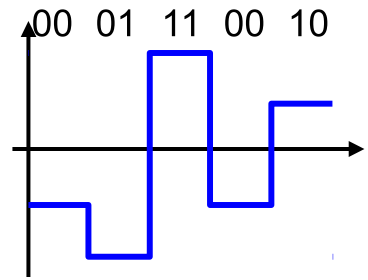
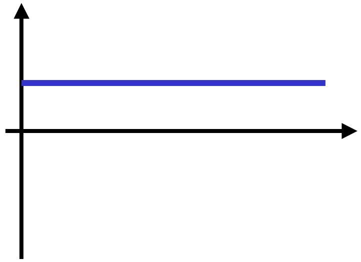
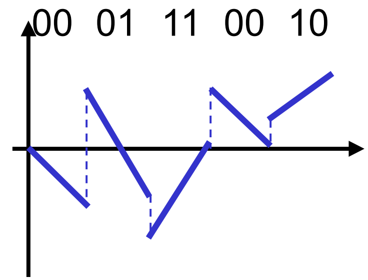
Transmitted radio signal

$$\begin{aligned} s(t) &= \mathrm{Re}\left\{ \tilde{s}(t)\, e^{j2\pi f_c t} \right\} \\ &= \mathrm{Re}\left\{ A(t)\, e^{j\phi(t)} e^{j2\pi f_c t} \right\} \\ &= \mathrm{Re}\left\{ A(t)\, e^{j(2\pi f_c t + \phi(t))} \right\} \\ &= A(t) \cos(2\pi f_c t + \phi(t)) \end{aligned}$$

By manipulating the amplitude *A*(t) and the phase *Φ*(t) of the complex envelope (phasor), we can create any type of modulation/radio signal.

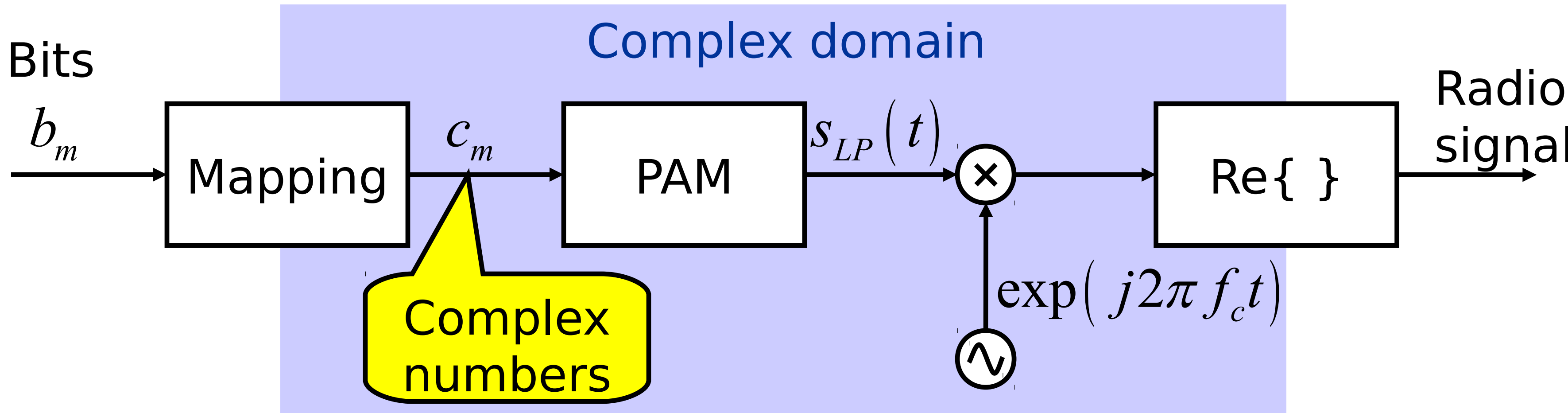
Example: Amplitude, phase and frequency modulation

$$s(t) = A(t)\cos\left(2\pi f_c t + \phi(t)\right)$$

	$A(t)$	$\phi(t)$	Comment:
4ASK			- Amplitude carries information - Phase constant (arbitrary)
4PSK			- Amplitude constant (arbitrary) - Phase carries information
4FSK			- Amplitude constant (arbitrary) - Phase slope (frequency) carries information

MODULATION BASICS

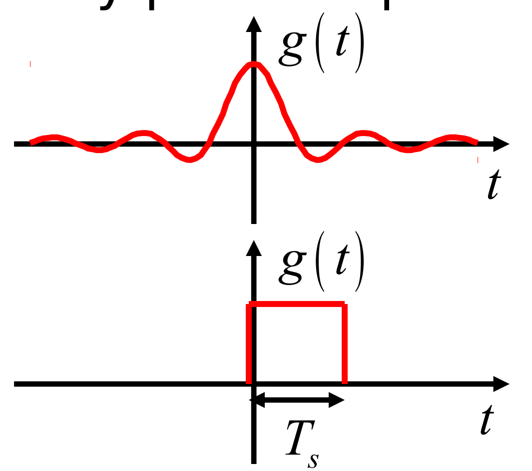
Pulse amplitude modulation (PAM)
The modulation process



PAM: $s_{\mathrm{LP}}(t) = \sum_{m=-\infty}^{\infty} c_m g(t - mT_s)$

Symbol time

Many possible pulses



"Standard" basis pulse criteria

$\int_{-\infty}^{\infty} |g(t)|^2 dt = 1 \text{ or } = T_s$ (energy norm.)

$\int_{-\infty}^{\infty} g(t) g^*(t - mT_s) dt = 0 \text{ for } m \neq 0$ (orthogonality)

Pulse amplitude modulation (PAM)
Basis pulses and spectrum

Assuming that the complex numbers c_m representing the data are independent, then the **power spectral density** of the base band PAM signal becomes:

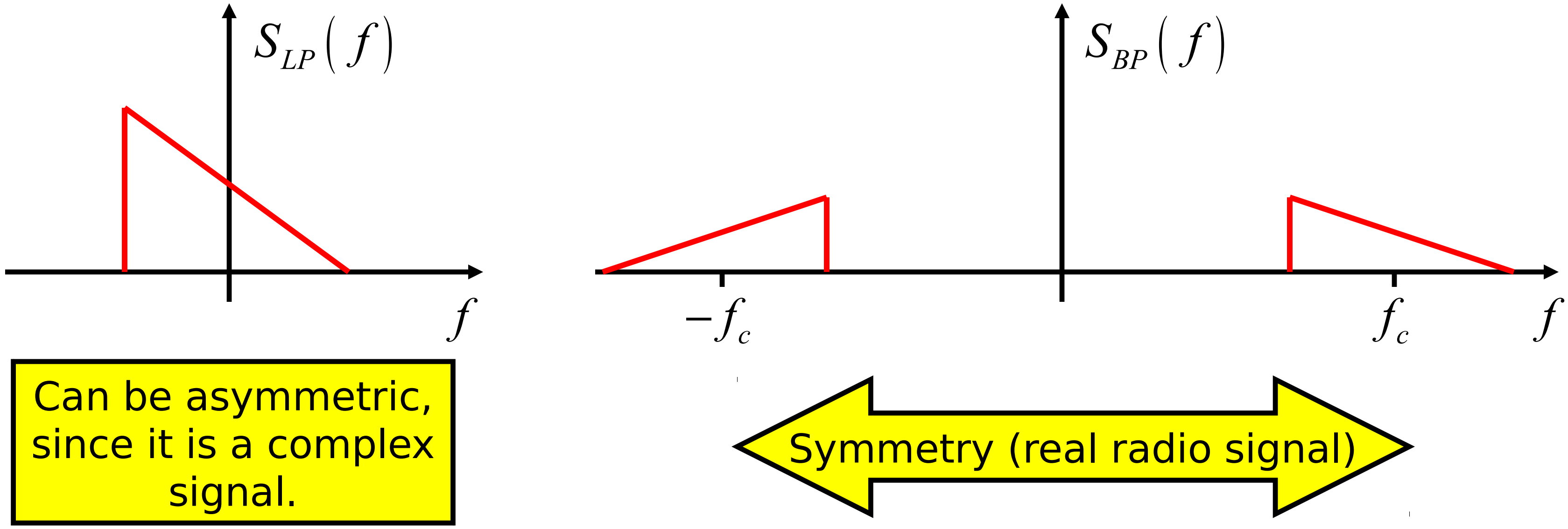
$$S_{\mathrm{LP}}(f) \sim \left| \int_{-\infty}^{\infty} g(t) e^{-j2\pi f t} dt \right|^2$$

which translates into a radio signal (band pass) with

$$S_{BP}(f) = \frac{1}{2}\Big(S_{LP}(f - f_c) + S_{LP}(-f - f_c) \Big)$$

Pulse amplitude modulation (PAM)
Basis pulses and spectrum

Illustration of power spectral density of the (complex) base-band signal, $S_{LP}(f)$, and the (real) radio signal, $S_{BP}(f)$.



What we need are basis pulses $g(t)$ with nice properties like:

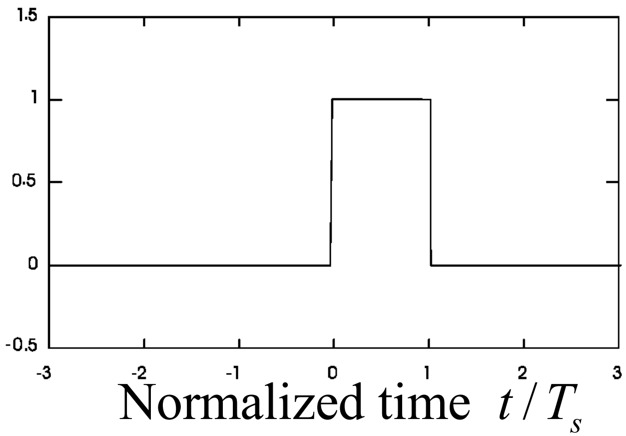
- Narrow spectrum (low side-lobes)

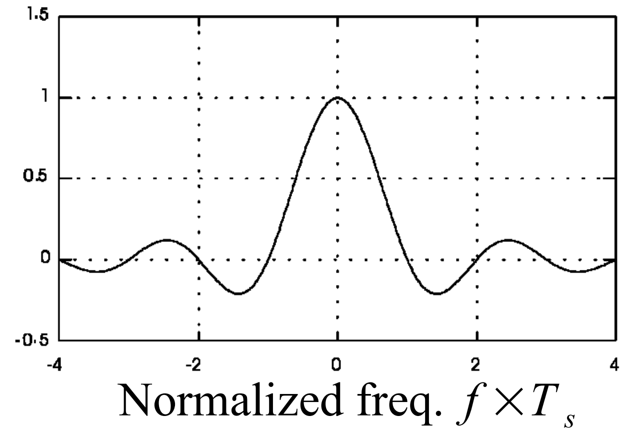
- Relatively short in time (low delay)

Pulse amplitude modulation (PAM)
Basis pulses

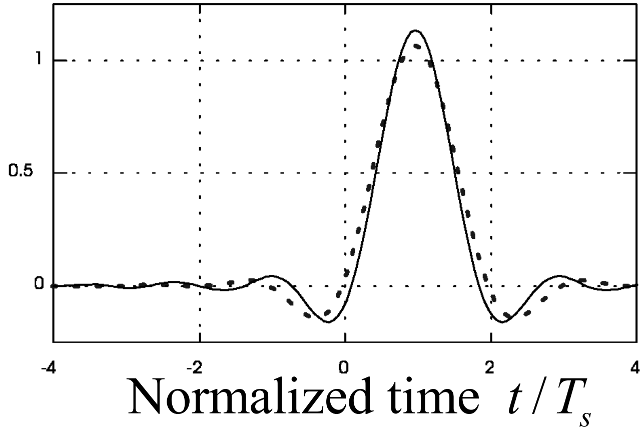
TIME DOMAIN | FREQ. DOMAIN

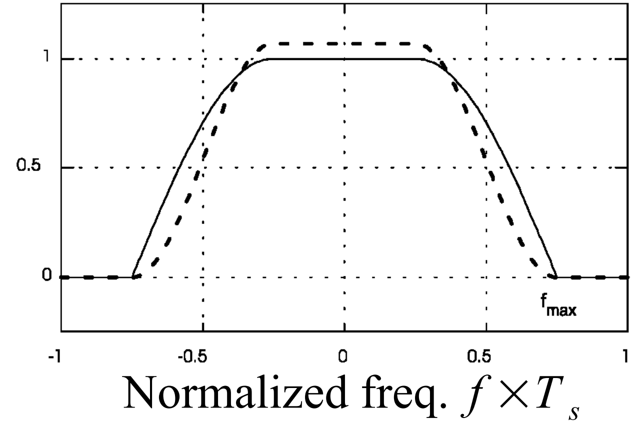
Rectangular [in time]





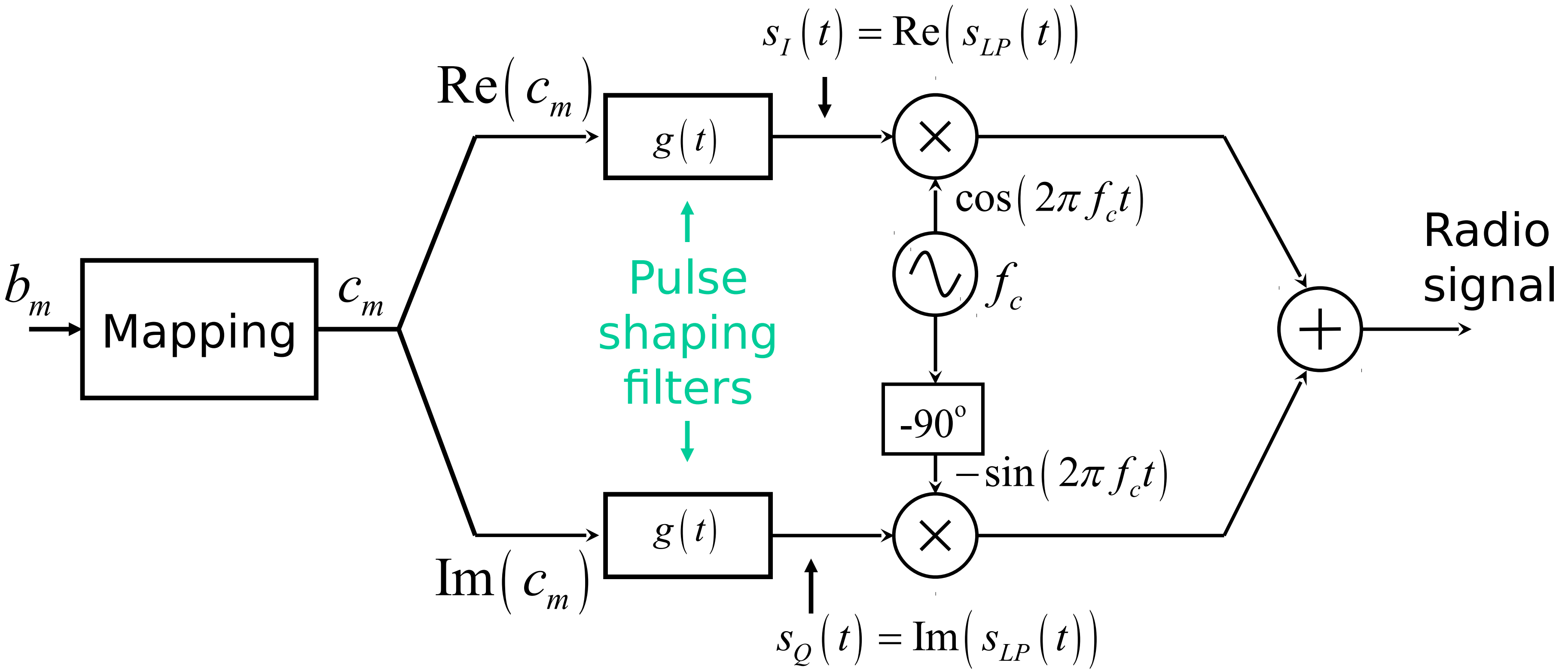
(Root-) Raised-cosine [in freq.]





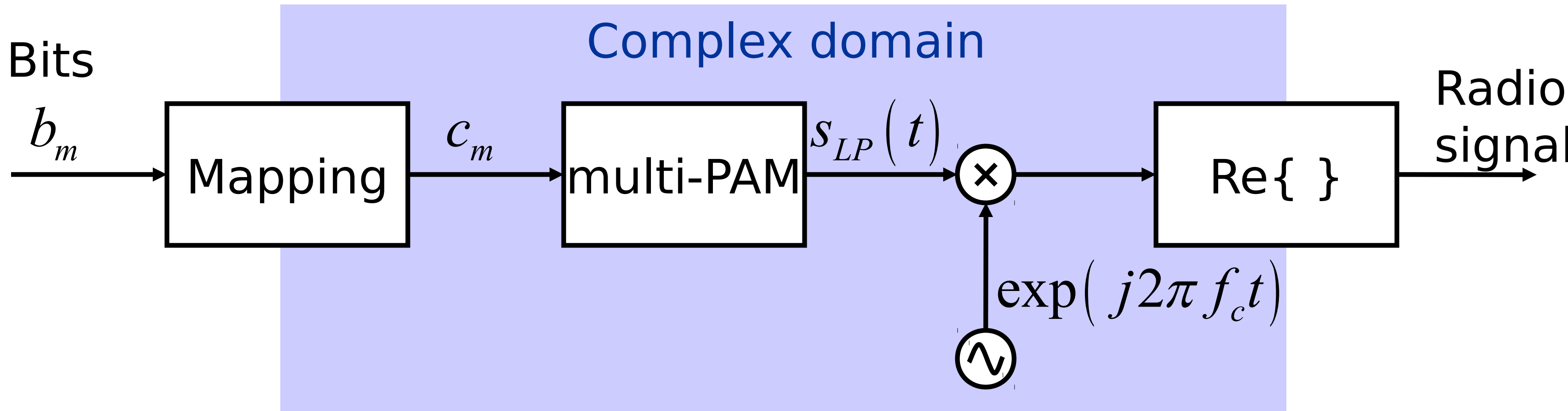
Pulse amplitude modulation (PAM)
Interpretation as IQ-modulator

For real valued basis functions *g*(*t*) we can view PAM as:



(Both the rectangular and the (root-) raised-cosine pulses are real valued.)

Multi-PAM
Modulation with multiple pulses



multi-PAM: $s_{\text{LP}}(t) = \sum_{m-\infty}^{\infty} g_{c_m}(t - mT_s)$

Several different pulses

"Standard" basis pulse criteria

$\int \left| g_{c_m}(t) \right|^2 dt = 1 \text{ or } = T_s$ (energy norm.)

$\int g_{c_m}(t)\, g^*_{c_m}(t - kT_s)\, dt = 0 \text{ for } k \neq 0$ (orthogonality)

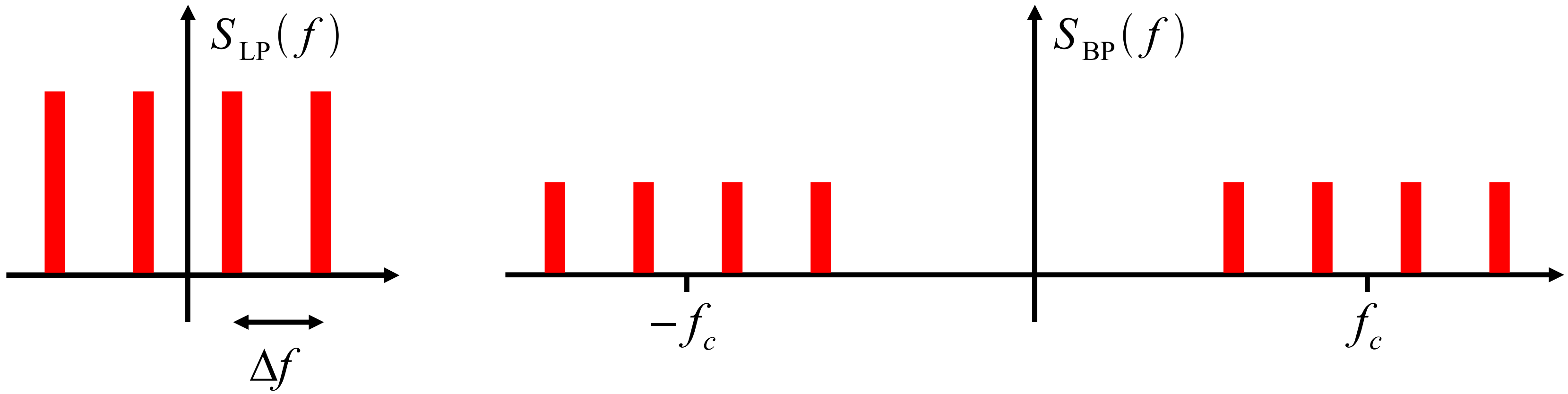
$\int g_{c_m}(t)\, g^*_{c_n}(t)\, dt = 0 \text{ for } c_m \neq c_n$ (orthogonality)

Multi-PAM
Modulation with multiple pulses

Frequency-shift keying (FSK) with *M* (even) different transmission frequencies can be interpreted as multi-PAM if the basis functions are chosen as:

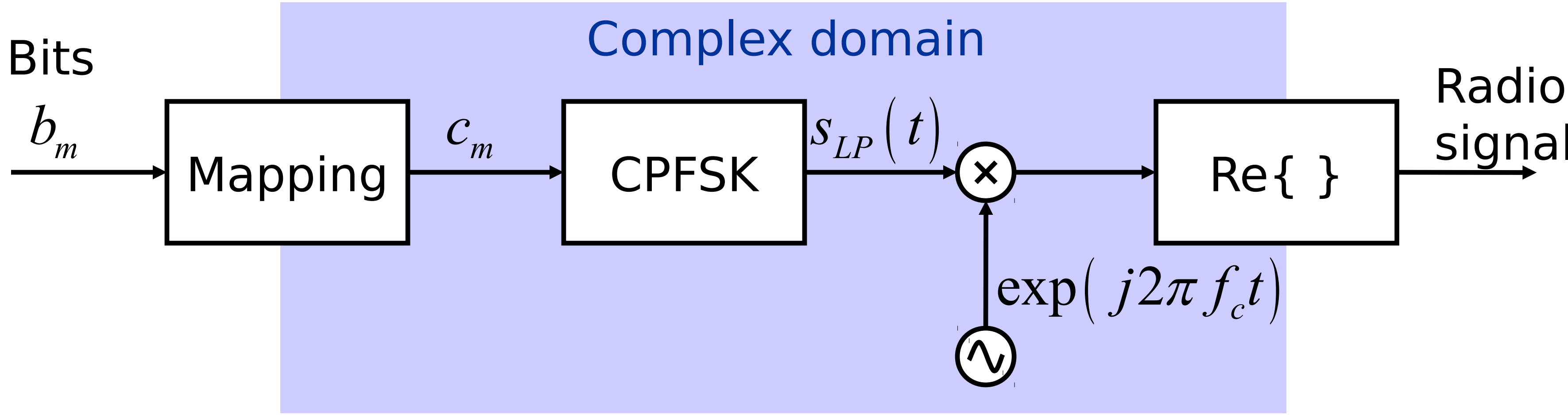
$$g_k(t) = e^{-j\pi k \Delta f t} \text{ for } 0 \leq t \leq T_s$$

and for *k* = +/- 1, +/- 3, ... , +/- M/2



Bits: 00 01 10 11

Continuous-phase FSK (CPFSK) The modulation process



CPFSK: $$s_{\mathrm{LP}}(t) = A\exp\left(j\Phi_{\mathrm{CPFSK}}(t)\right)$$

where the amplitude *A* is constant and the phase is

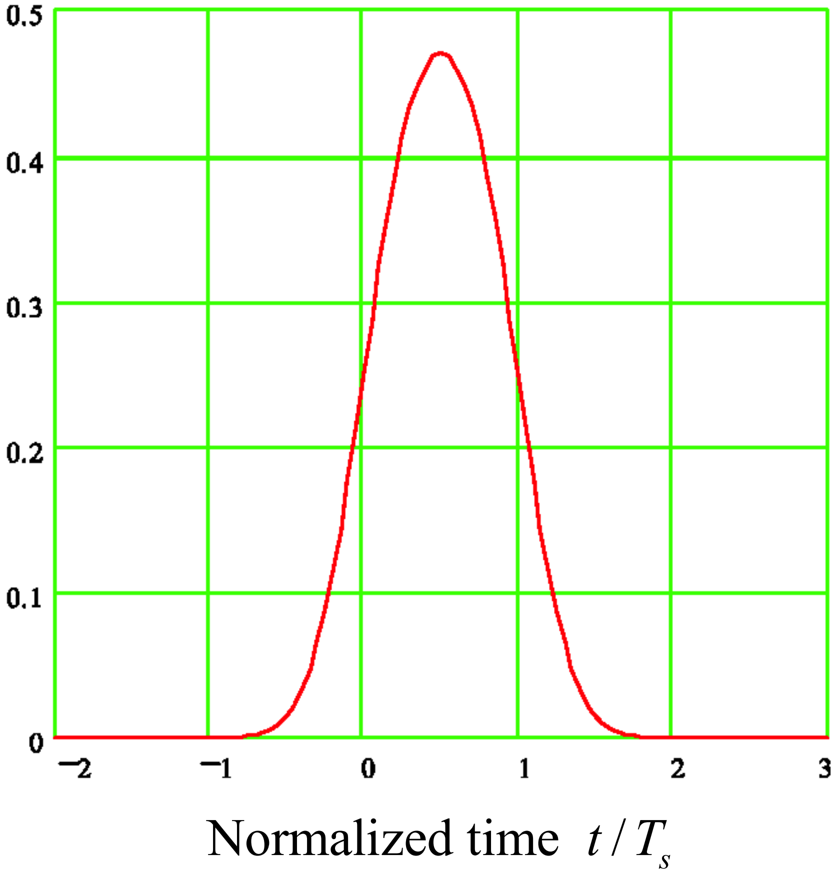
$$\Phi_{\mathrm{CPFSK}}(t) = 2\pi\, h_{\mathrm{mod}} \sum_{m=-\infty}^{\infty} c_m \int_{-\infty}^{t} \tilde{g}(u - mT)\, du$$

Phase basis pulse

where h_{mod} is the modulation index.

Continuous-phase FSK (CPFSK)
The Gaussian phase basis pulse

In addition to the rectangular phase basis pulse, the Gaussian is the most common.

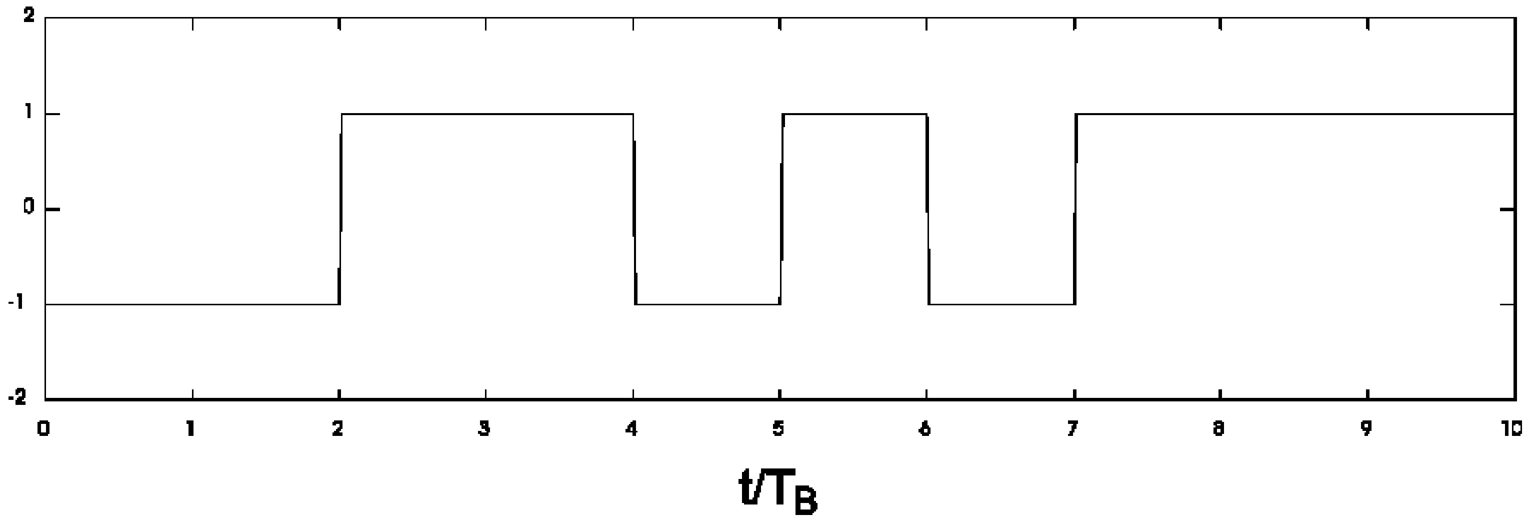


$BT_s = 0.5$

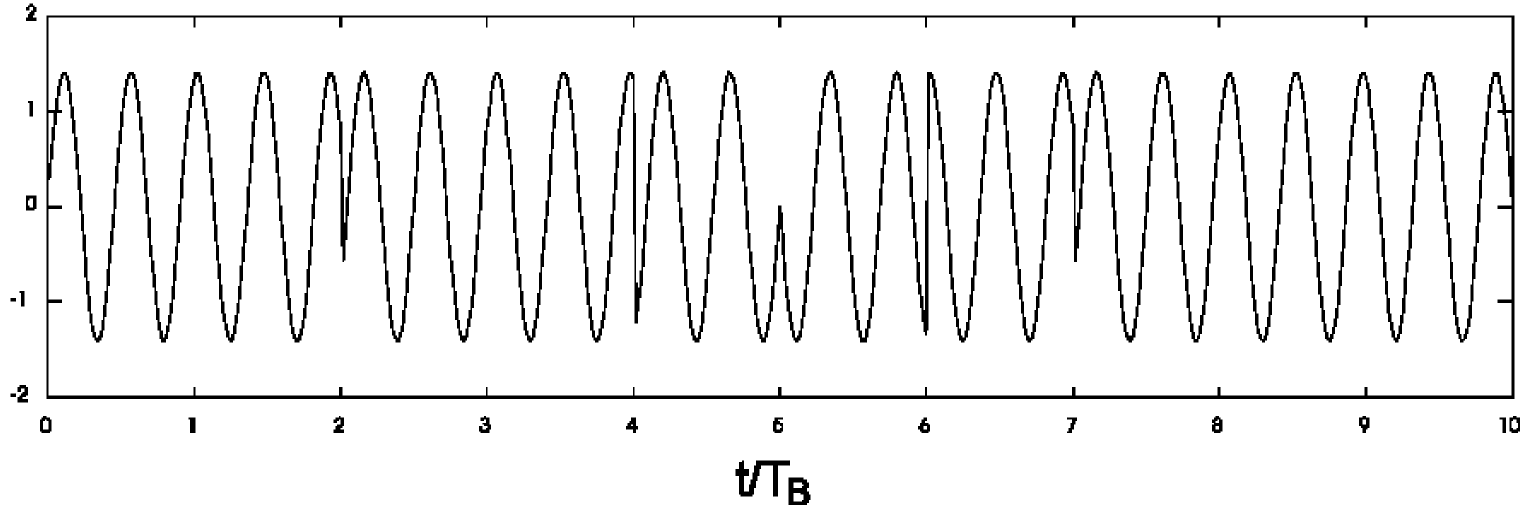
IMPORTANT MODULATION FORMATS

Binary phase-shift keying (BPSK)
Rectangular pulses

Base-band

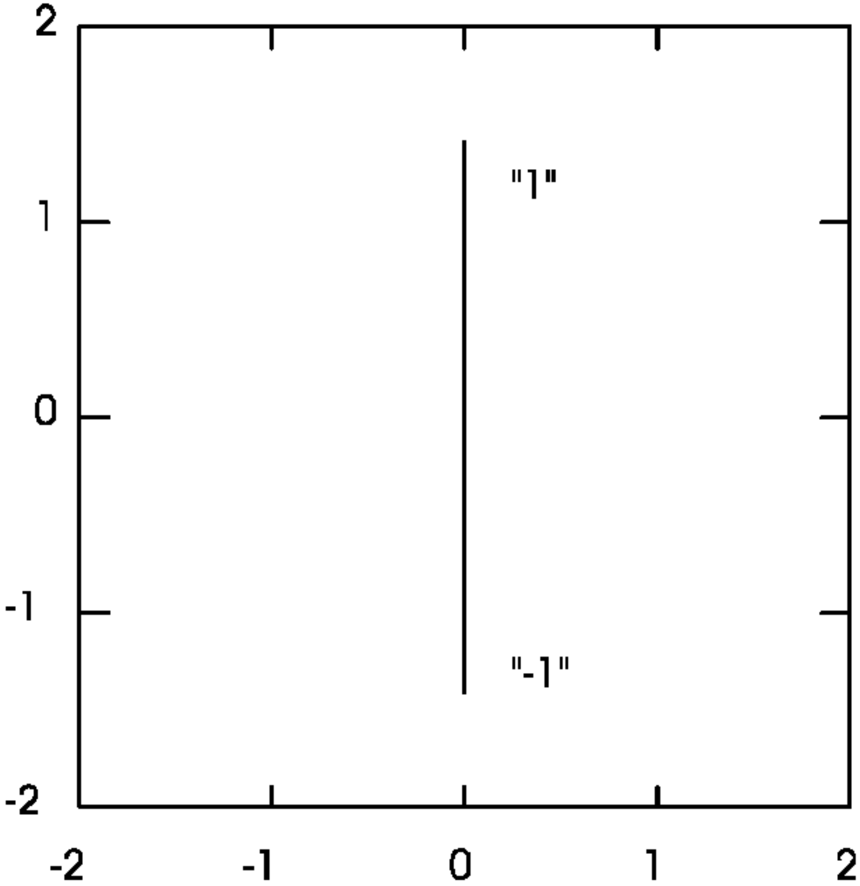


Radio signal

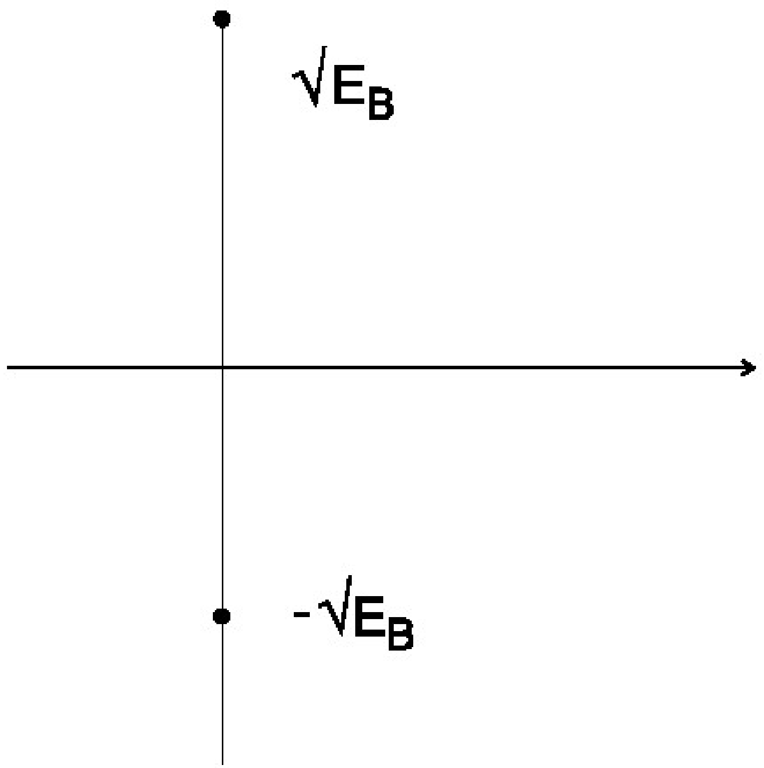


Binary phase-shift keying (BPSK) Rectangular pulses

Complex representation

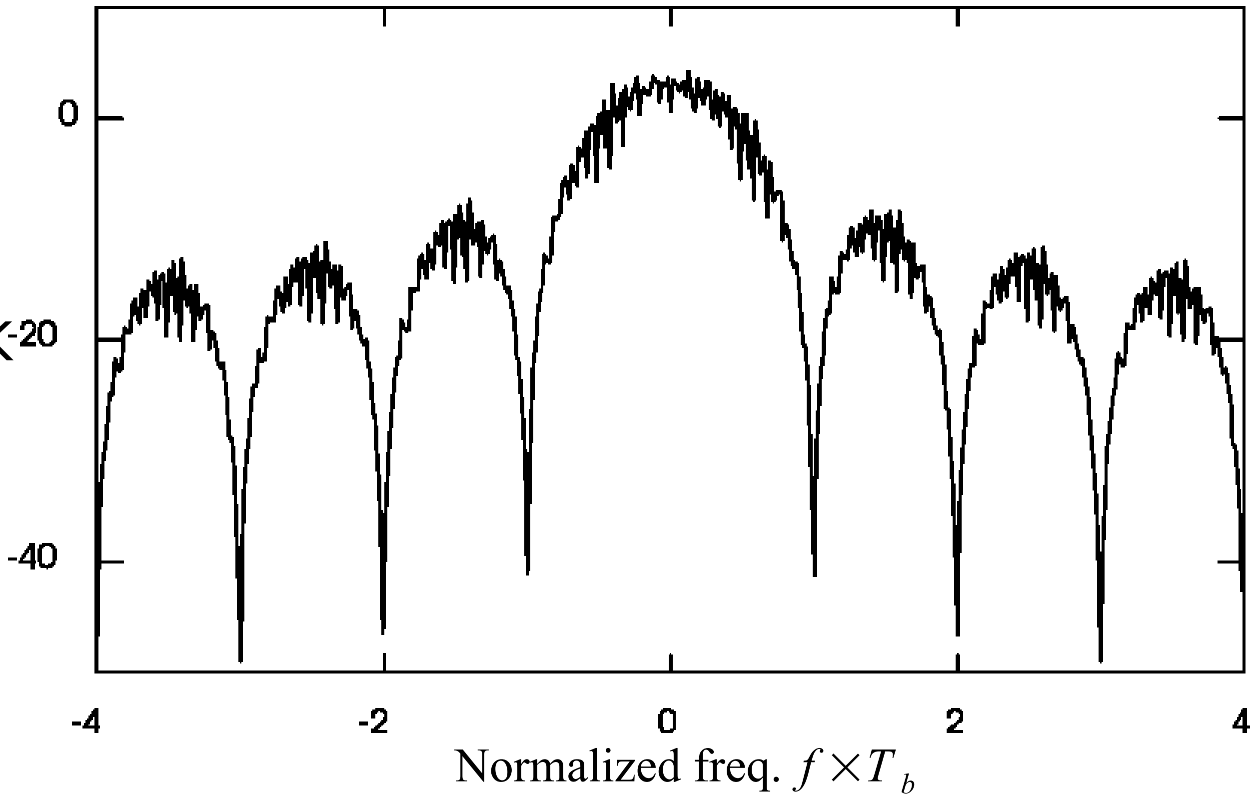


Signal constellation diagram



Binary phase-shift keying (BPSK) Rectangular pulses

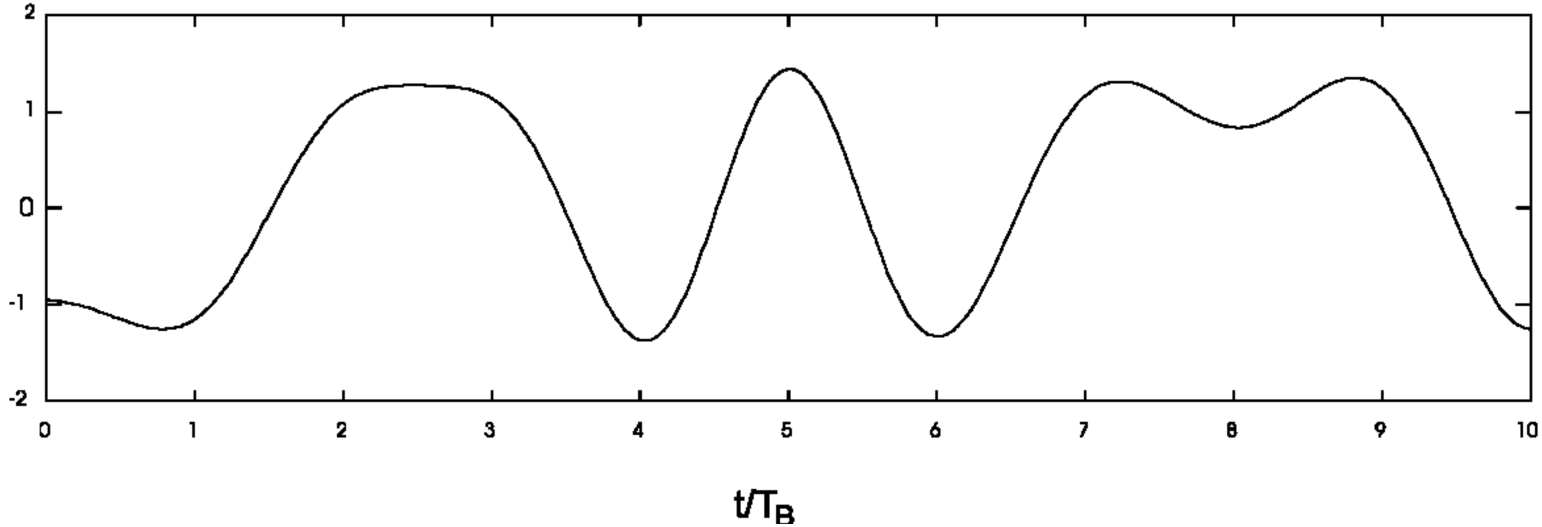
Power spectral density for BPSK



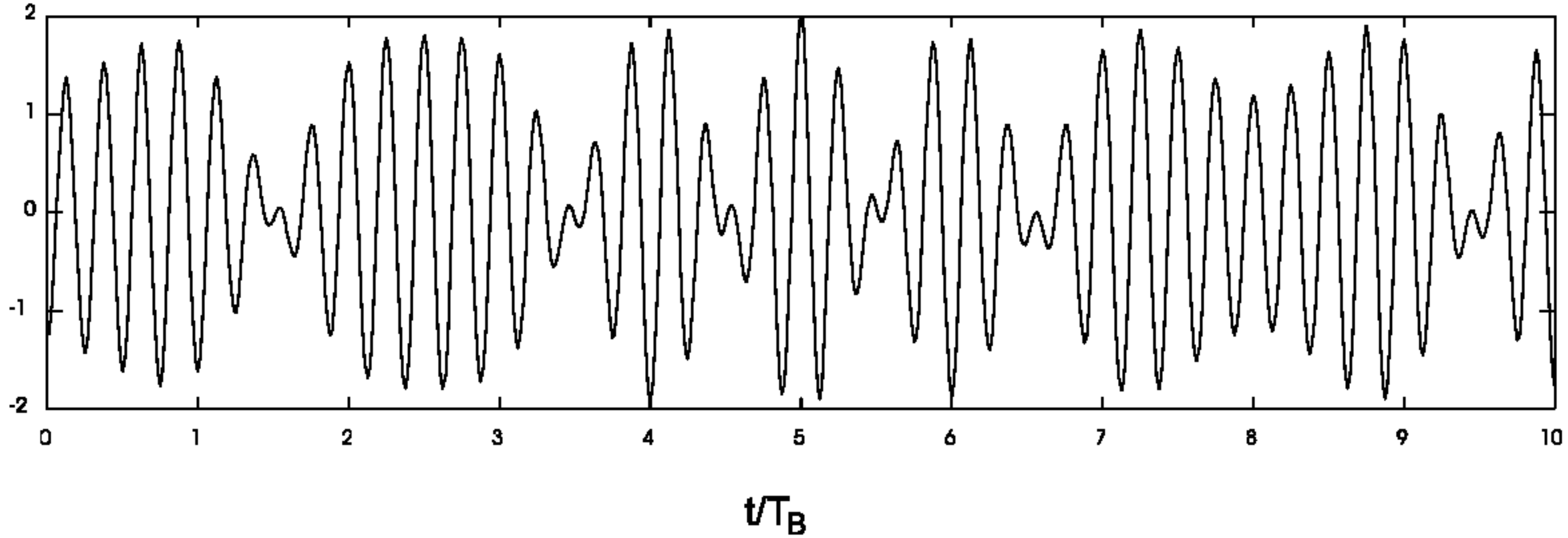
Contained percentage of total energy	spectral efficiency
90%	$0.59 Bit/s/Hz$
99%	$0.05 Bit/s/Hz$

Binary phase-shift keying (BPSK)
Raised-cosine pulses (roll-off 0.5)

Base-band

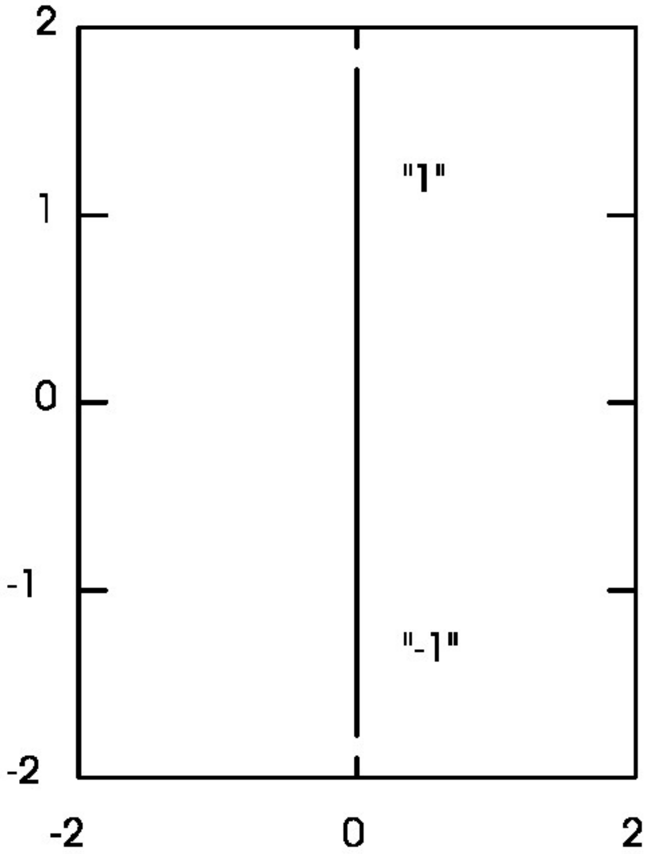


Radio signal

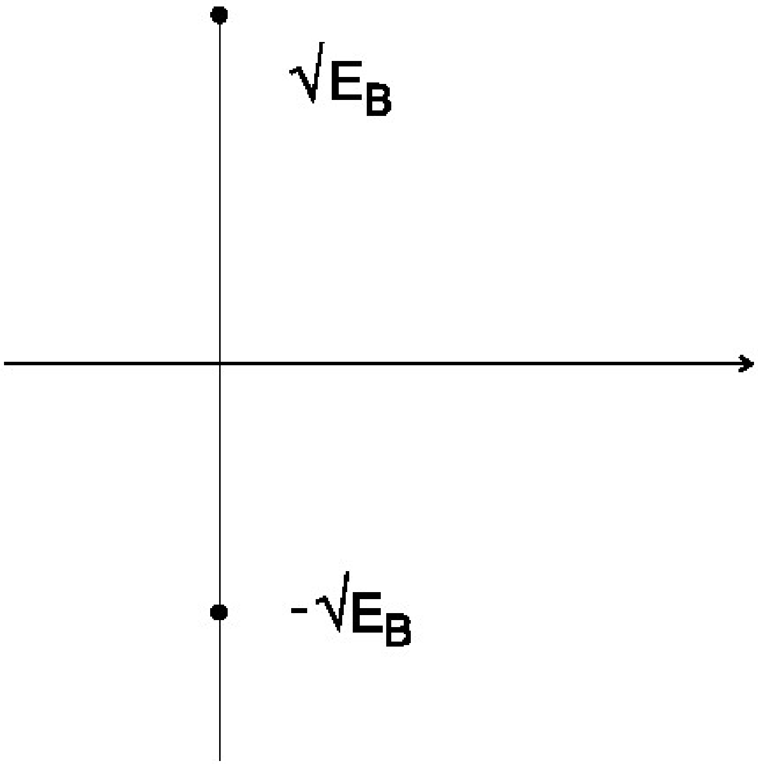


Binary phase-shift keying (BPSK)
Raised-cosine pulses (roll-off 0.5)

Complex representation

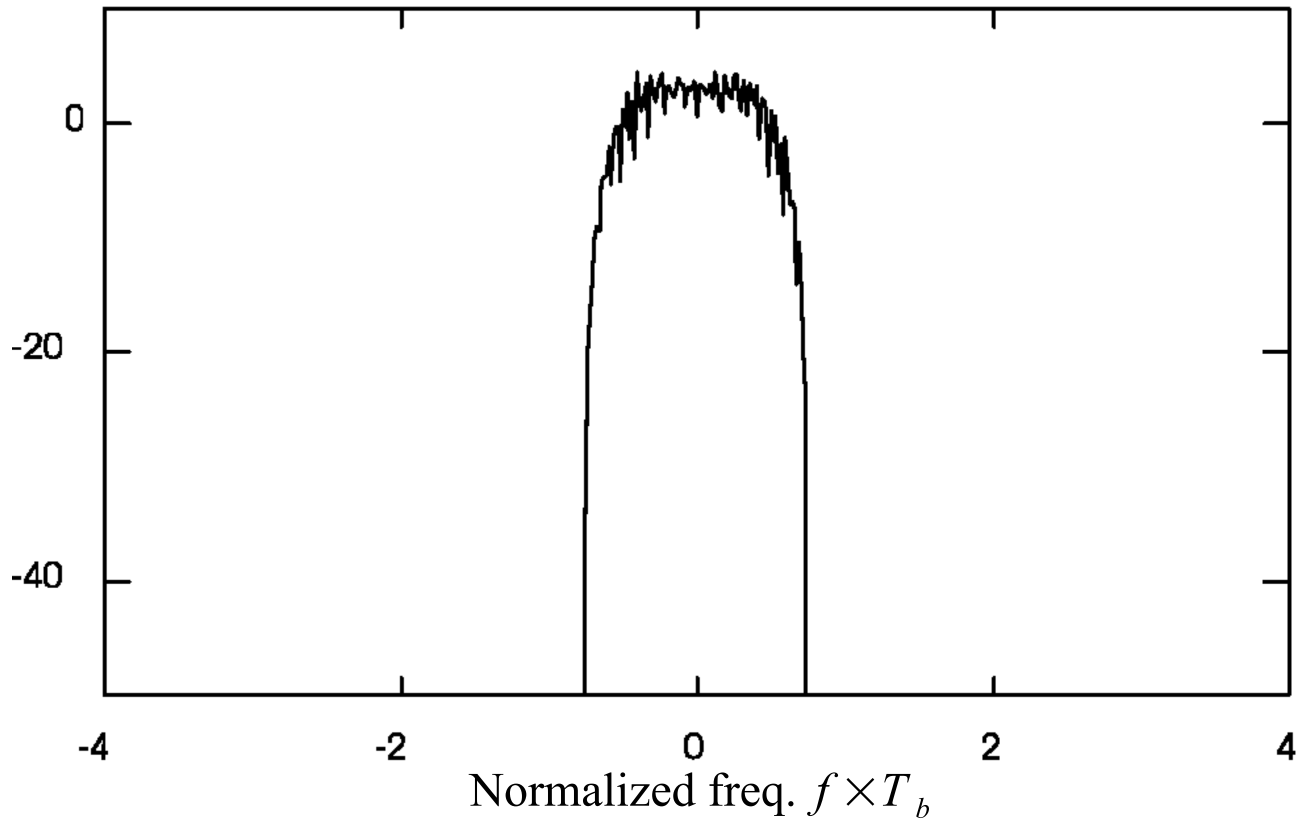


Signal constellation diagram



Binary phase-shift keying (BPSK) Raised-cosine pulses (roll-off 0.5)

Power spectral density for BAM

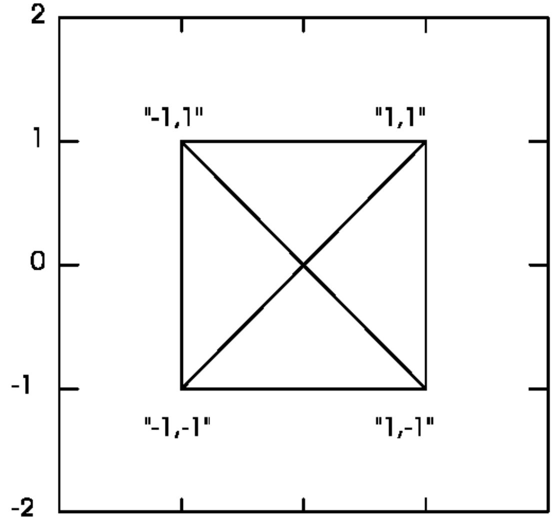


Contained percentage of total energy	spectral efficiency
90%	$1.02 Bit/s/Hz$
99%	$0.79 Bit/s/Hz$

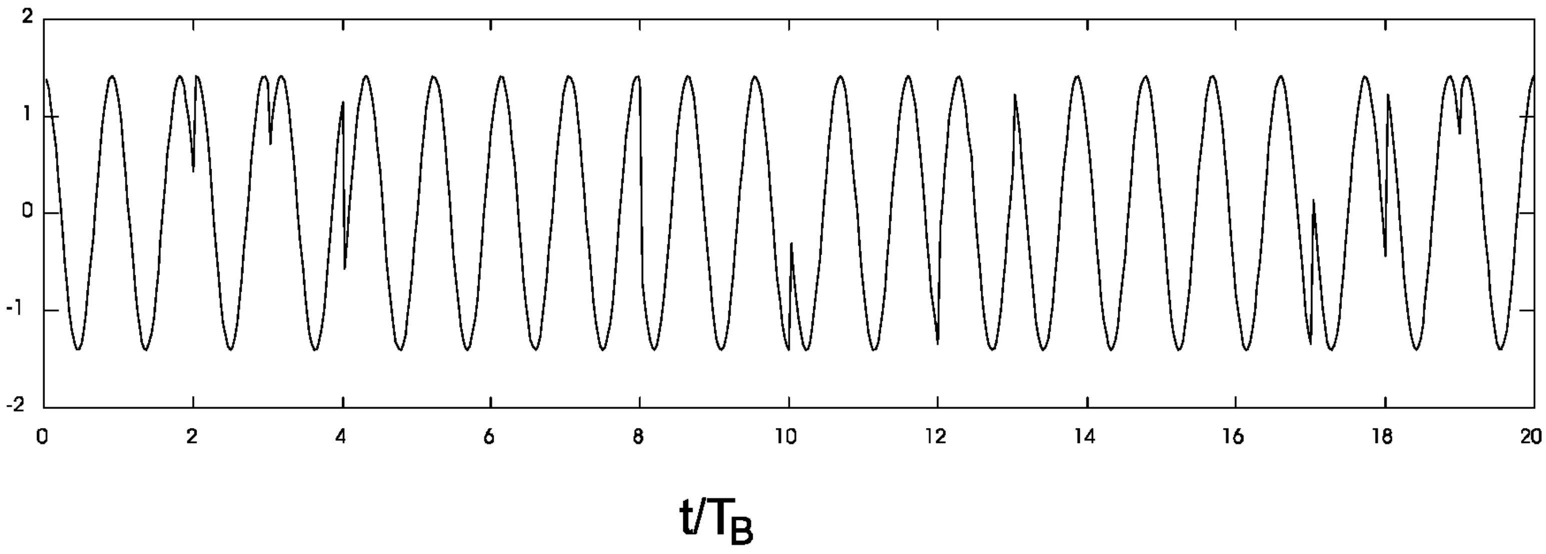
Much higher spectral efficiency than BPSK (with rectangular pulses).

Quaternary PSK (QPSK or 4-PSK)
Rectangular pulses

Complex representation

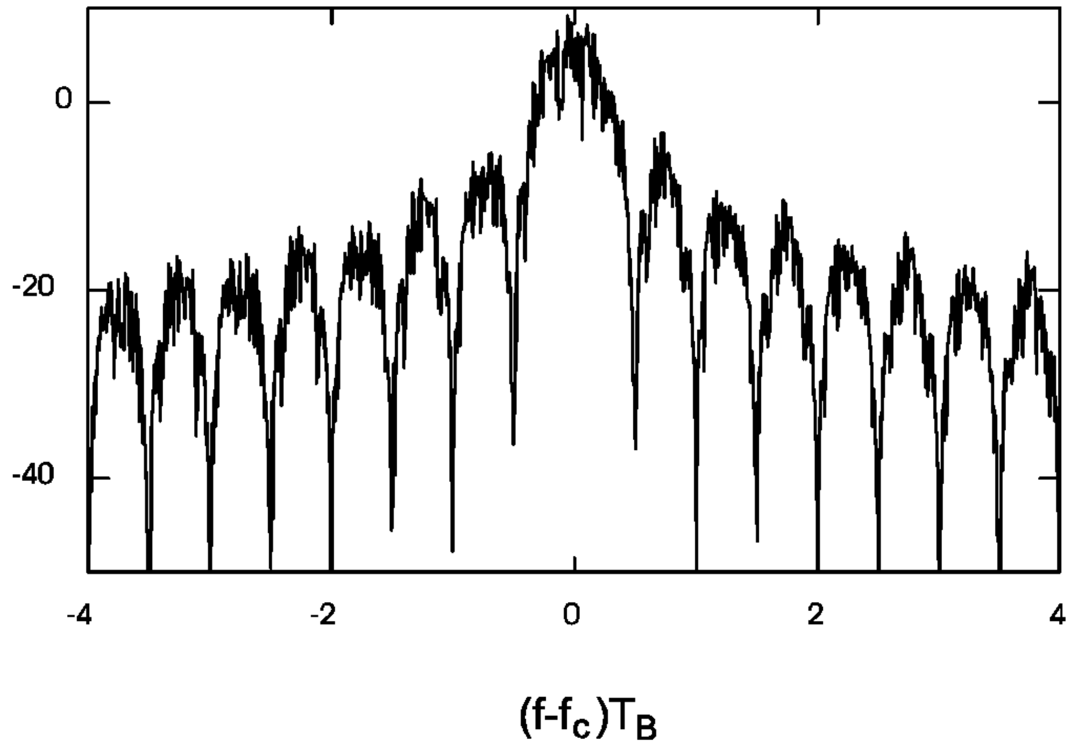


Radio signal



Quaternary PSK (QPSK or 4-PSK) Rectangular pulses

Power spectral density for QPSK

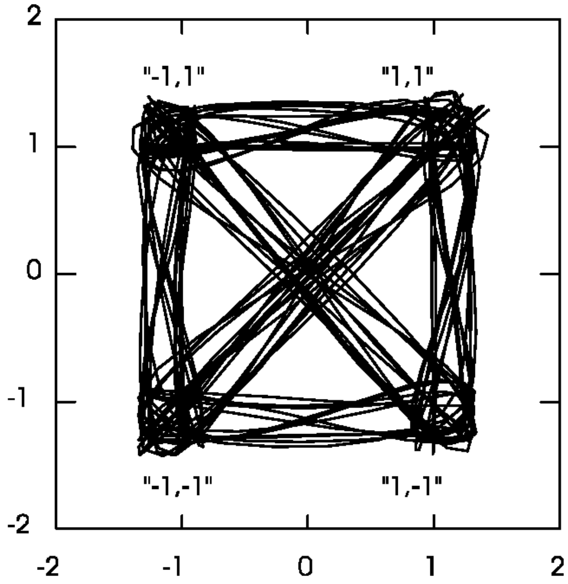


Contained percentage of total energy	spectral efficiency
90%	$1,18 Bit/s/Hz$
99%	$0.10 Bit/s/Hz$

Twice the spectrum efficiency of BPSK (with rect. pulses). TWO bits/pulse instead of one.

Quadrature ampl.-modulation (QAM)
Root raised-cos pulses (roll-off 0.5)

Complex representation



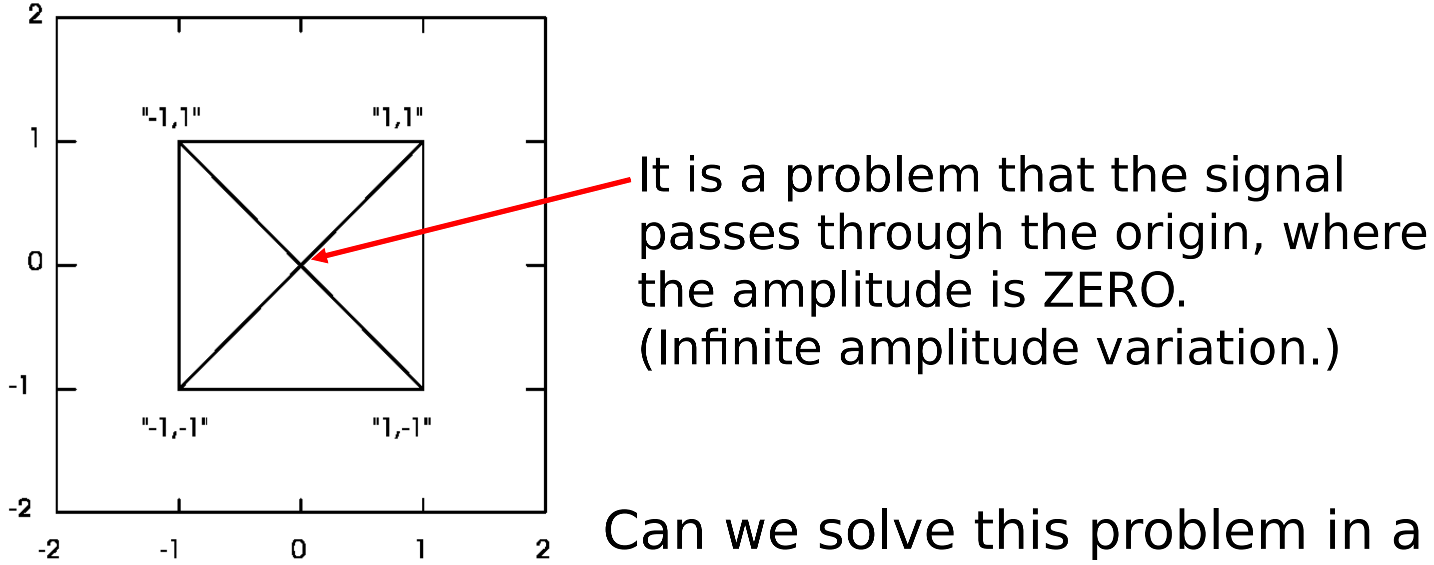
Contained percentage of total energy	spectral efficiency
90%	$2.04 Bit/s/Hz$
99%	$1.58 Bit/s/Hz$

Much higher spectral efficiency than QPSK (with rectangular pulses).

Amplitude variations
The problem

Signals with high amplitude variations leads to less efficient amplifiers.

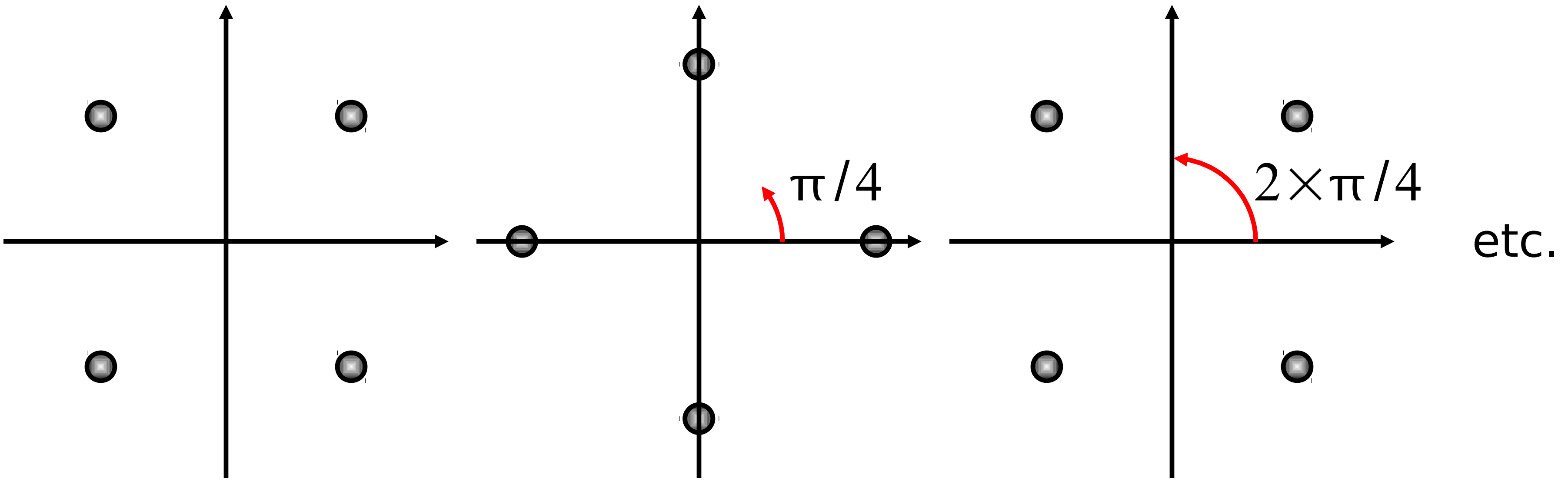
Complex representation of QPSK



Can we solve this problem in a simple way?

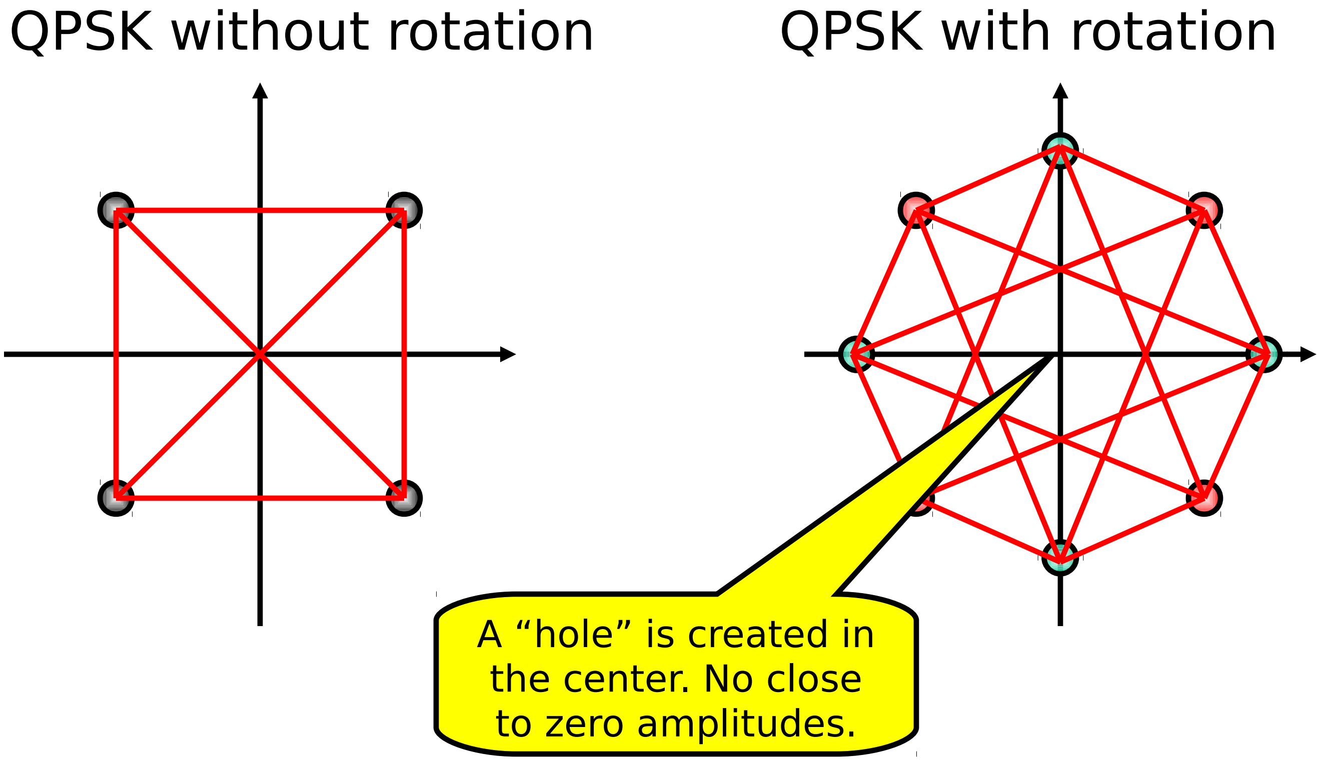
Amplitude variations
A solution

Let's rotate the signal constellation diagram for each transmitted symbol!



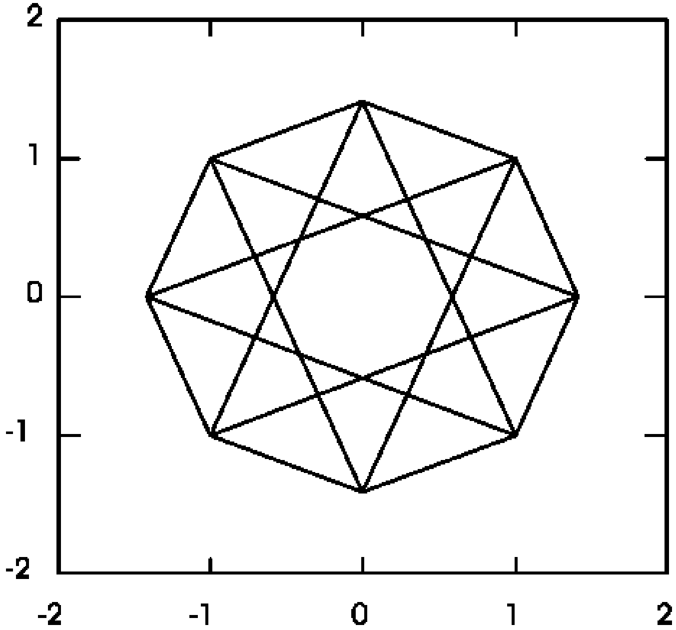
Amplitude variations
A solution

Looking at the complex representation ...



$\pi/4$ - Differential QPSK (DQPSK)

Complex representation



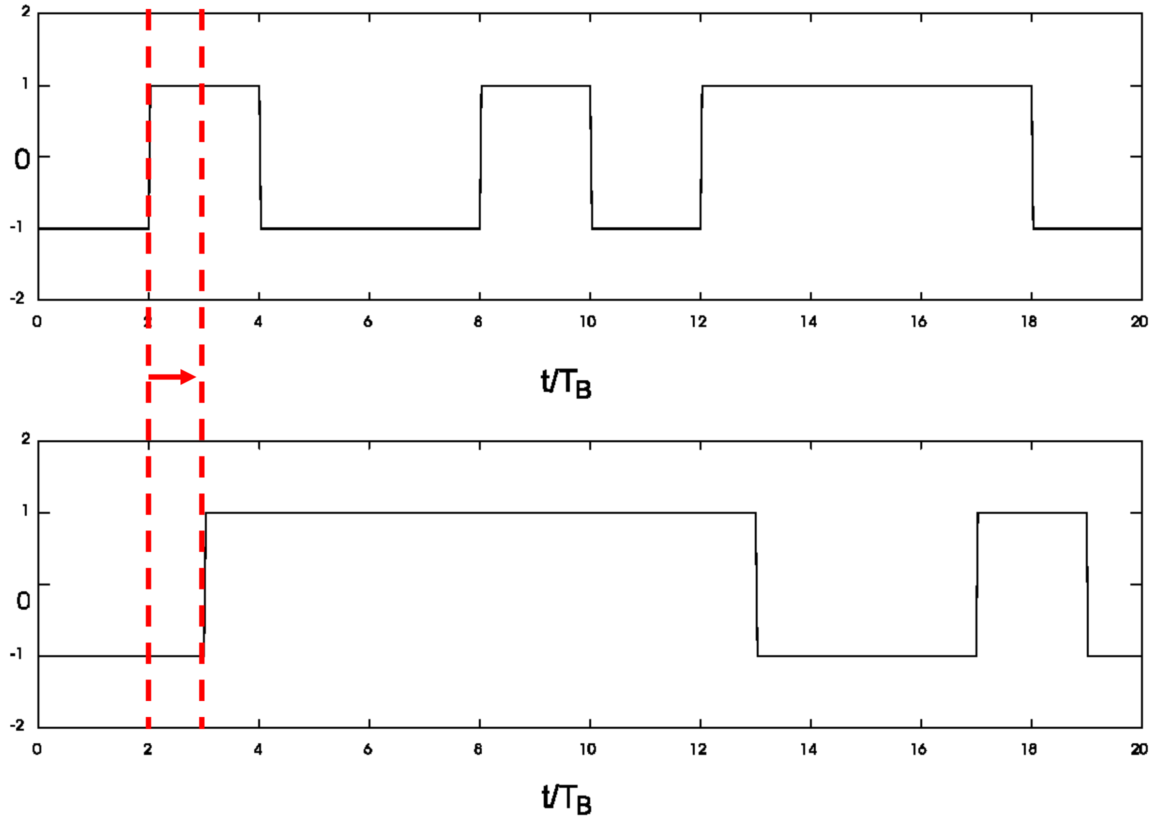
Still uses the same rectangular pulses as QPSK - the power spectral density and the spectral efficiency are the same.

This modulation type is used in several standards for mobile communications (due to it's low amplitude variations).

Offset QPSK (OQPSK)
Rectangular pulses

In-phase signal

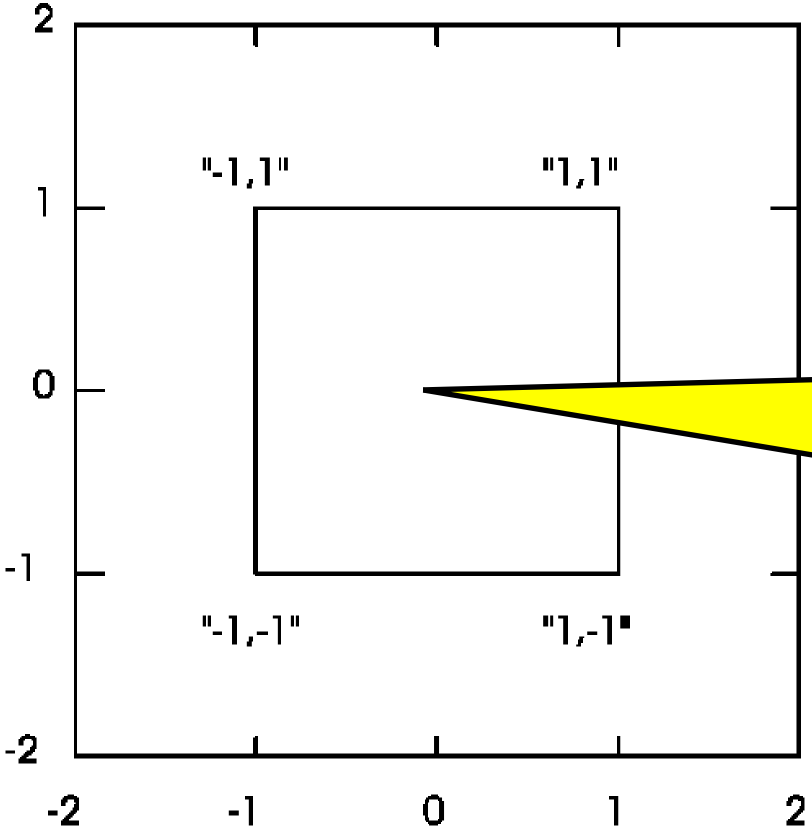
Quadrature signal



There is **one bit-time** offset between the in-pase and the quadrature part of the signal (a delay on the Q channel). This makes the transitions between pulses take place at different times!

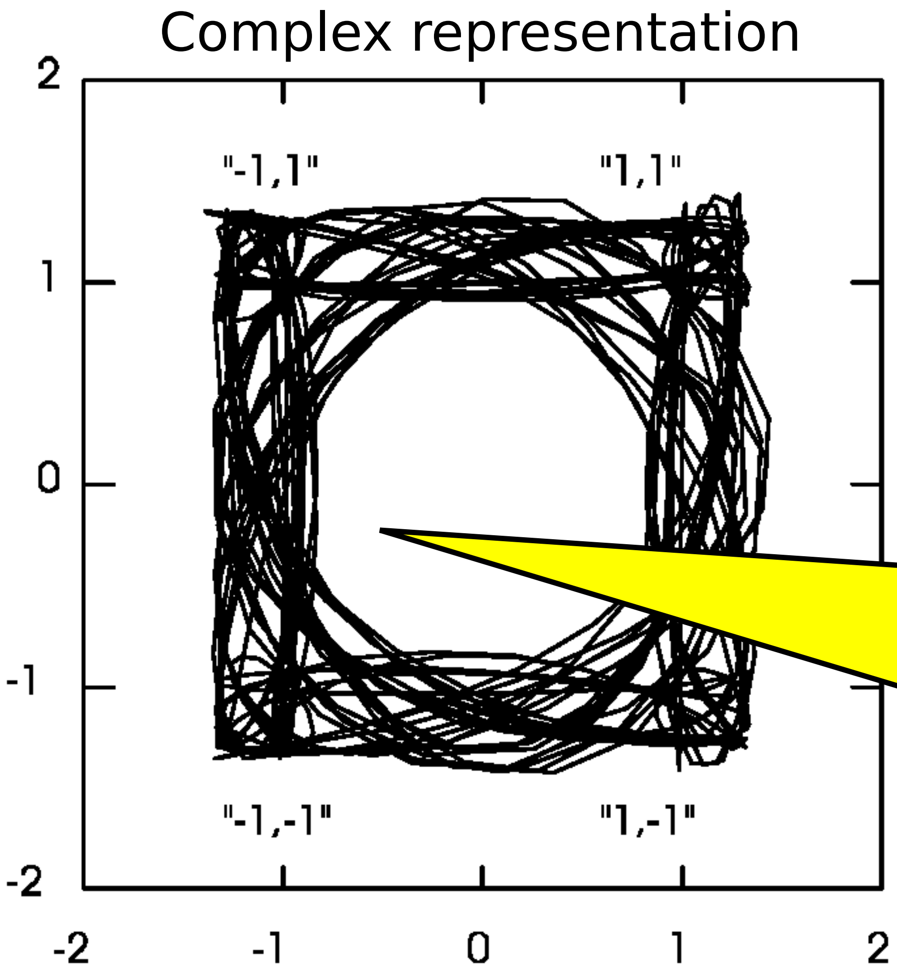
Offset QPSK
Rectangular pulses

Complex representation



This method also creates a hole in the center, giving less amplitude variations.

Offset QAM (OQAM)
Raised-cosine pulses



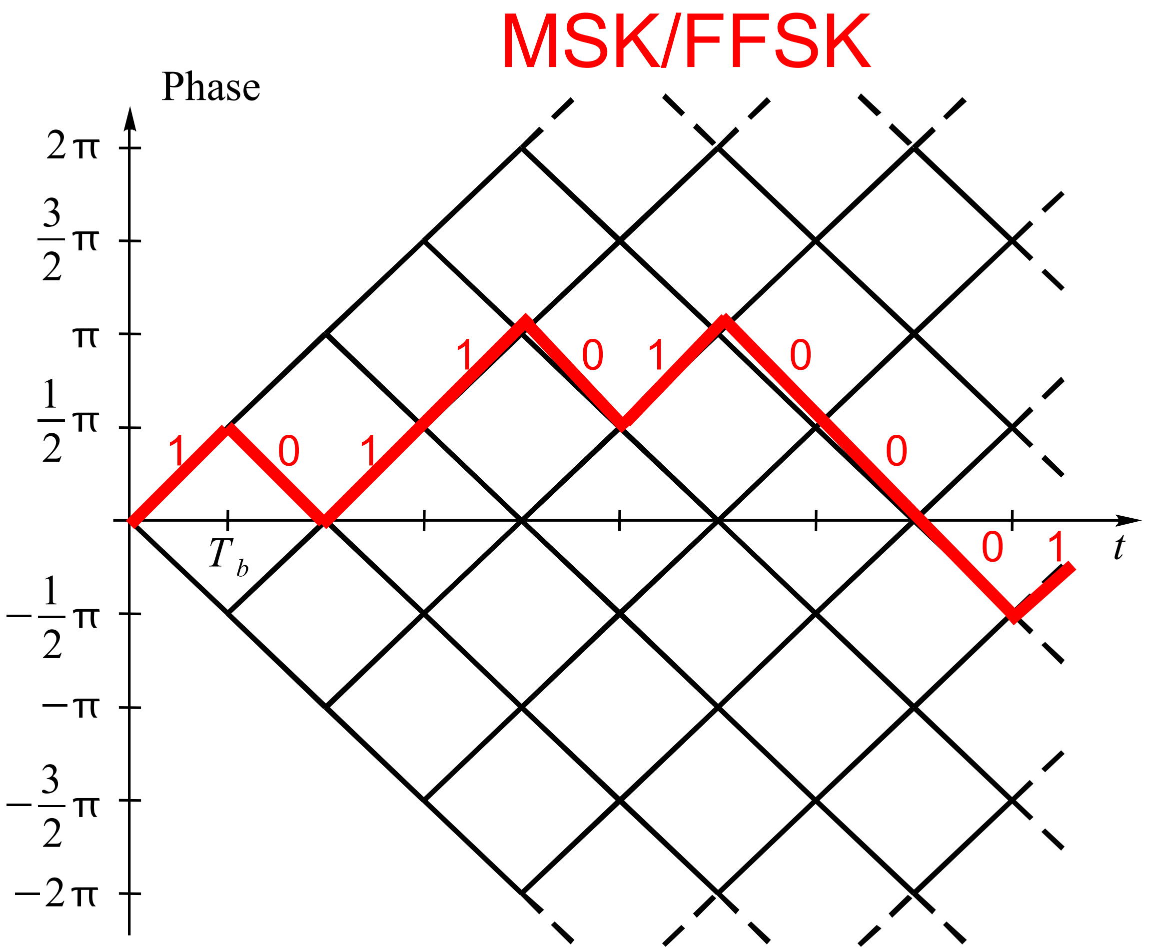
This method also creates a hole in the center, but has larger amplitude variations than OQPSK.

Continuous-phase modulation

Basic idea:
- Keep **amplitude constant**
- Change phase continuously

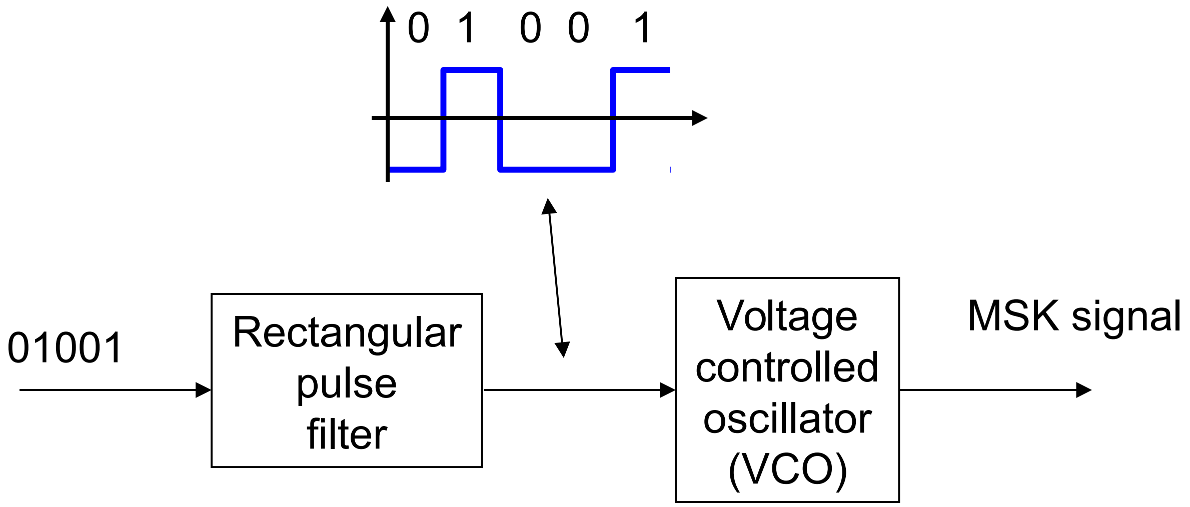
In this particular example we change the phase in a piecewise linear fashion by +/- $\pi/2$, depending on the data transmitted.

This type of modulation can be interpreted both as phase and frequency modulation. It is called **MSK** (minimum shift keying) or **FFSK** (fast frequency shift keying).



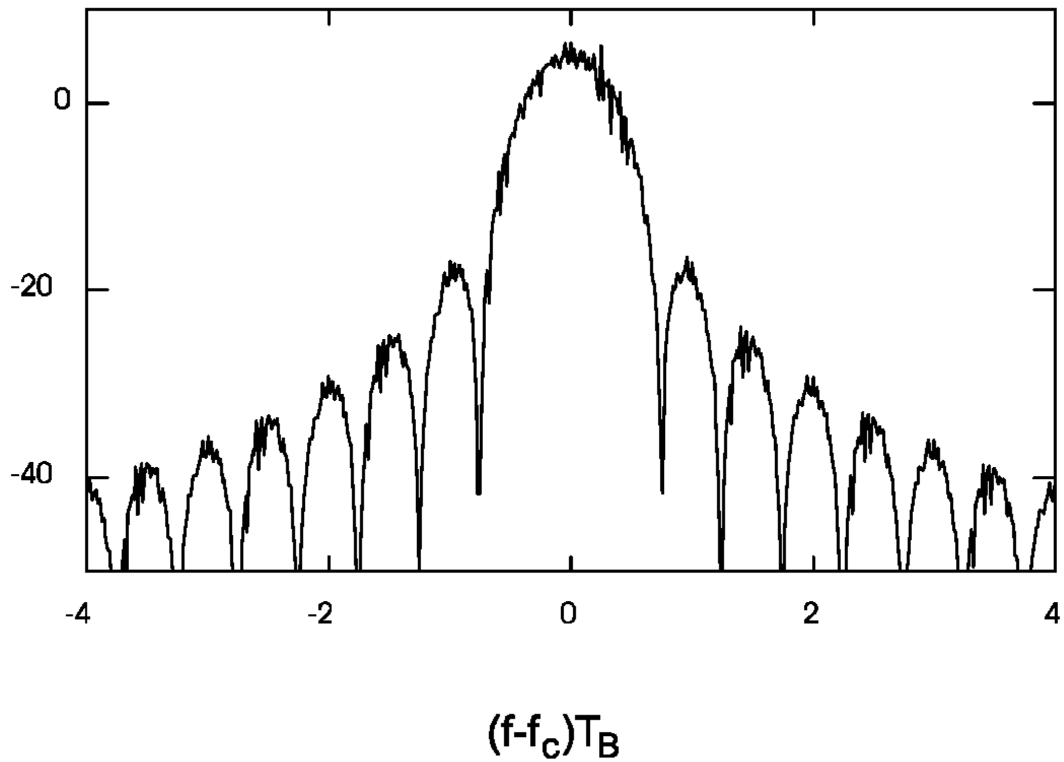
Minimum shift keying (MSK)

Simple MSK implementation



Minimum shift keying (MSK)

Power spectral density of MSK

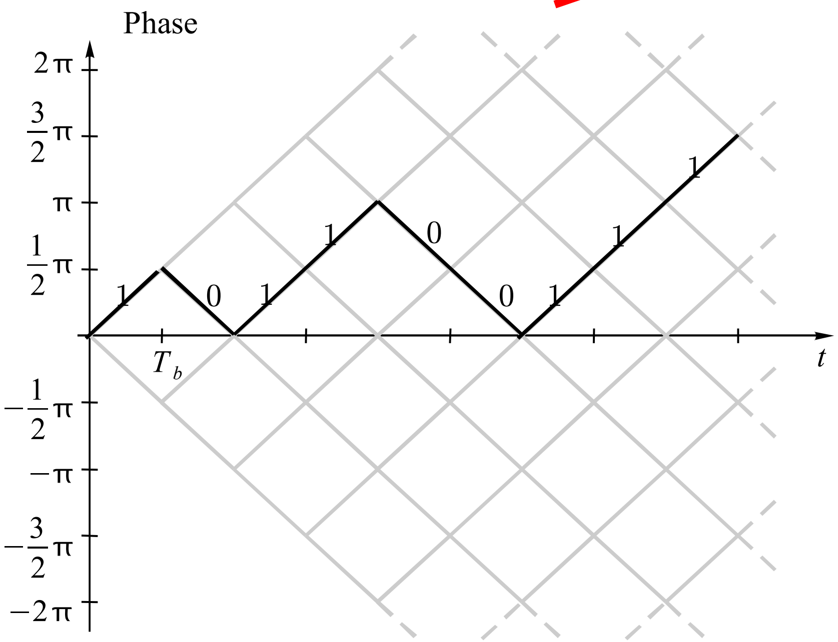


Contained percentage of total energy	spectral efficiency
90 %	1,29 Bit / s / Hz
99 %	0,85 Bit / s / Hz

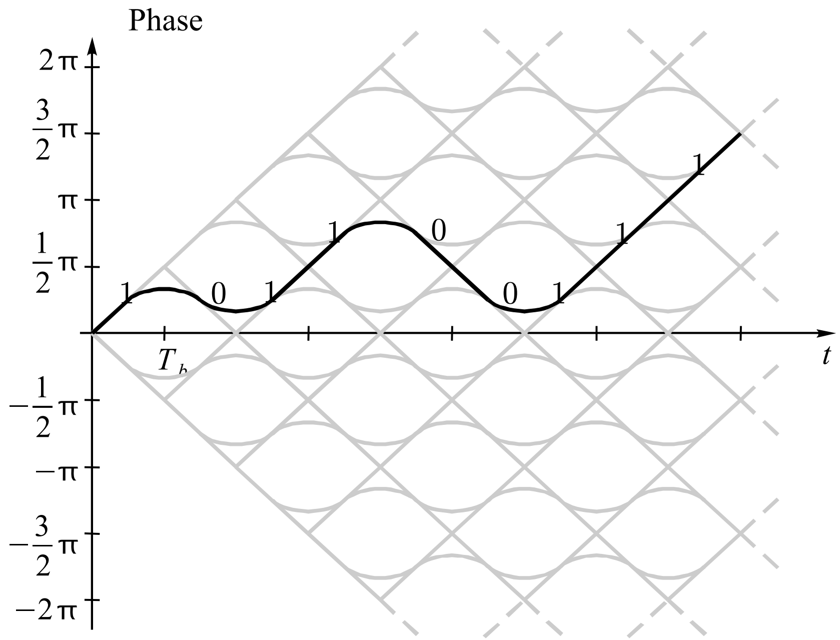
Gaussian filtered MSK (GMSK)

Further improvement of the phase: Remove 'corners'

(Simplified figure)



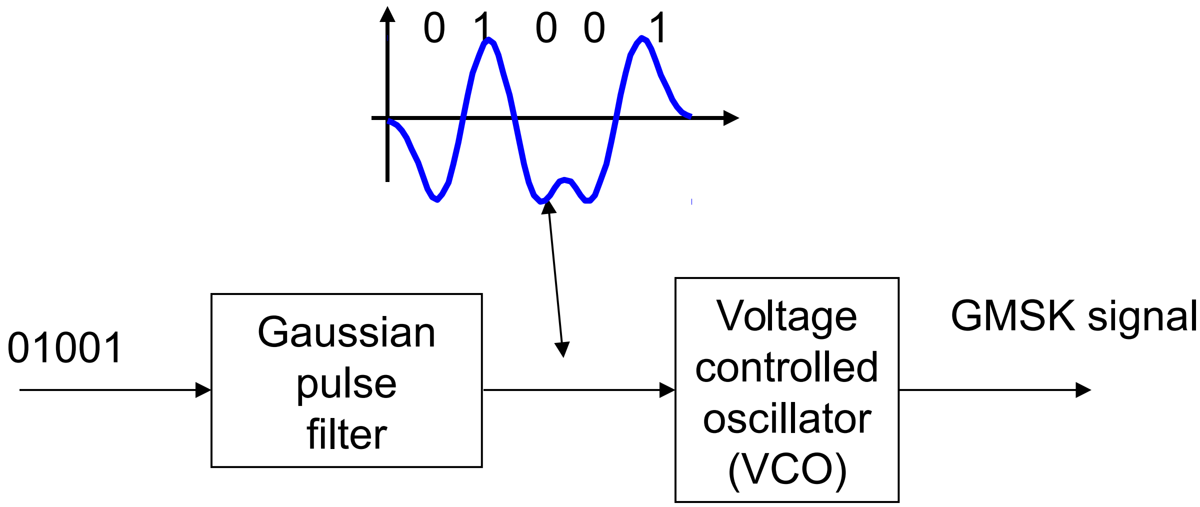
MSK
(Rectangular pulse filter)



Gaussian filtered MSK - GMSK
(Gaussian pulse filter)

Gaussian filtered MSK (GMSK)

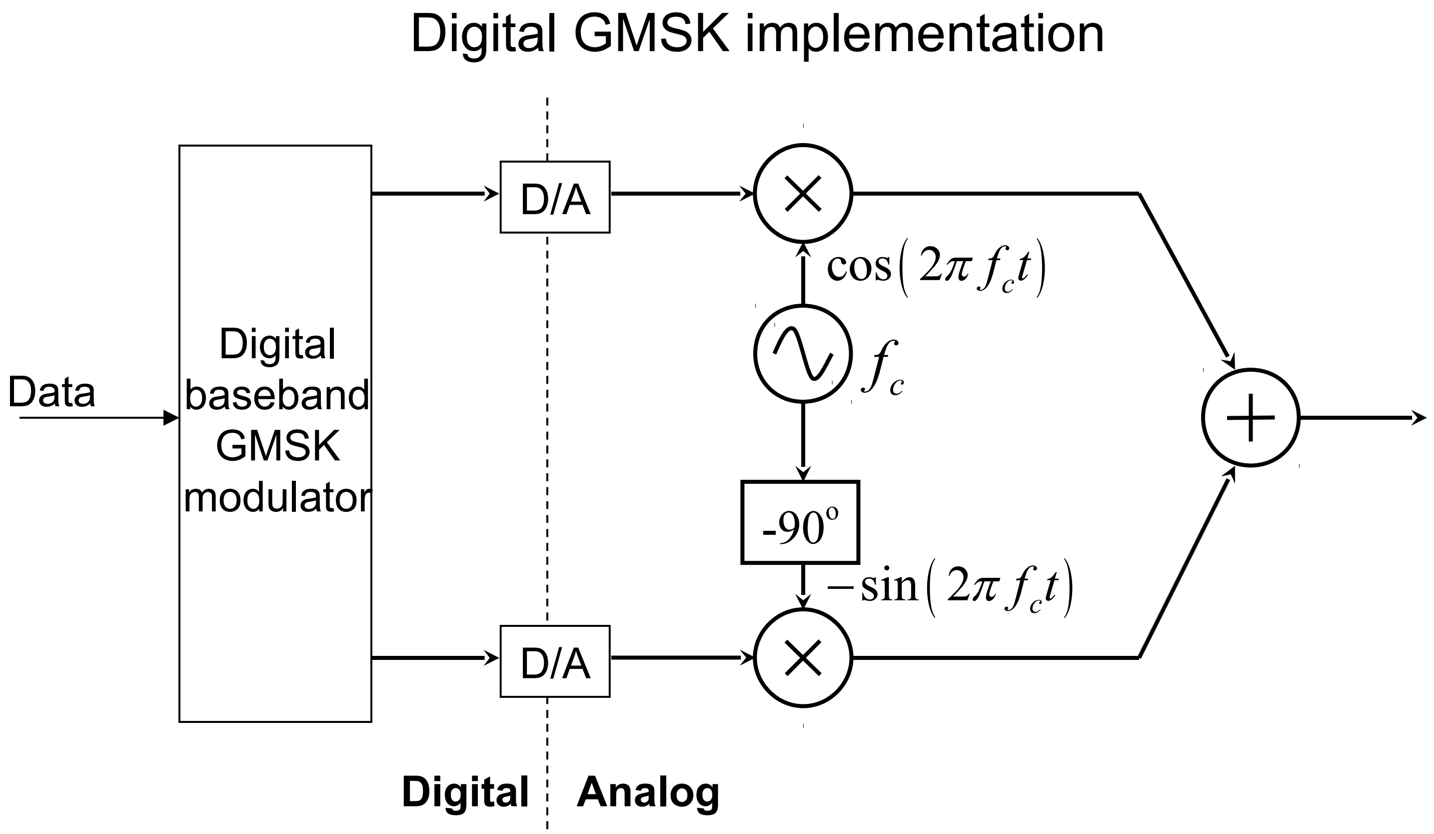
Simple GMSK implementation



When implemented this “simple” way, it is usually called Gaussian filtered frequency shift keying (GFSK).

GSFK is used in e.g. Bluetooth.

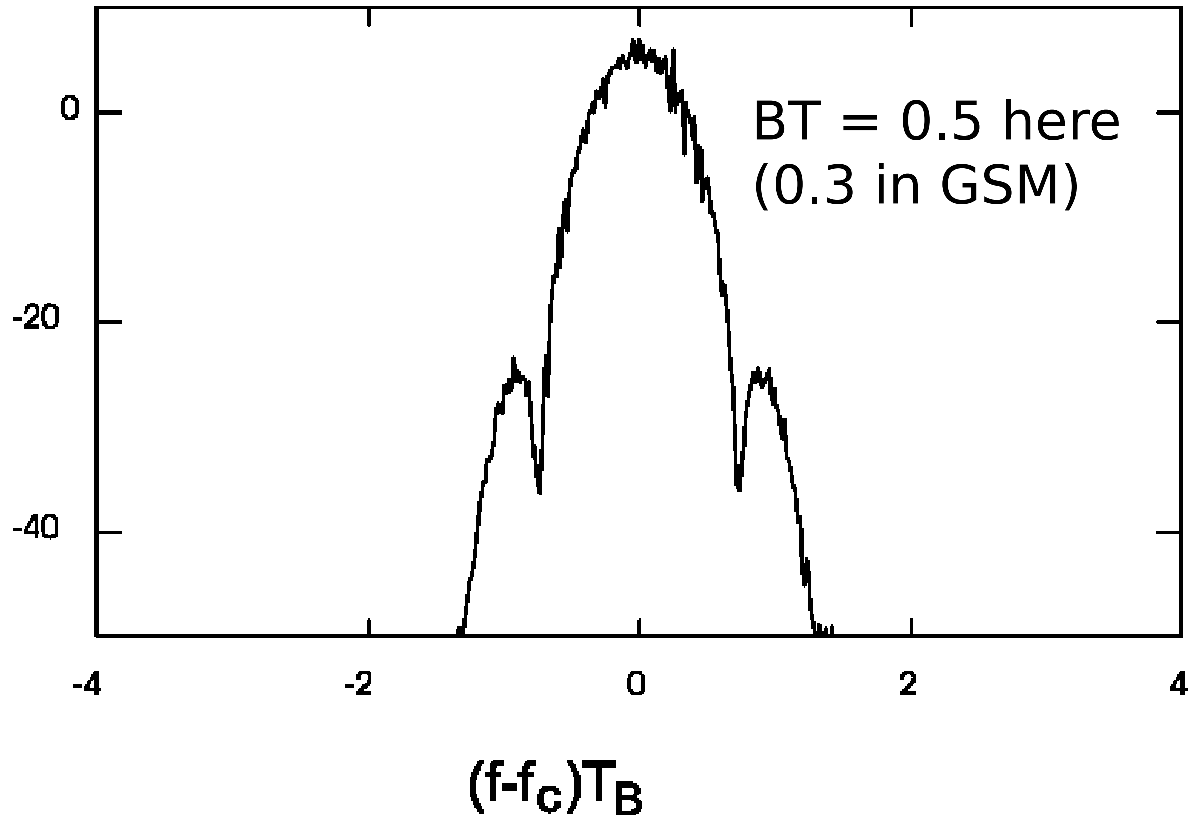
Gaussian filtered MSK (GMSK)



This is a more precise implementation of GMSK, which is used in e.g. GSM.

Gaussian filtered MSK (GMSK)

Power spectral density of GMSK.



Contained percentage of total energy	spectral efficiency
90 %	1,45 Bit / s / Hz
99 %	0,97 Bit / s / Hz

How do we use all these spectral efficiencies?

Example: Assume that we want to use MSK to transmit 50 kbit/sec, and want to know the required transmission bandwidth.

Take a look at the spectral efficiency table:

Contained percentage of total energy	spectral efficiency
90 %	1,29 Bit / s / Hz
99 %	0,85 Bit / s / Hz

The 90% and 99% bandwidths become:

$$B_{90\%} = 50000 / 1.29 = 38.8 \text{ kHz}$$

$$B_{99\%} = 50000 / 0.85 = 58.8 \text{ kHz}$$

Summary

BPSK with root-raised cosine pulses → BAM (α=0.5)

Modulation method	spectral efficiency	spectral efficiency
	for 90 % of	for 99 % of
	total energy	total energy
	Bit / s / Hz	Bit / s / Hz
BPSK	0,59	0,05
BAM (α=0.5)	1,02	0,79
QPSK, OQPSK,	1,18	0,10
MSK	1,29	0,85
GMSK (B_G T= 0.5)	1,45	0,97
QAM ($\alpha = 0.5$)	2,04	1,58

TABLE 11.1 in textbook.