



Lecture no: **3**

## Narrow- and wideband channels

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## **NARROW-BAND CHANNELS**

- Radio signals and complex notation
- Large-scale fading
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- Combining large- and small-scale fading
- Noise- and interference-limited links

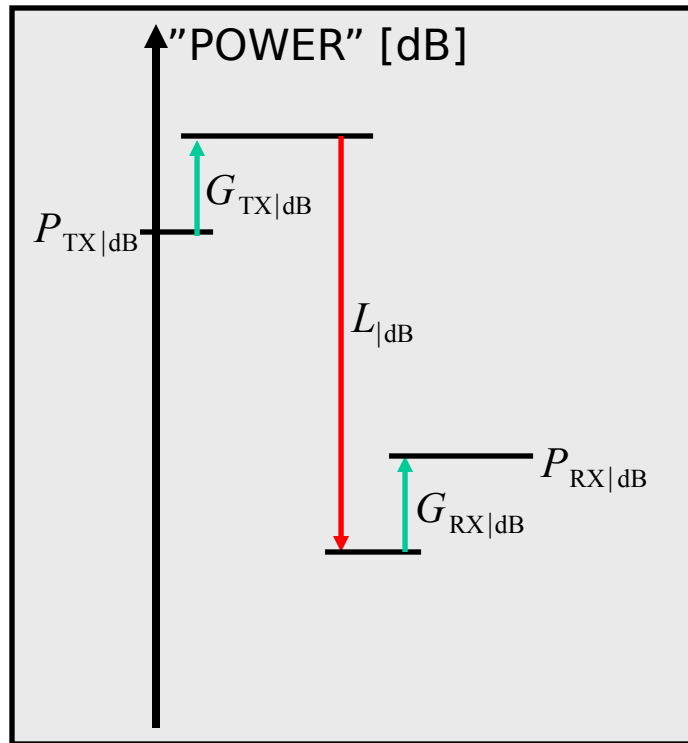
## **WIDE-BAND CHANNELS**

- What makes a channel wide-band?
- Delay (time) dispersion
- Narrow- versus wide-band channels



# SHORT REVIEW

# What do we know so far about propagation losses?



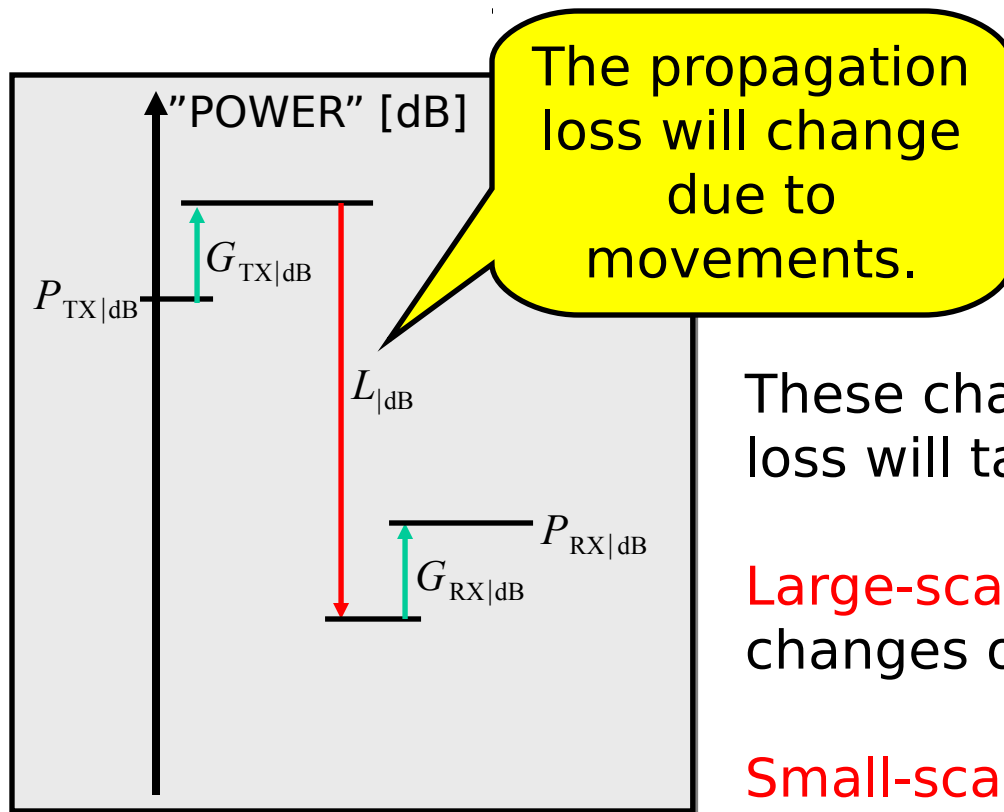
Two theoretical expressions for the deterministic propagation loss as functions of distance:

$$L_{|dB}(d) = \begin{cases} 20 \log_{10} \left( \frac{4\pi d}{\lambda} \right), & \text{free space} \\ 20 \log_{10} \left( \frac{d^2}{h_{TX} h_{RX}} \right), & \text{ground plane} \end{cases}$$

There are other models, which we will discuss later.

We have discussed shadowing/ diffraction and reflections, but not really made any detailed calculations.

# Statistical descriptions of the mobile radio channel



These changes of the propagation loss will take place in two scales:

**Large-scale:** shadowing, "slow" changes over *many wavelengths*.

**Small-scale:** interference, "fast" changes on the scale of *a wavelength*.

Now we are going to approach these variations from a statistical point of view.



# RADIO SIGNALS AND COMPLEX NOTATION



# Simple model of a radio signal

- A transmitted radio signal can be written

$$s(t) = A \cos(2\pi ft + \phi)$$

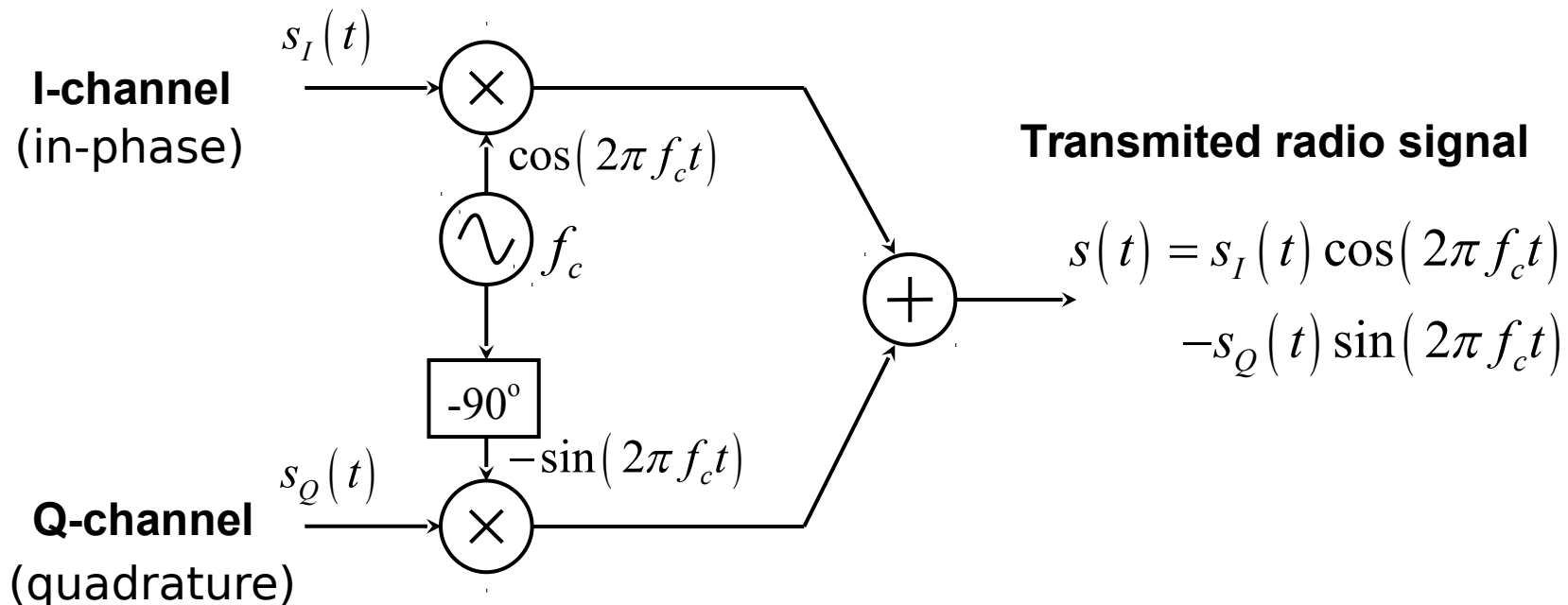
Amplitude      Frequency      Phase

- By letting the transmitted information change the amplitude, the frequency, or the phase, we get the three basic types of digital modulation techniques

- **ASK** (Amplitude Shift Keying)
  - **FSK** (Frequency Shift Keying)
  - **PSK** (Phase Shift Keying)
- Constant amplitude
-



# The IQ modulator

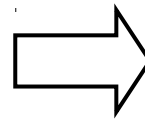


**Take a step into the complex domain:**

Complex envelope  $\tilde{s}(t) = s_I(t) + j s_Q(t)$

Carrier factor

$$e^{j2\pi f_c t}$$



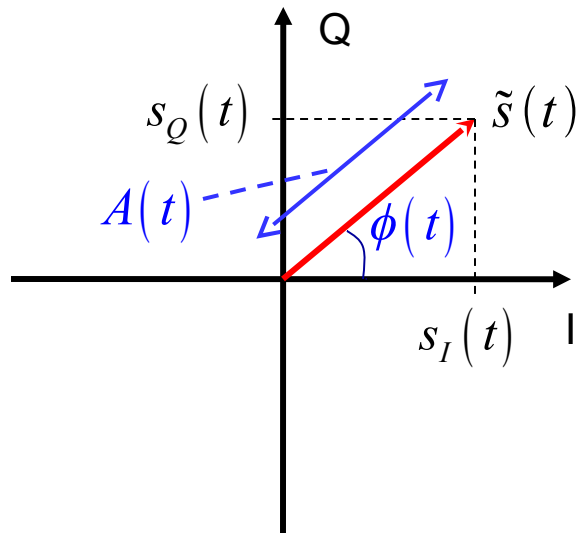
$$s(t) = \text{Re}\{\tilde{s}(t) e^{j2\pi f_c t}\}$$



# Interpreting the complex notation



Complex envelope (phasor)



Polar coordinates:

$$\tilde{s}(t) = s_I(t) + j s_Q(t) = A(t) e^{j\phi(t)}$$

Transmitted radio signal

$$\begin{aligned} s(t) &= \operatorname{Re} \left\{ \tilde{s}(t) e^{j2\pi f_c t} \right\} \\ &= \operatorname{Re} \left\{ A(t) e^{j\phi(t)} e^{j2\pi f_c t} \right\} \\ &= \operatorname{Re} \left\{ A(t) e^{j(2\pi f_c t + \phi(t))} \right\} \\ &= A(t) \cos(2\pi f_c t + \phi(t)) \end{aligned}$$

By manipulating the amplitude  $A(t)$  and the phase  $\phi(t)$  of the complex envelope (phasor), we can create any type of modulation/radio signal.

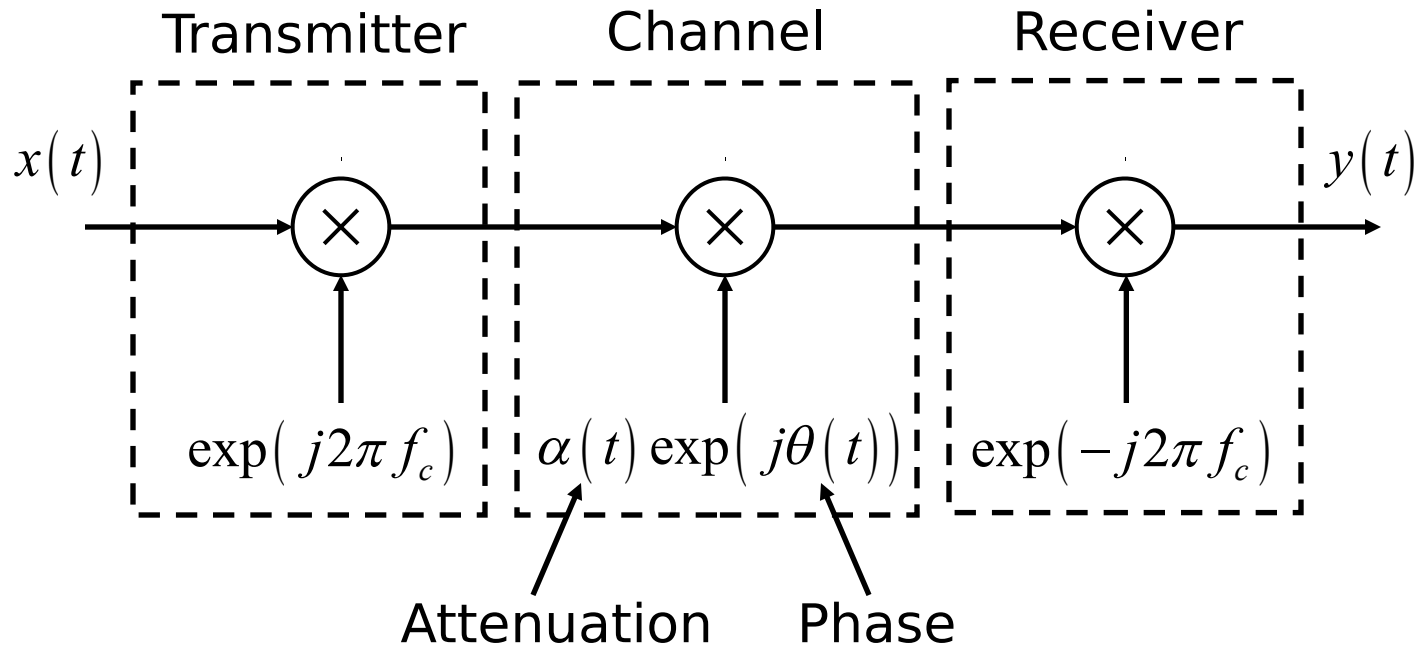
# Example: Amplitude, phase and frequency modulation



$$s(t) = A(t) \cos(2\pi f_c t + \phi(t))$$

	$A(t)$	$\phi(t)$	Comment:
4ASK			<ul style="list-style-type: none"> <li>- Amplitude carries information</li> <li>- Phase constant (arbitrary)</li> </ul>
4PSK			<ul style="list-style-type: none"> <li>- Amplitude constant (arbitrary)</li> <li>- Phase carries information</li> </ul>
4FSK			<ul style="list-style-type: none"> <li>- Amplitude constant (arbitrary)</li> <li>- Phase slope (frequency) carries information</li> </ul>

# A narrowband system described in complex notation (noise free)



In:  $x(t) = A(t) \exp(j\phi(t))$

Out:  $y(t) = A(t) \exp(j\phi(t)) \cancel{\exp(-j2\pi f_c t)} \alpha(t) \exp(j\theta(t)) \cancel{\exp(-j2\pi f_c t)}$   
 $= A(t) \alpha(t) \exp(j(\phi(t) + \theta(t)))$

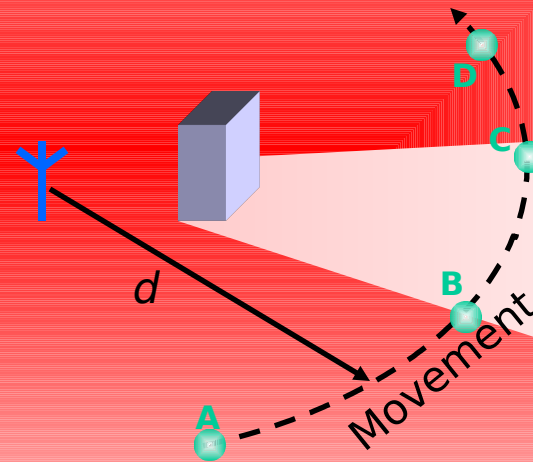
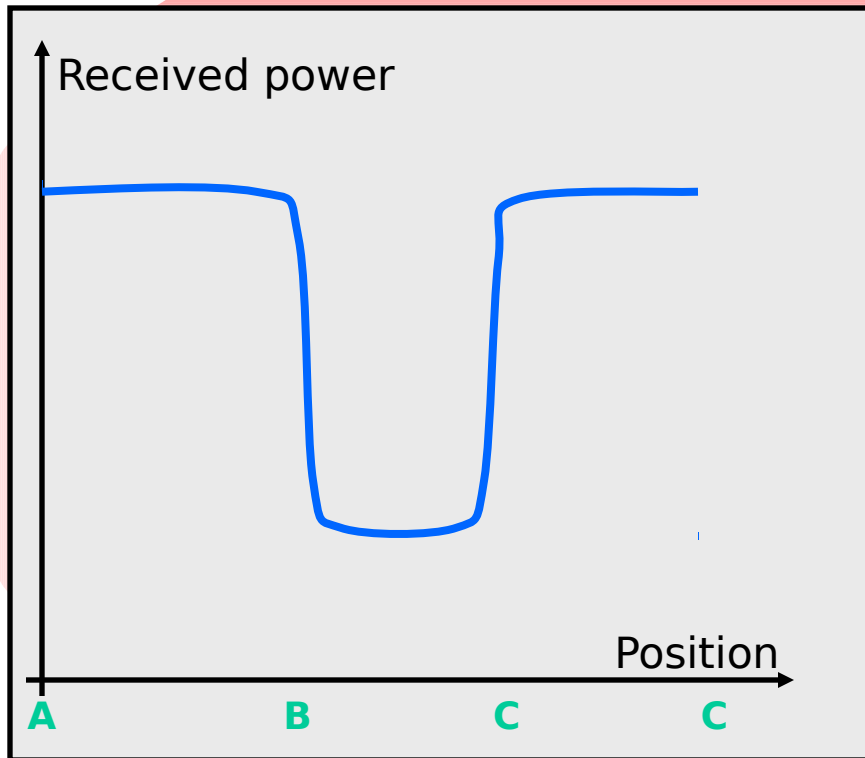
It is the behaviour of the channel attenuation and phase we are going to model.



# LARGE-SCALE FADING

# Large-scale fading

## Basic principle

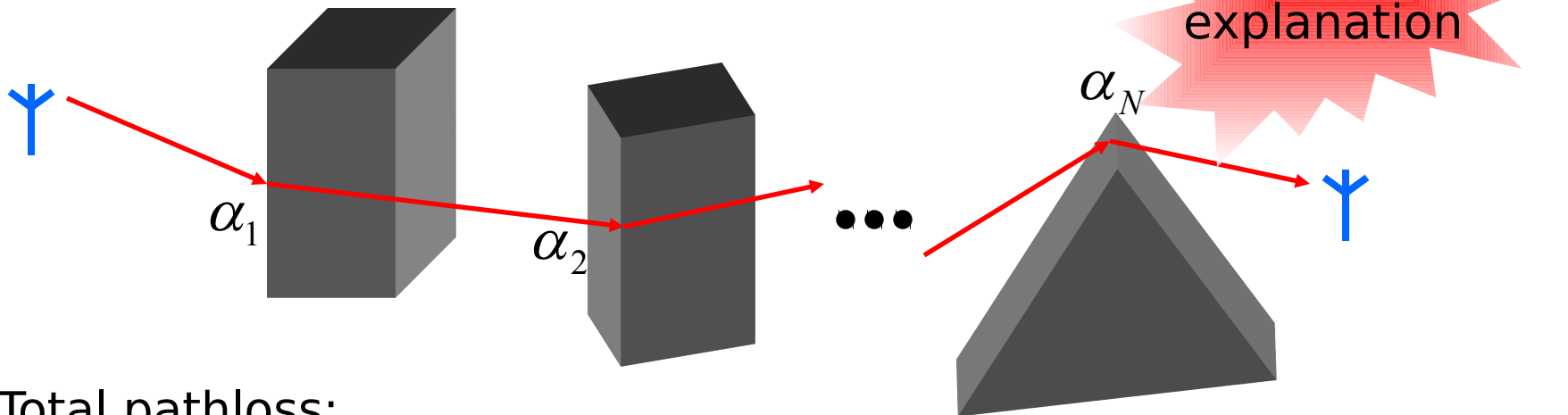




# Large-scale fading

## More than one shadowing object

Signal path in terrain with several diffraction points adding extra attenuation to the pathloss.



Total pathloss:

$$L_{\text{tot}} = L(d) \times \alpha_1 \times \alpha_2 \times \dots \times \alpha_N$$

$$L_{\text{tot}|dB} = L(d)|_{dB} + \alpha_1|_{dB} + \alpha_2|_{dB} + \dots + \alpha_N|_{dB}$$

Deterministic

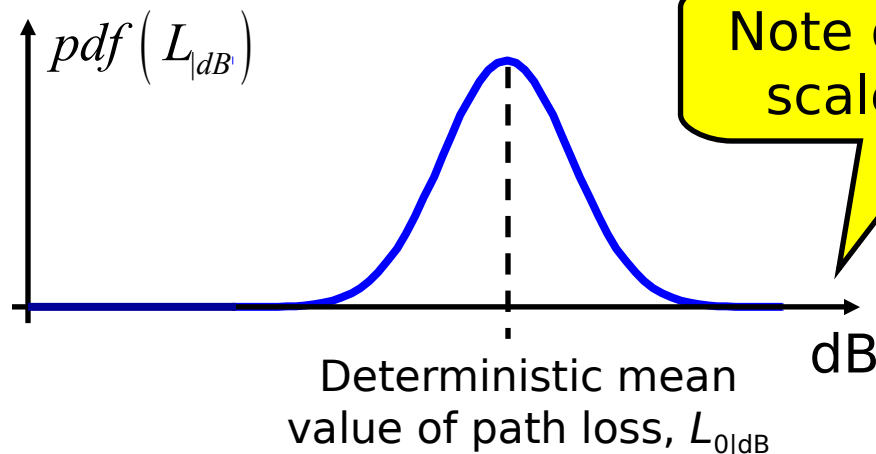
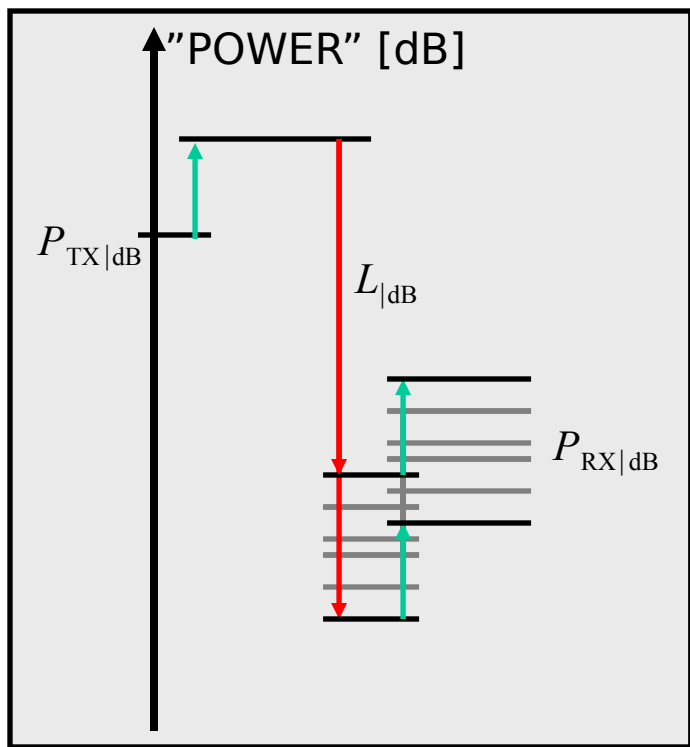
If these are considered random and independent, we should get a normal distribution in the dB domain.



# Large-scale fading

## Log-normal distribution

Measurements confirm that in many situations, the large-scale fading of the received signal strength has a normal distribution in the dB domain.



$$pdf(L_{|dB}) = \frac{1}{\sqrt{2\pi}\sigma_{F|dB}} \exp\left(-\frac{(L_{|dB} - L_{0|dB})^2}{2\sigma_{F|dB}^2}\right)$$

Standard deviation  $\sigma_{F|dB} \approx 4 - 10$  dB

# Large-scale fading

## Fading margin



We know that the path loss will vary around the deterministic value predicted.

We need to design our system with a “margin” allowing us to handle higher path losses than the deterministic prediction. This margin is called a **fading margin**.

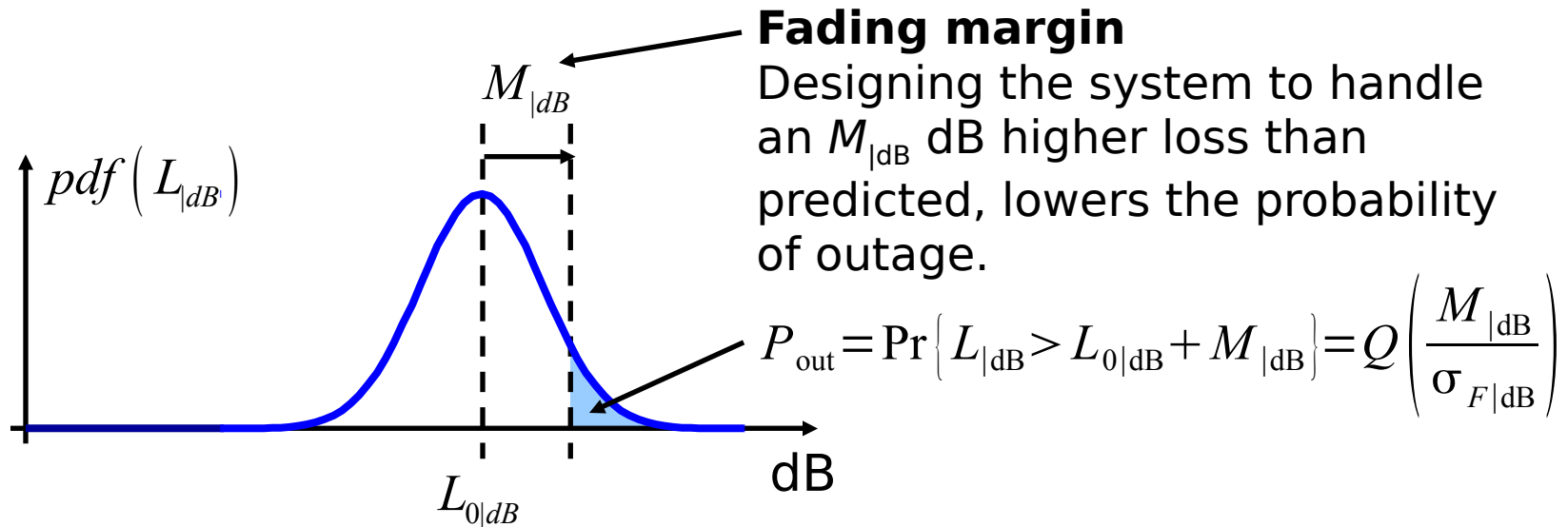
Increasing the fading margin decreases the **probability of outage**, which is the probability that our system receive a too low power to operate correctly.





# Large-scale fading

## Fading margin (cont.)



The upper tail probability of a unit variance, zero-mean, Gaussian (normal) variable:

$$Q(y) = \int_y^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right) dx = \frac{1}{2} \text{erfc}\left(\frac{y}{\sqrt{2}}\right)$$

The complementary error-function can be found in e.g. MATLAB

# The Q(.)-function

## Upper-tail probabilities



x	Q(x)
4.265	0.00001
4.107	0.00002
4.013	0.00003
3.944	0.00004
3.891	0.00005
3.846	0.00006
3.808	0.00007
3.775	0.00008
3.746	0.00009
3.719	0.00010
3.540	0.00020
3.432	0.00030
3.353	0.00040
3.291	0.00050
3.239	0.00060
3.195	0.00070
3.156	0.00080
3.121	0.00090

x	Q(x)
3.090	0.00100
2.878	0.00200
2.748	0.00300
2.652	0.00400
2.576	0.00500
2.512	0.00600
2.457	0.00700
2.409	0.00800
2.366	0.00900
2.326	0.01000
2.054	0.02000
1.881	0.03000
1.751	0.04000
1.645	0.05000
1.555	0.06000
1.476	0.07000
1.405	0.08000
1.341	0.09000

x	Q(x)
1.282	0.10000
0.842	0.20000
0.524	0.30000
0.253	0.40000
0.000	0.50000

# Large-scale fading

## A numeric example



How many dB fading margin, against  $\sigma_{F|\text{dB}} = 7$  dB log-normal fading, do we need to obtain an outage probability of 0.5%?

$$P_{\text{out}} = Q\left(\frac{M_{|\text{dB}}}{\sigma_{F|\text{dB}}}\right) = 0.5\% = 0.005$$

Consulting the  $Q(\cdot)$ -function table (or using a numeric software), we get

$$\frac{M_{|\text{dB}}}{\sigma_{F|\text{dB}}} = 2.576 \Rightarrow \frac{M_{|\text{dB}}}{7} = 2.576 \Rightarrow M_{|\text{dB}} = 18$$



# SMALL-SCALE FADING

# Small-scale fading

## Illustration shown during Lecture 1

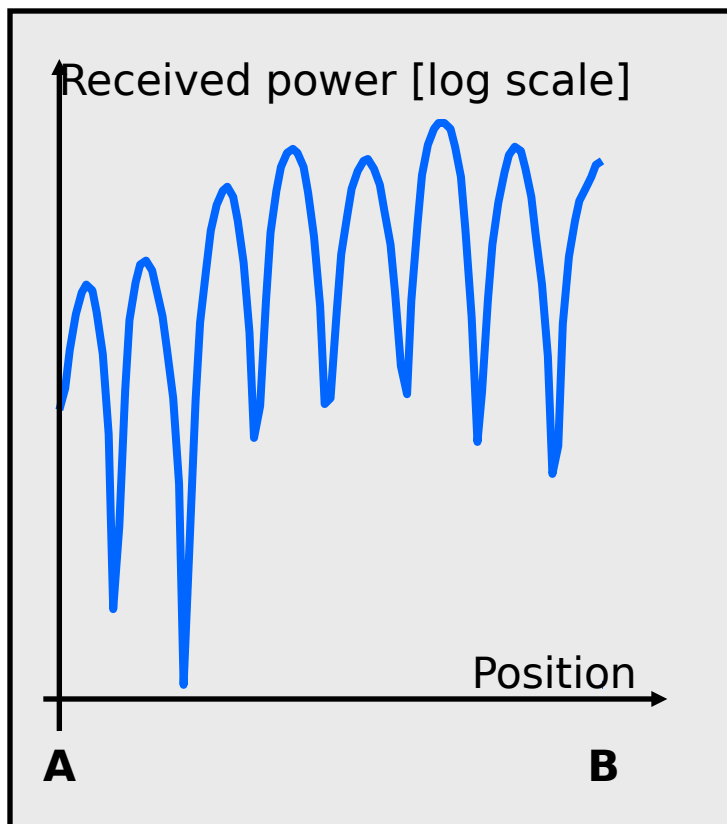
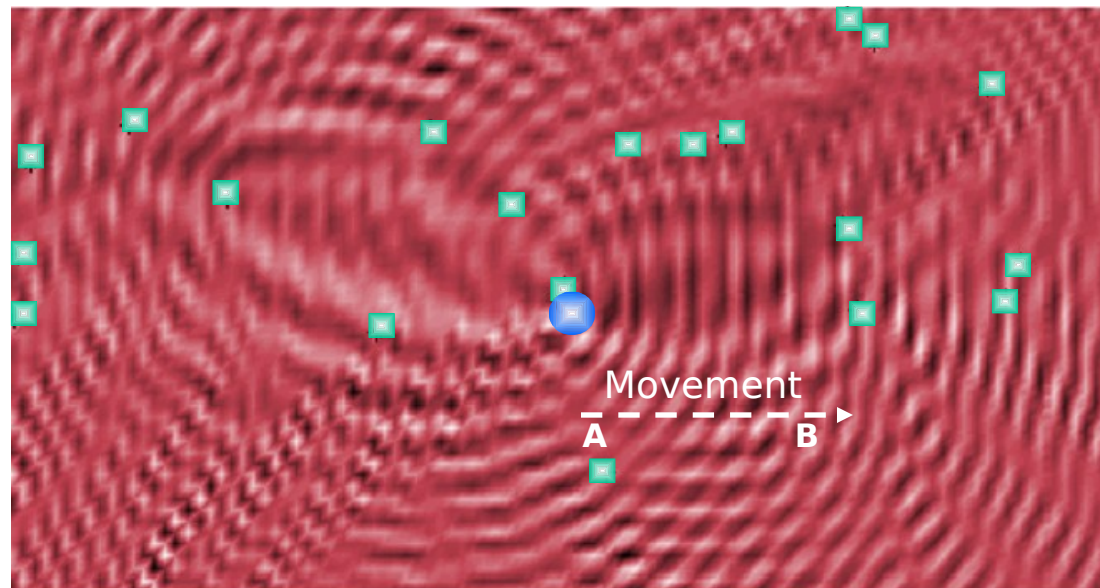


Illustration of interference pattern from above



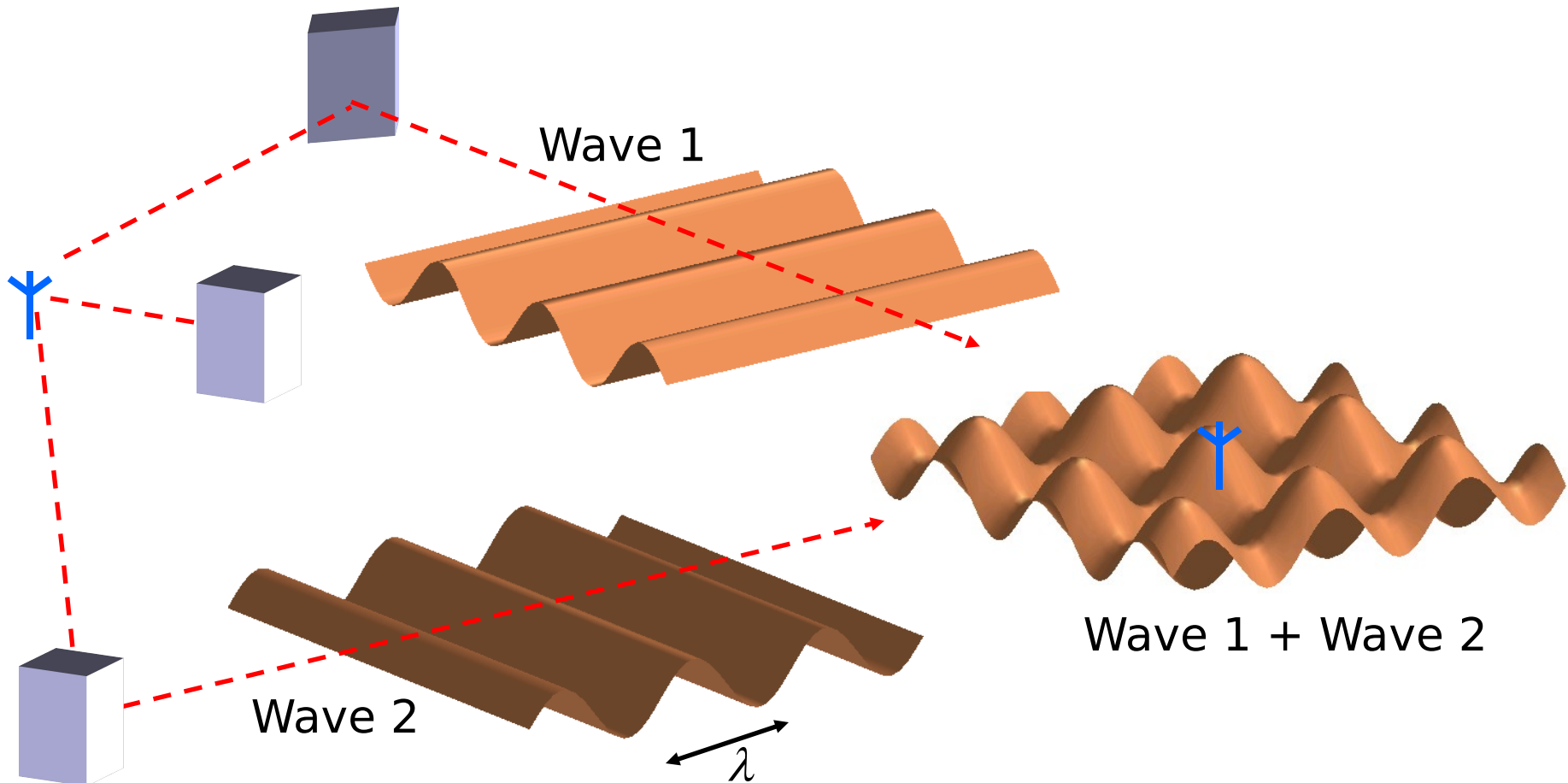
● Transmitter

■ Reflector

Many reflectors ... let's look at a simpler case!

# Small-scale fading

## Two waves



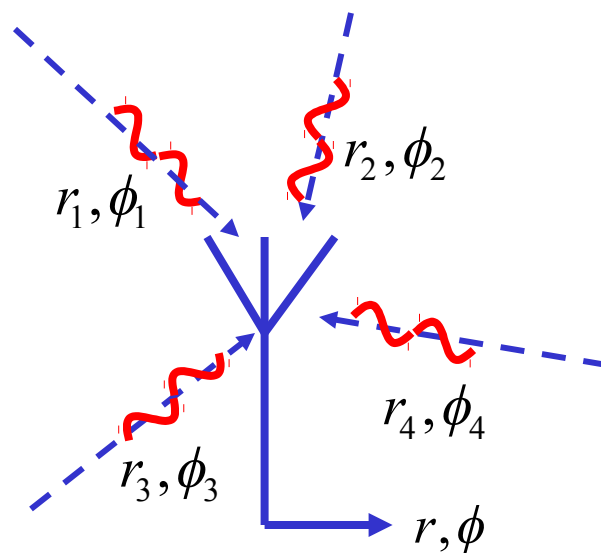
At least in this case, we can see that the interference pattern changes on the wavelength scale.

# Small-scale fading

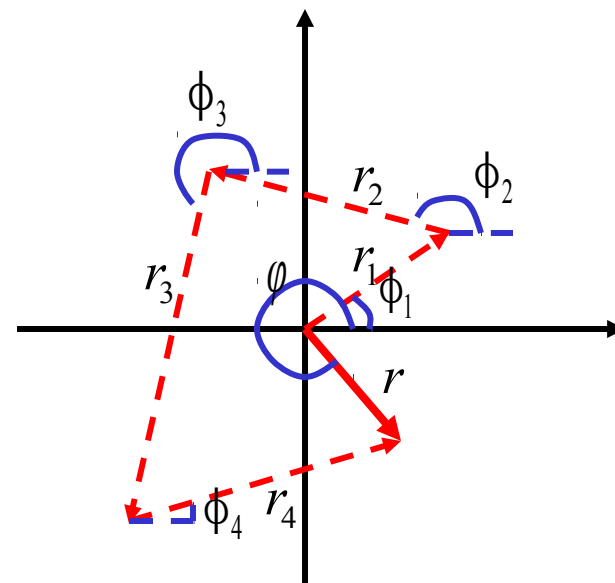
## Many incoming waves



Many incoming waves with independent amplitudes and phases



Add them up as phasors



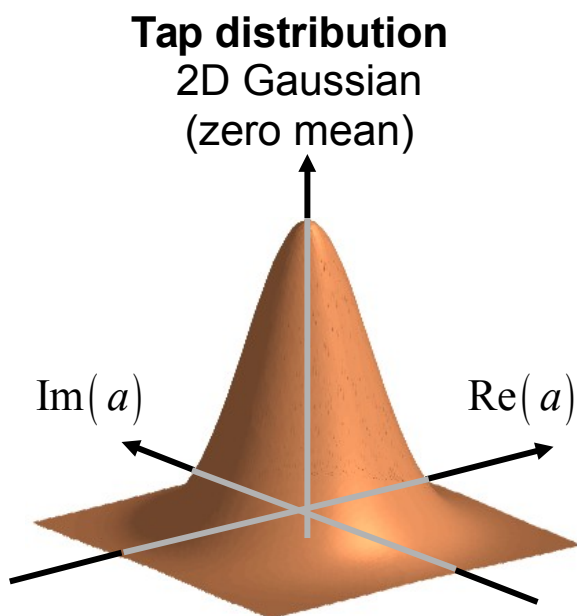
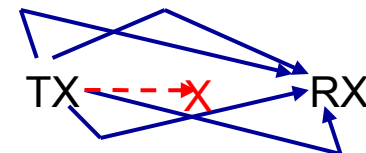
$$r \exp(j\phi) = r_1 \exp(j\phi_1) + r_2 \exp(j\phi_2) + r_3 \exp(j\phi_3) + r_4 \exp(j\phi_4)$$

# Small-scale fading

## Rayleigh fading

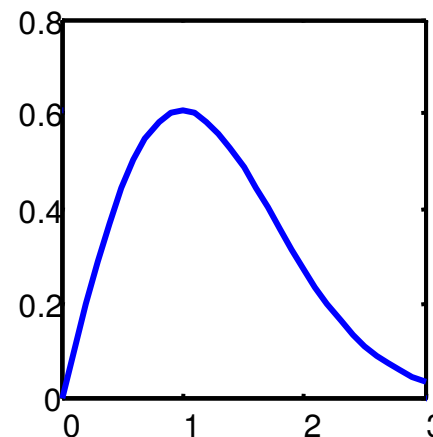


No dominant component  
(no line-of-sight)



$$r = |a|$$

Amplitude distribution  
Rayleigh



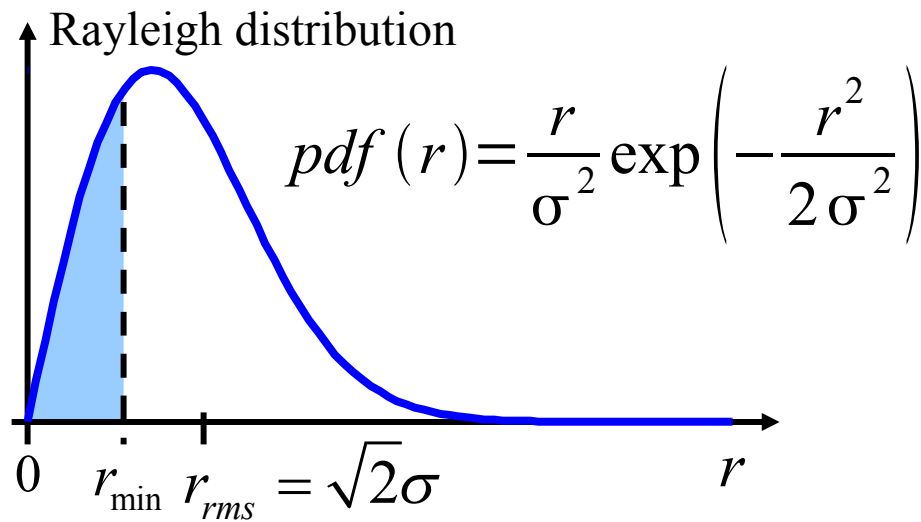
$$pdf(r) = \frac{r}{\sigma^2} \exp\left(-\frac{r^2}{2\sigma^2}\right)$$

No line-of-sight  
component



# Small-scale fading

## Rayleigh fading - Fading margin



### Fading margin

$$M = \frac{r_{\text{rms}}^2}{r_{\text{min}}^2}$$

$$M_{|\text{db}} = 10 \log_{10} \left( \frac{r_{\text{rms}}^2}{r_{\text{min}}^2} \right)$$

Probability that the amplitude  $r$  is below some threshold  $r_{\text{min}}$ :

$$\Pr(r < r_{\text{min}}) = \int_0^{r_{\text{min}}} pdf(r) dr = 1 - \exp\left(-\frac{r_{\text{min}}^2}{r_{\text{rms}}^2}\right)$$

# Small-scale fading

## A numeric example



How many dB fading margin, against Rayleigh fading, do we need to obtain an outage probability of 1%?

$$\Pr(r < r_{\min}) = 1 - \exp\left(-\frac{r_{\min}^2}{r_{\text{rms}}^2}\right) = 1\% = 0.01$$

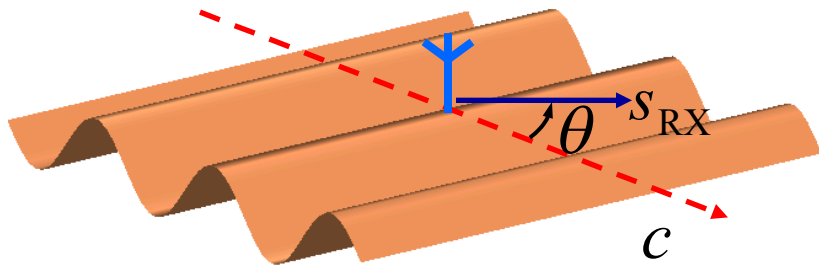
Some manipulation gives

$$1 - 0.01 = \exp\left(-\frac{r_{\min}^2}{r_{\text{rms}}^2}\right) \Rightarrow \ln(0.99) = -\frac{r_{\min}^2}{r_{\text{rms}}^2}$$

$$\Rightarrow \frac{r_{\min}^2}{r_{\text{rms}}^2} = -\ln(0.99) = 0.01 \Rightarrow M = \frac{r_{\text{rms}}^2}{r_{\min}^2} = 1/0.01 = 100$$

$$M_{\text{dB}} = 20$$

# Small-scale fading Doppler shifts



Receiving antenna moves with speed  $s_{RX}$  at an angle  $\theta$  relative to the propagation direction of the incoming wave, which has frequency  $f_0$ .

[ $c$  = speed of light =  $3 \times 10^8$  m/s]

Frequency of received signal:

$$f = f_0 + \nu$$

where the Doppler shift is

$$\nu = -f_0 \frac{s_{RX}}{c} \cos(\theta)$$

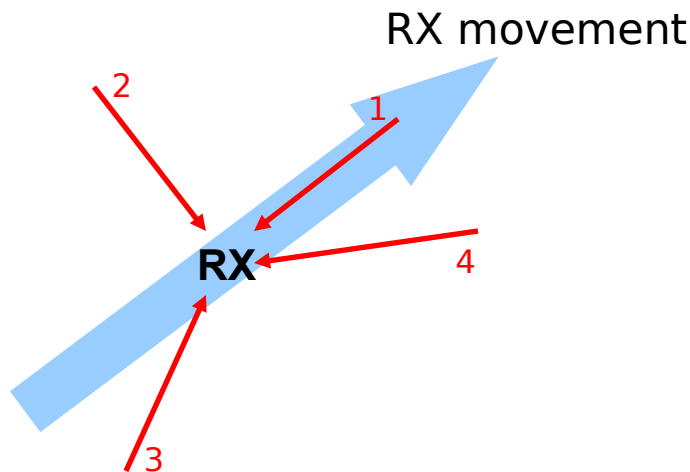
The maximal Doppler shift is

$$\nu_{\max} = f_0 \frac{s_{RX}}{c}$$

# Small-scale fading Doppler spectrum

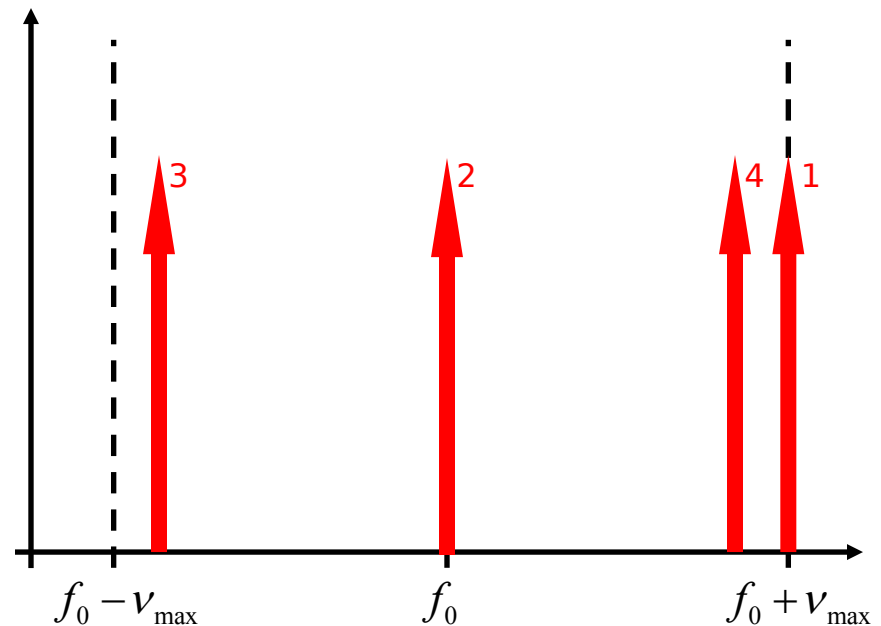


Incoming waves from several directions  
(relative to movement or RX)

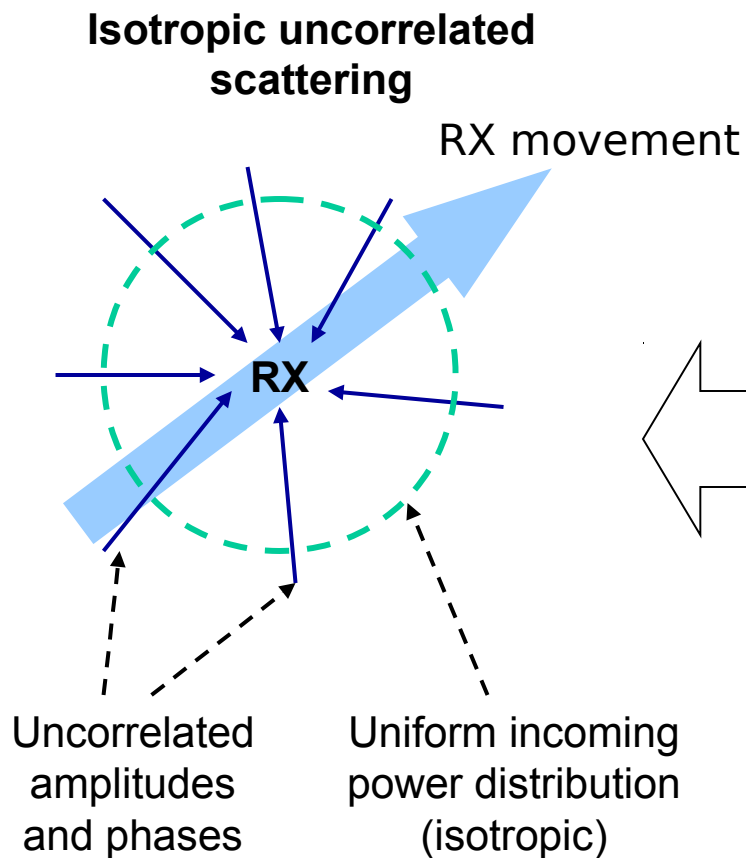


All waves of equal strength in this example, for simplicity.

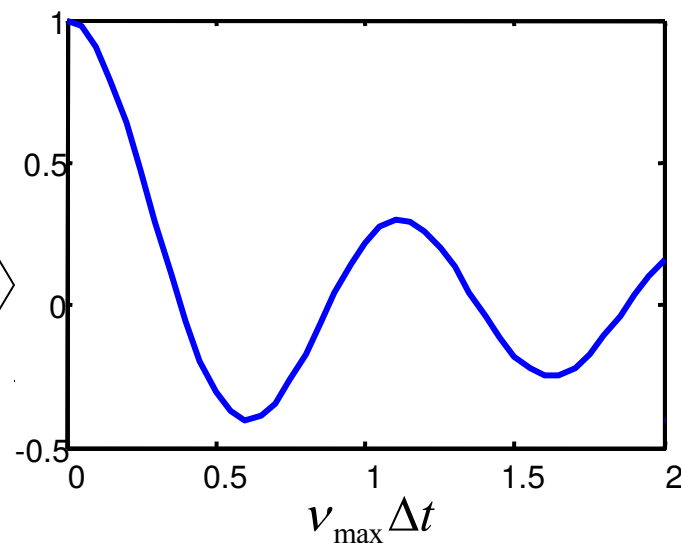
**Spectrum of received signal  
when a  $f_0$  Hz signal is transmitted.**



# Small-scale fading Doppler spectrum



**Time correlation**



$$\rho(\Delta t) = E\{a(t)a^*(t+\Delta t)\} \sim J_0(2\pi v_{\max} \Delta t)$$



# Small-scale fading

## The Doppler spectrum

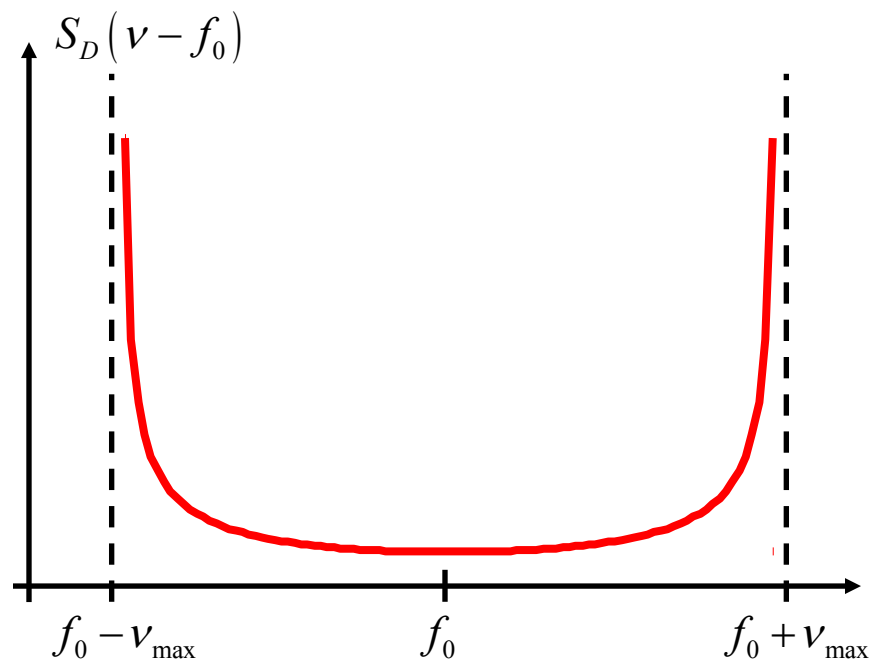
For the uncorrelated scattering with uniform angular distribution of incoming power (isotropic scattering), we obtain the Doppler spectrum by Fourier transformation of the time correlation of the signal:

$$S_D(\nu) = \int \rho(\Delta t) e^{-j2\pi\nu\Delta t} d\Delta t$$

$$\sim \frac{1}{\pi \sqrt{\nu_{\max}^2 - \nu^2}}$$

for  $-\nu_{\max} < \nu < \nu_{\max}$

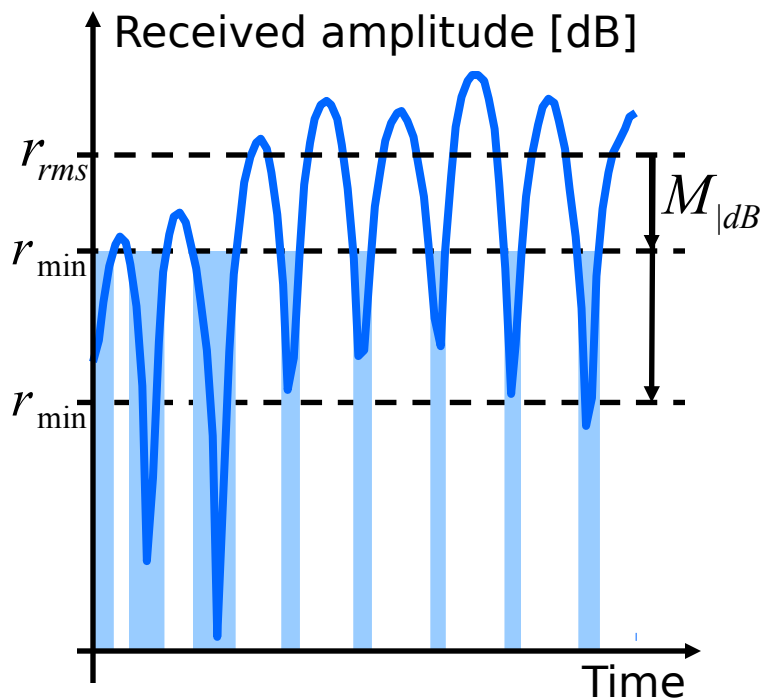
Doppler spectrum at center frequency  $f_0$ .



This is the "classical" Doppler spectrum, a.k.a. the **Jakes' doppler spectrum**.

# Small-scale fading

## Fading dips



The larger the fading margin, the rarer the fading dips, and the shorter they are.

The length and the frequency of fading dips can be important for the functionality of a radio system.

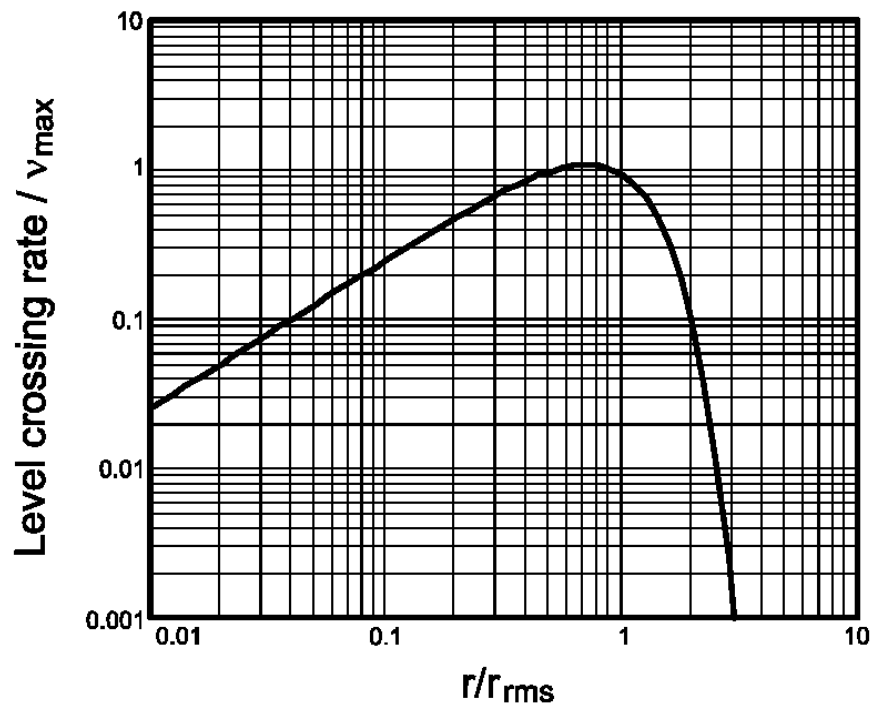
Can we quantify these?

# Small-scale fading

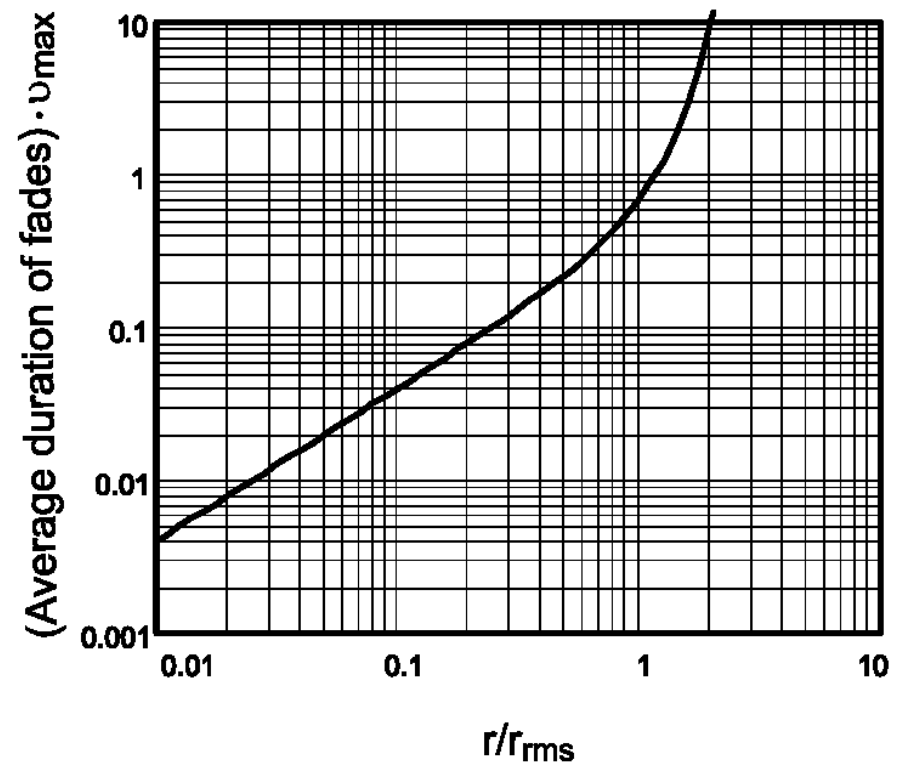
## Statistics of fading dips



Frequency of the fading dips  
(normalized dips/second)



Length of fading dips  
(normalized dip-length)



These curves are for Rayleigh fading and isotropic uncorrelated scattering (**Jakes' doppler spectrum**).

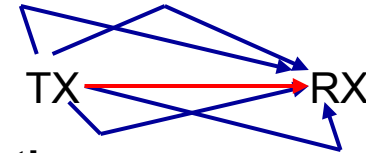


# Small-scale fading

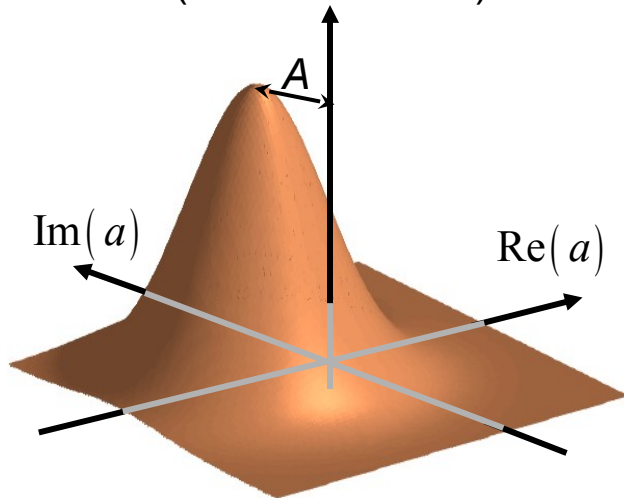
## Rice fading



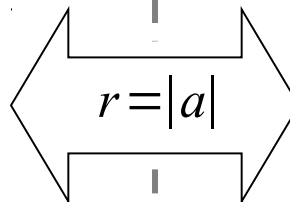
**A dominant component  
(line of sight)**



**Tap distribution**  
2D Gaussian  
(non-zero mean)

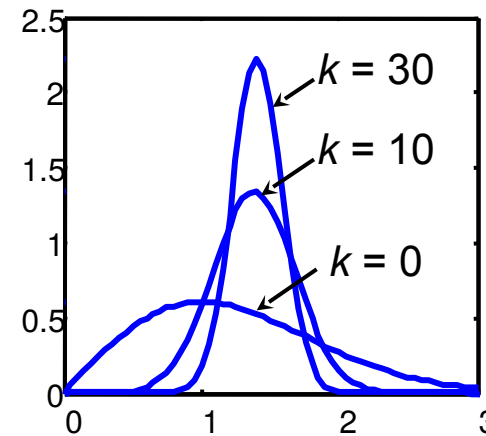


Line-of-sight (LOS)  
component with  
amplitude  $A$ .



**Amplitude distribution**

Rice



$$pdf(r) = \frac{r}{\sigma^2} \exp\left(-\frac{r^2 + A^2}{2\sigma^2}\right) I_0\left(\frac{rA}{\sigma^2}\right)$$

$$k = \frac{\text{Power in LOS component}}{\text{Power in random components}} = \frac{A^2}{2\sigma^2}$$



# COMBINING LARGE- AND SMALL-SCALE FADING

# Large- and small-scale fading

## Combining the two



We will start using  
Alternative 1

We have seen examples of how we can compute the required fading margins, due to large- and small-scale fading, given certain criteria (e.g. outage probability).

If we have both types of fading, how do we combine them into a "total" fading margin?

There are basically two options:

- 1) Calculate the fading margins separately and add them up.
- 2) Derive the pdf (or cdf) of the total fading and calculate a single fading margin for both.

Alternative 1 is the simple solution, but it will overdimension the system a bit.

Alternative 2 is a much more complex operation.

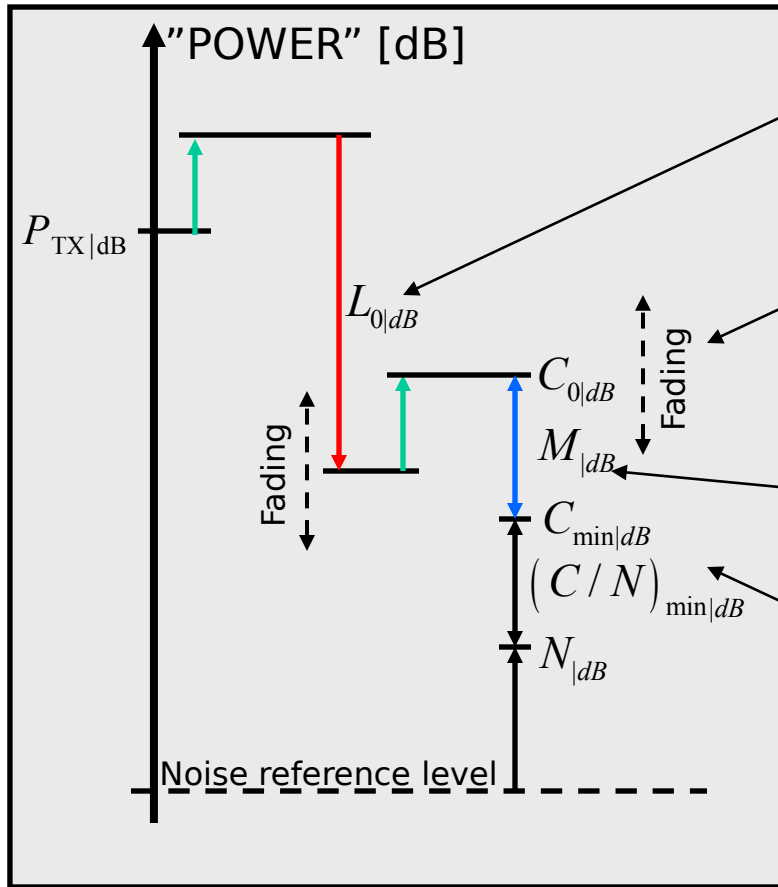


# NOISE- AND INTERFERENCE- LIMITED LINKS



# Noise-limited system

## Fading margin and the link budget



We use some propagation model to calculate a deterministic propagation loss.

Variations in the environment and movements will cause variations in the the propagation loss, which will influence the instantaneous received power.

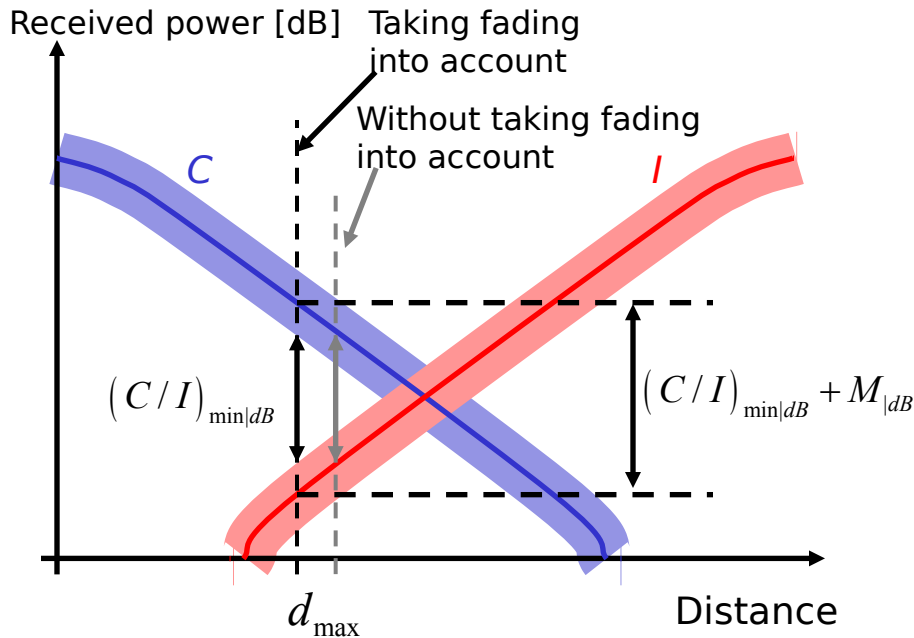
To protect the receiver from too low received power, we add a fading margin.

Requirement for the receiver to operate properly.



# Interference-limited system

## Interference fading margin



In interference limited systems, we are preliminary interested in how far from the transmitter we can be, without receiving too much interference.

Depending on the system design and requirements on quality, our receiver can tolerate a certain  $(C/I)_{\min}$ .

Assuming fading on the wanted and interfering signal we can calculate a fading margin  $M_{|dB}$  required to fulfill some criterion on e.g. outage.

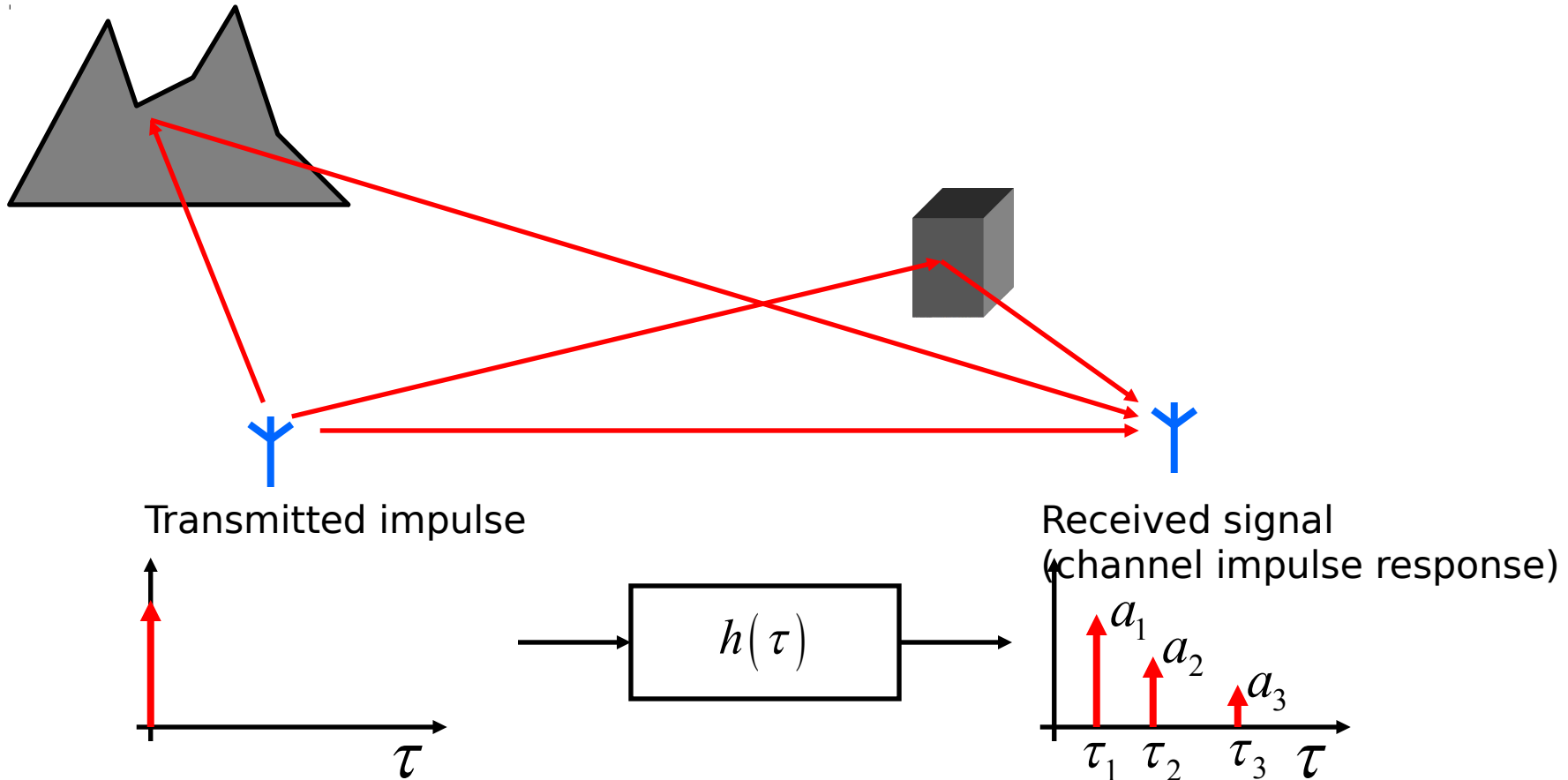
For independent log-normal fading, we can add the variances of the two fading characteristics and get a "total" lognormal fading with standard deviation:  $\sigma_{tot|dB} = \sqrt{\sigma_{C|dB}^2 + \sigma_{I|dB}^2}$



# DELAY (TIME) DISPERSION

# Delay (time) dispersion

## A simple case

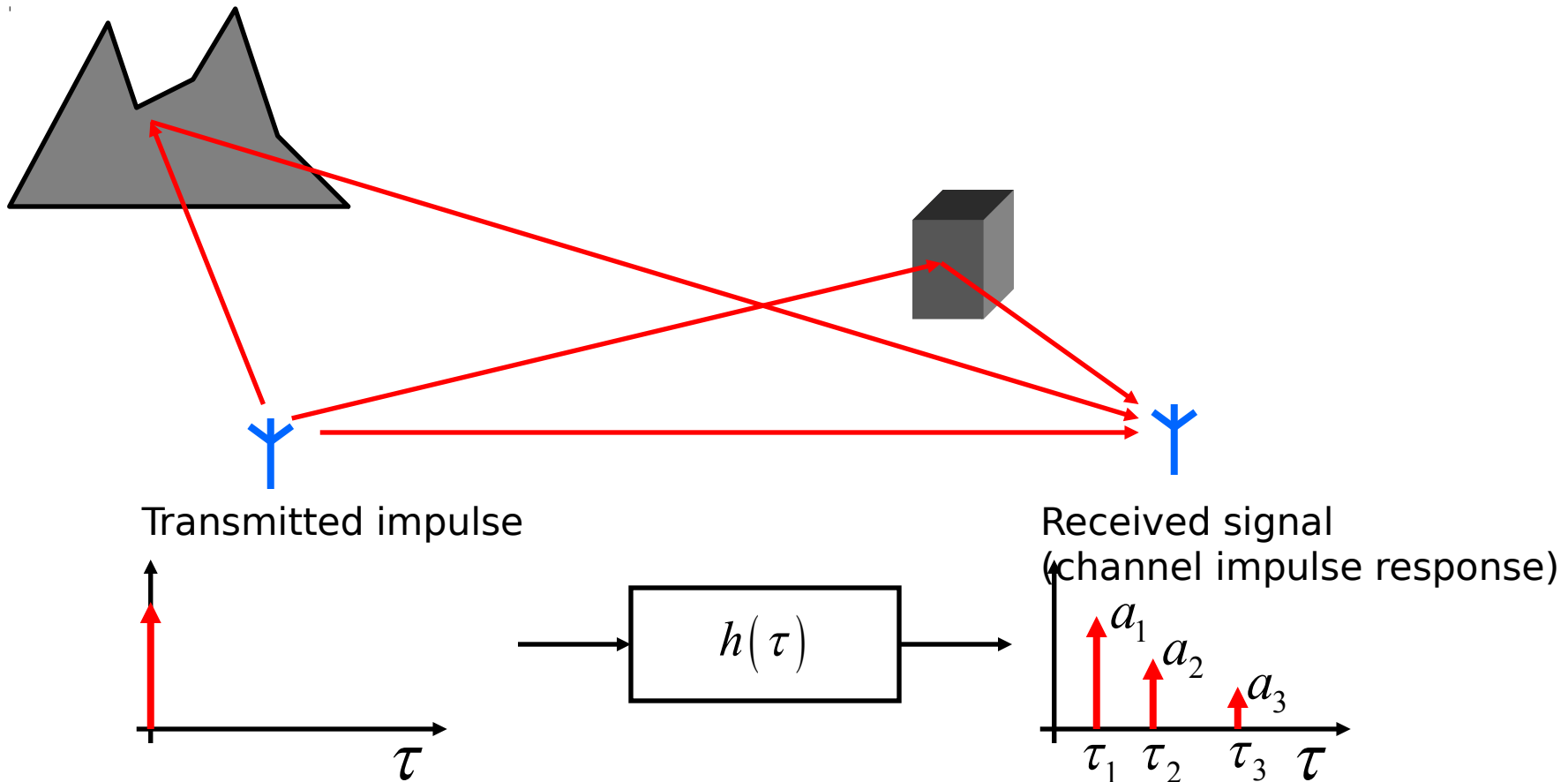


$$h(\tau) = a_1\delta(\tau - \tau_1) + a_2\delta(\tau - \tau_2) + a_3\delta(\tau - \tau_3)$$



# Delay (time) dispersion

## A simple case

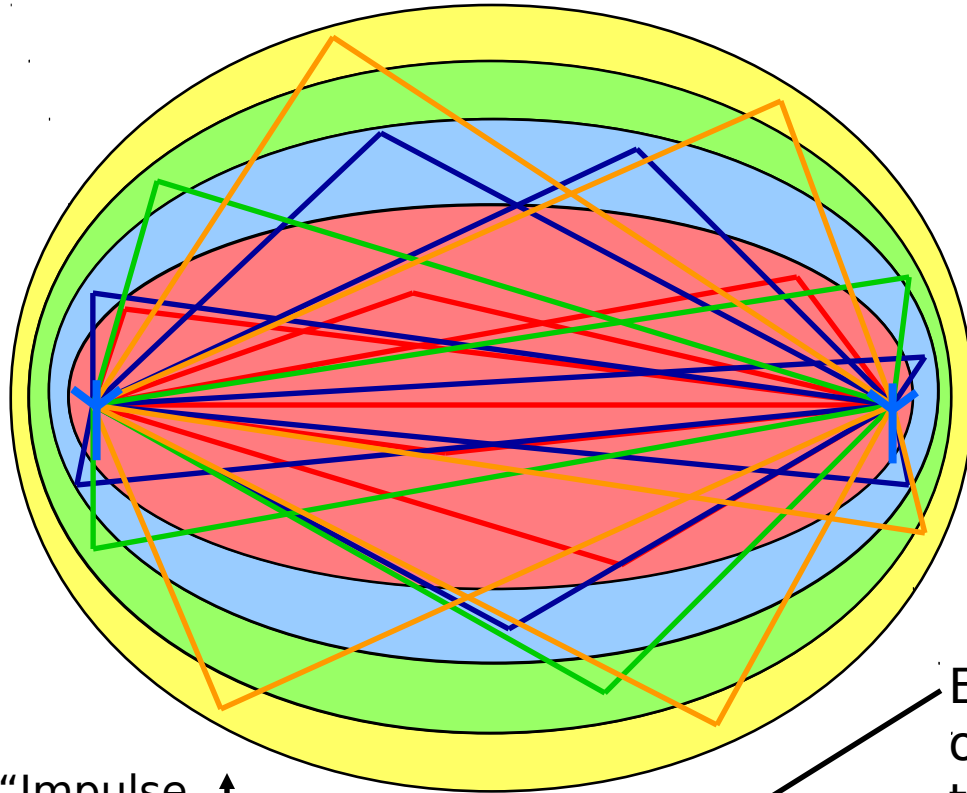


$$h(\tau) = a_1\delta(\tau - \tau_1) + a_2\delta(\tau - \tau_2) + a_3\delta(\tau - \tau_3)$$

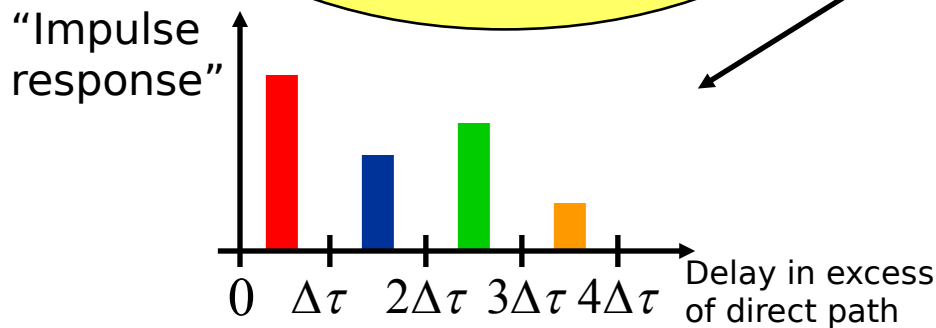


# Delay (time) dispersion

## One reflection/path, many paths



Since each bin consists of contributions from several waves, each bin will fade if we introduce movement.



Each bin consists of incoming waves that are too close in time to resolve.

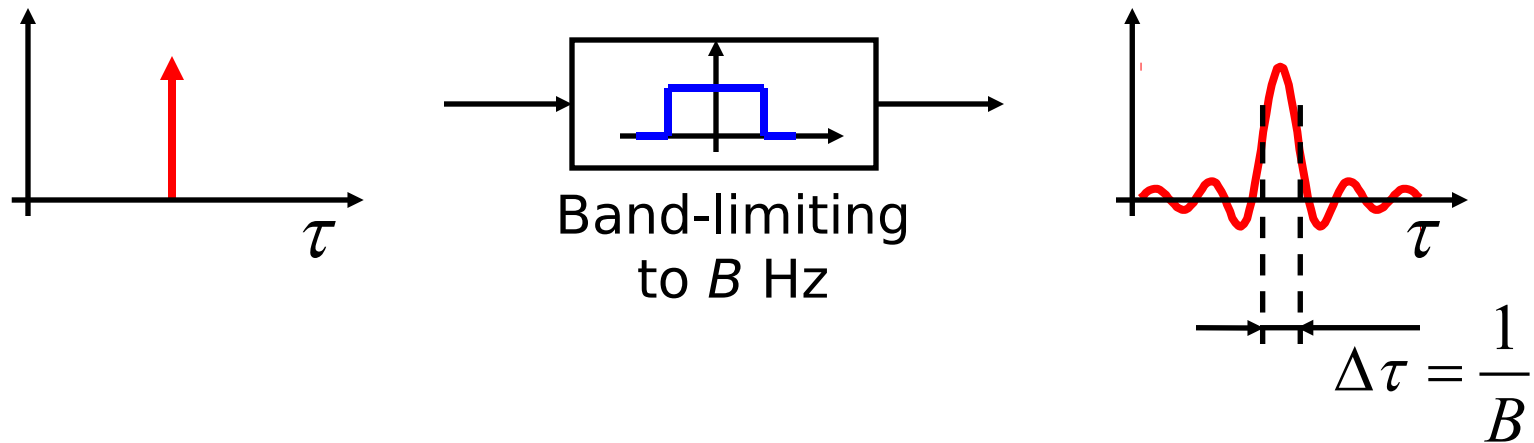
What do we mean by “too close in time”?



# Delay (time) dispersion

## Bandwidth and time-resolution

Radio systems are band-limited, which makes our infinitely short impulses become waveforms with a certain width in time.



The time-width of the pulses is inversely proportional to the bandwidth.

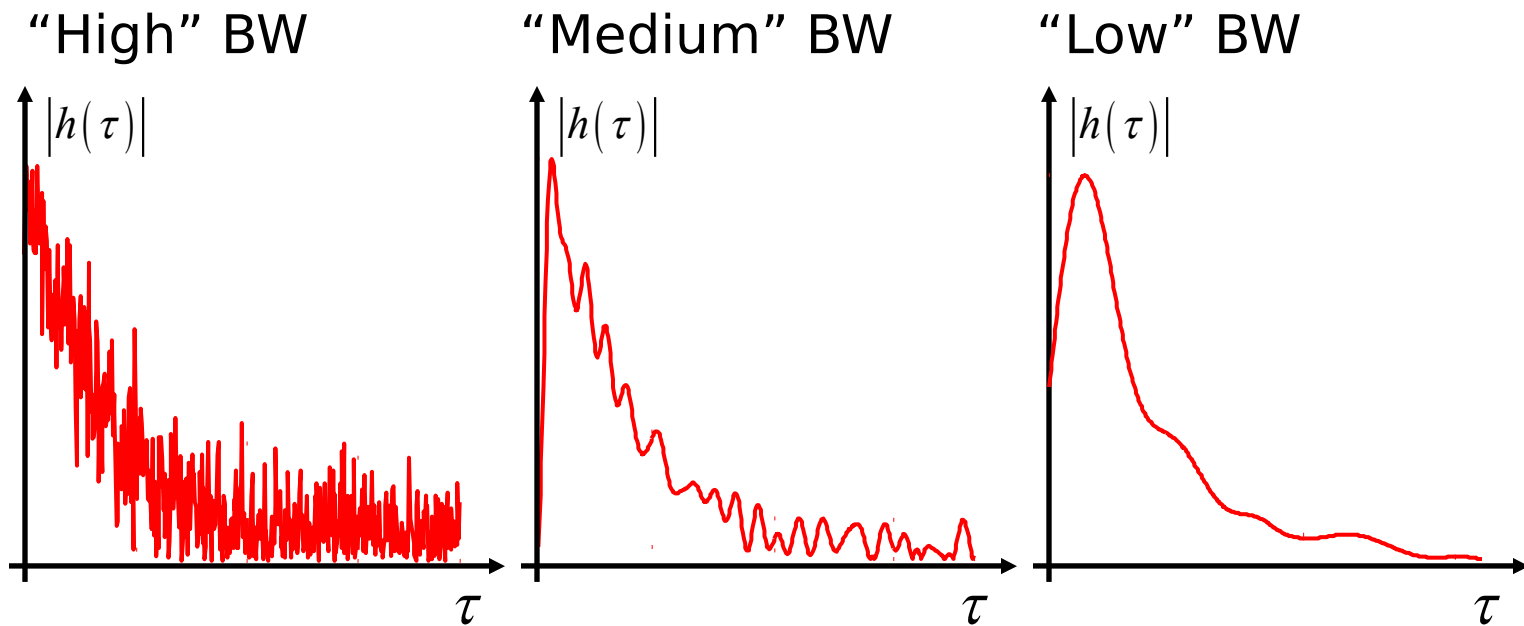


# NARROW- VERSUS WIDE-BAND

# Narrow- versus wide-band Channel impulse response

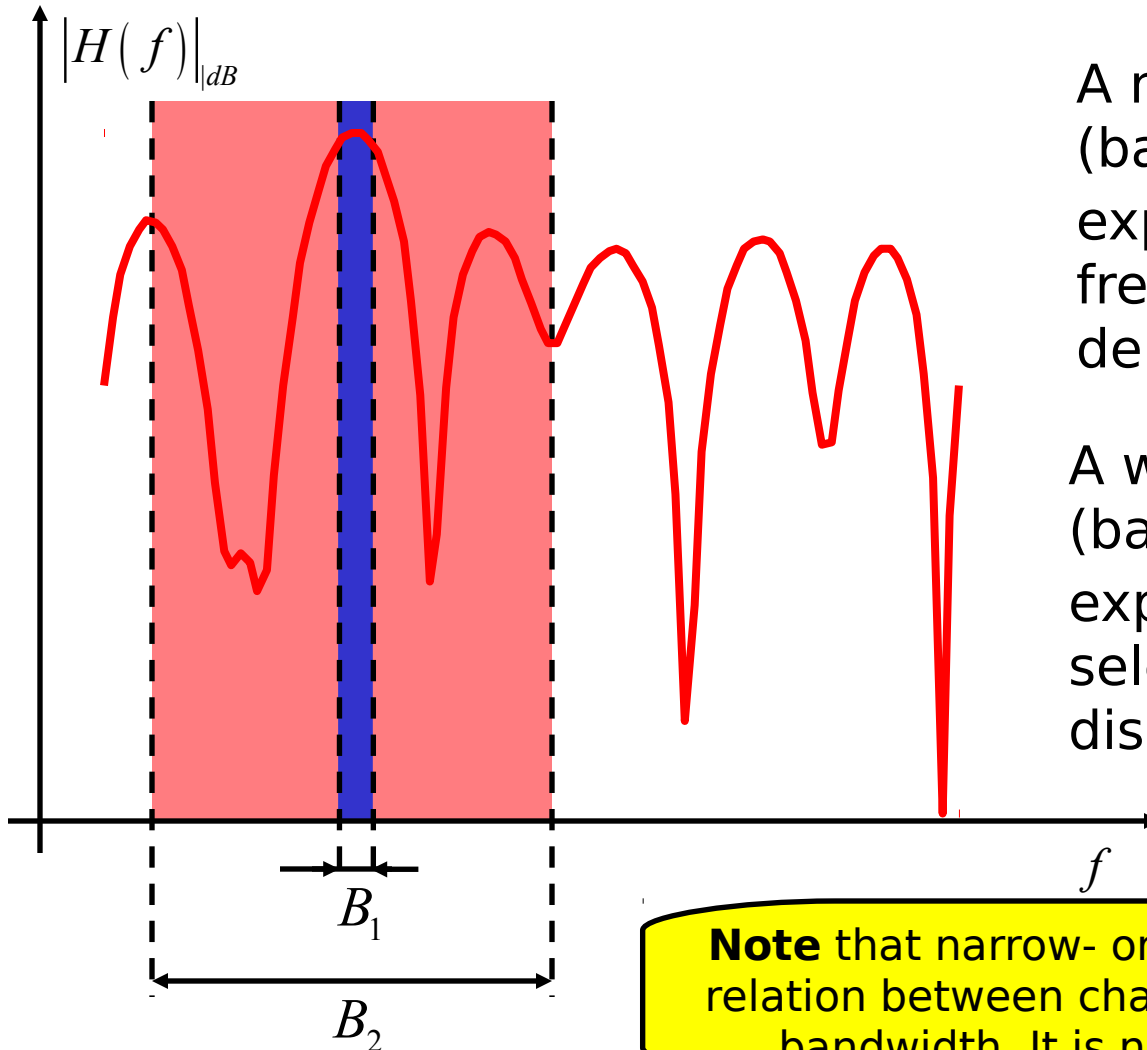


The same radio propagation environment is experienced differently, depending on the system bandwidth.





# Narrow- versus wide-band Channel frequency response



A narrow-band system (bandwidth  $B_1$ ) will not experience any significant frequency selectivity or delay dispersion.

A wide-band system (bandwidth  $B_2$ ) will however experience both frequency selectivity and delay dispersion.

**Note** that narrow- or wide-band depends on the relation between channel properties and system bandwidth. It is not an absolute measure.



# Narrow- versus wide-band

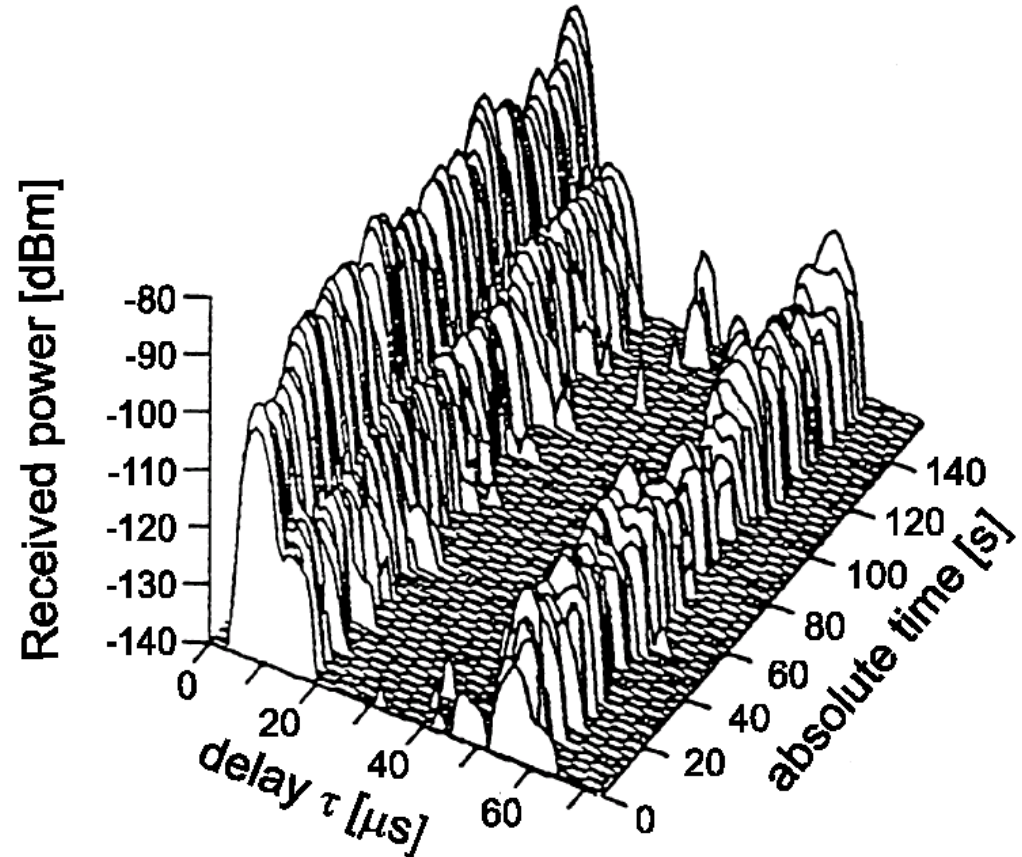
## Let's not forget the time-dependence!

We need to take **absolute time**  $t$  into consideration, as the channel will change when things move.

The channel impulse response becomes:

$$h(t, \tau)$$

Measurement in hilly terrain  
at 900 MHz.



[Liebenow & Kuhlmann 1993]



# Narrow- versus wide-band Doppler spectrum and delay

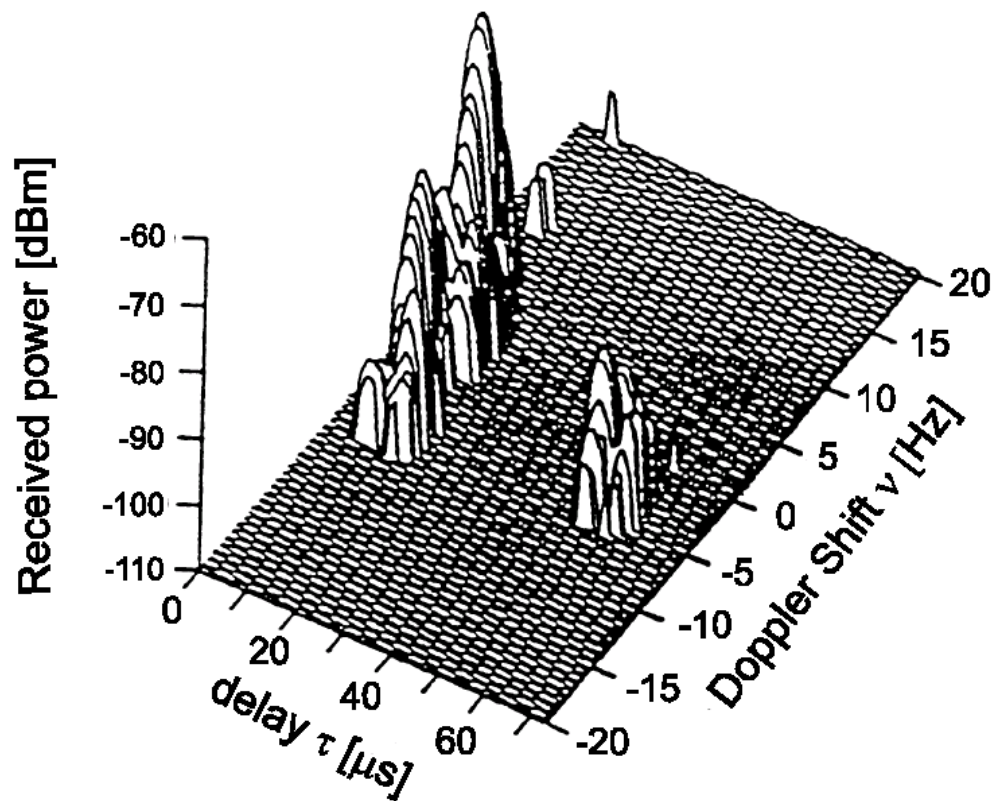
Since the channel at each delay  $\tau$  is the result of different propagation paths, we can have different Doppler spectra for each delay.

This effect is shown by the scattering function:

$$P_s(\nu, \tau)$$

(received power as function of doppler shift and delay)

## Measurement in hilly terrain at 900 MHz.







# Summary

## NARROW-BAND CHANNELS

- Complex notation with **amplitude**, **phase** and **complex envelope** (phasor).
- Large-scale fading with **log-normal distribution** and calculation of **fading margin**.
- Small-scale fading with **Rayleigh** and **Rice** distribution, calculation of **fading margin**.
- Received signal **maximal Doppler shift**, **Doppler spectrum** and **time characteristics**.
- Fading margin in the **link-budget** of noise limited systems.
- Fading margin and **maximal distance** for interference limited systems.

## WIDE-BAND CHANNELS

- Instead of one (time varying) channel coefficient, we have an entire **(time varying) impulse response**
- Channel **delay (time) dispersion** and **frequency selectivity**
- Doppler spectrum as function of delay, i.e. the **scattering function**

There is **MUCH MORE** to learn about this – which many of you have done in the Channel Modeling course (ETIN10)!