Lecture no: 3

Narrow- and wideband channels

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NARROW-BAND CHANNELS

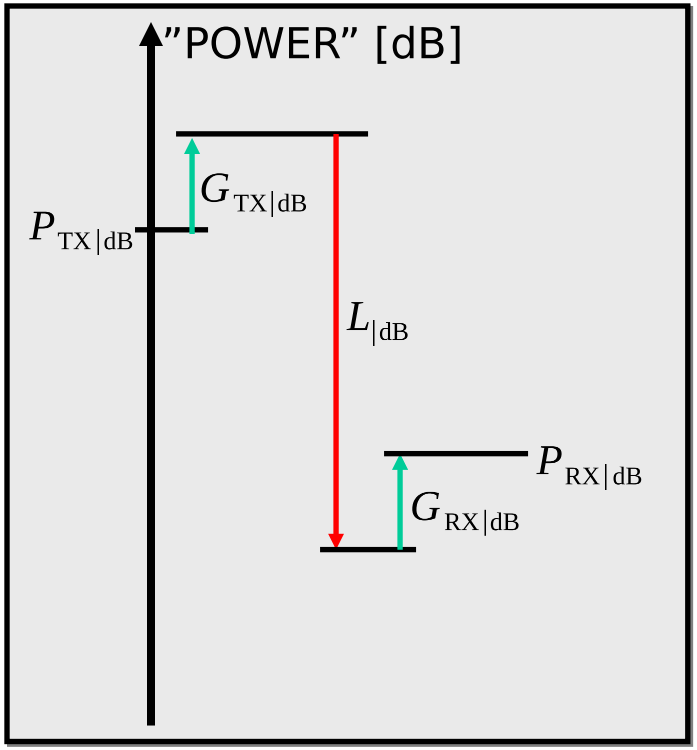
- Radio signals and complex notation
- Large-scale fading
- Small-scale fading
- Combining large- and small-scale fading
- Noise- and interference-limited links

WIDE-BAND CHANNELS

- What makes a channel wide-band?
- Delay (time) dispersion
- Narrow- versus wide-band channels

SHORT REVIEW

What do we know so far about propagation losses?



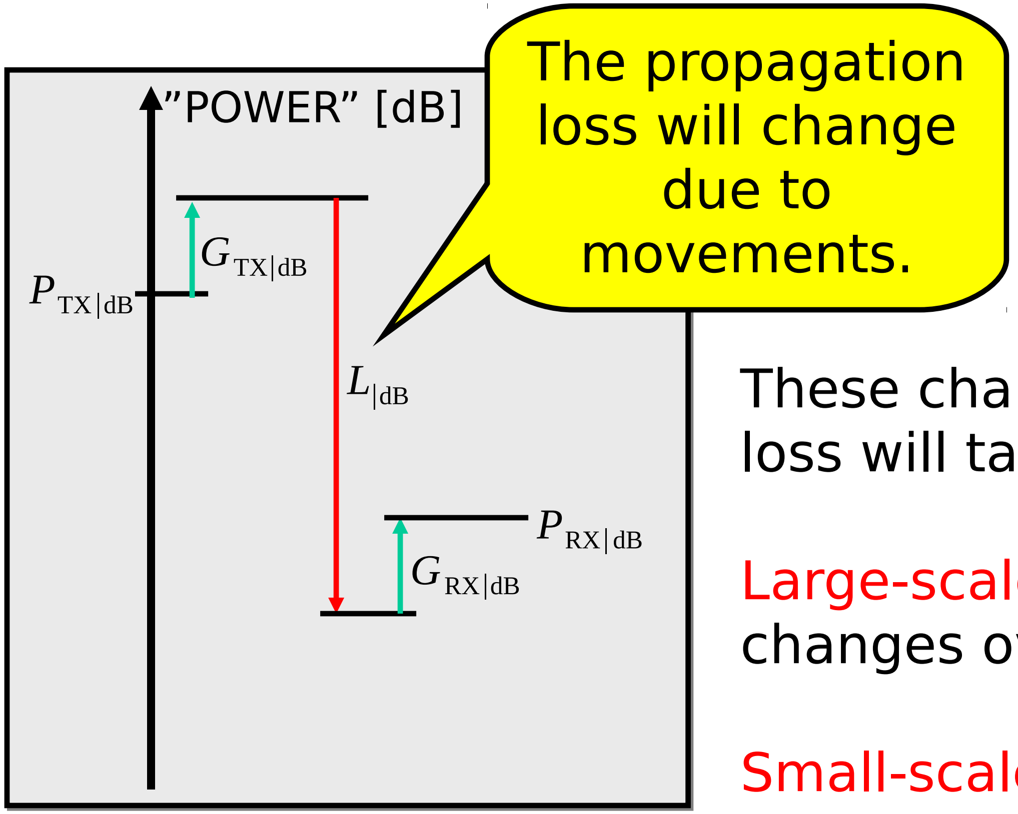
Two theoretical expressions for the deterministic propagation loss as functions of distance:

$$L_{|dB}(d) = \begin{cases} 20\log_{10}\left(\frac{4\pi d}{\lambda}\right), & \text{free space} \\ 20\log_{10}\left(\frac{d^2}{h_{TX}h_{RX}}\right), & \text{ground plane} \end{cases}$$

There are other models, which we will discuss later.

We have discussed shadowing/ diffraction and reflections, but not really made any detailed calculations.

Statistical descriptions of the mobile radio channel



These changes of the propagation loss will take place in two scales:

Large-scale: shadowing, “slow” changes over *many wavelengths*.

Small-scale: interference, “fast” changes on the scale of *a wavelength*.

Now we are going to approach these variations from a statistical point of view.

RADIO SIGNALS AND COMPLEX NOTATION

Simple model of a radio signal

- A transmitted radio signal can be written

$$s(t) = A\cos(2\pi ft + \phi)$$

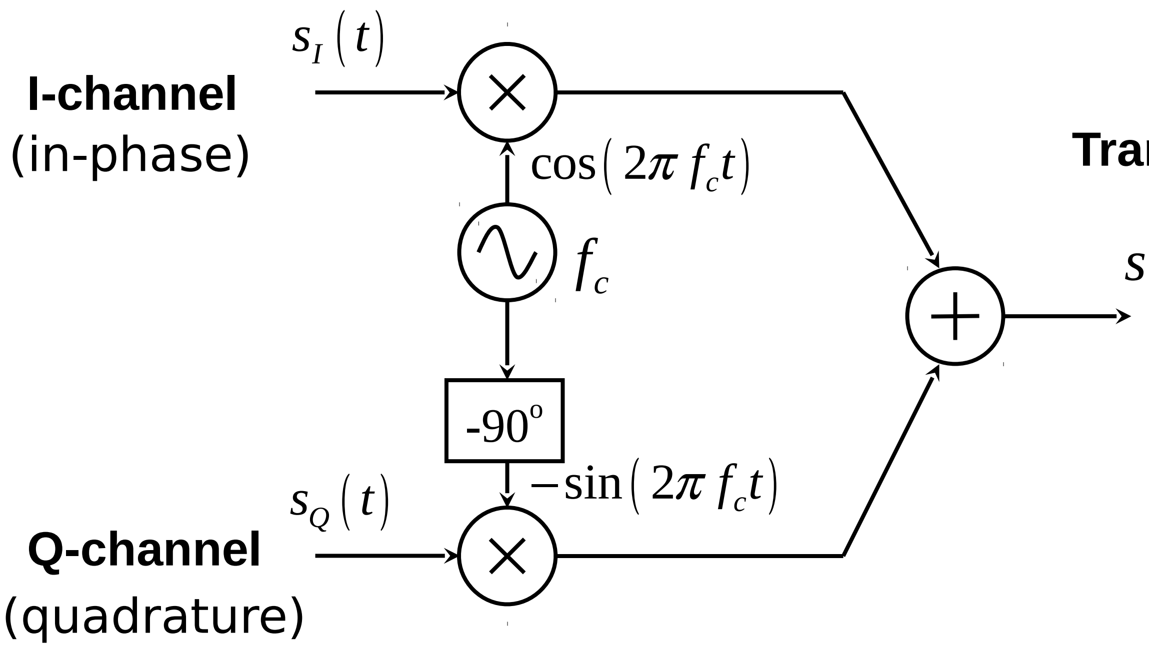
Amplitude Frequency Phase

- By letting the transmitted information change the amplitude, the frequency, or the phase, we get the tree basic types of digital modulation techniques

  - **ASK** (Amplitude Shift Keying)
  - **FSK** (Frequency Shift Keying)
  - **PSK** (Phase Shift Keying)

Constant amplitude (FSK, PSK)

The IQ modulator



Transmited radio signal

$$s(t) = s_I(t)\cos(2\pi f_c t) - s_Q(t)\sin(2\pi f_c t)$$

Take a step into the complex domain:

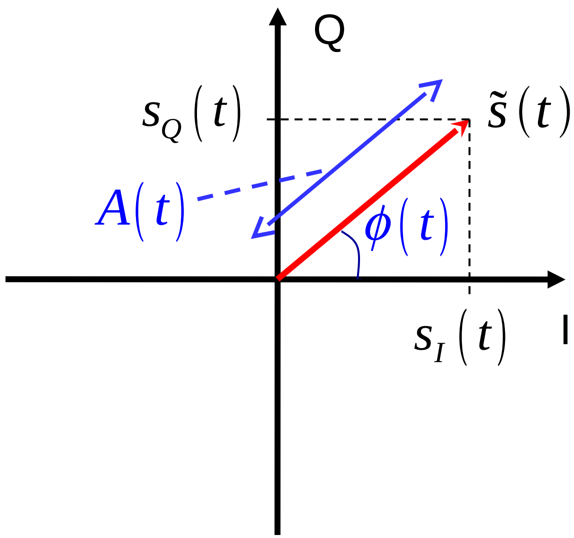
Complex envelope $\tilde{s}(t) = s_I(t) + j\, s_Q(t)$

Carrier factor $e^{j2\pi f_c t}$

$\Rightarrow$ $s(t) = \mathrm{Re}\left\{\tilde{s}(t)\, e^{j2\pi f_c t}\right\}$

Interpreting the complex notation

Complex envelope (phasor)



Polar coordinates:

$$\tilde{s}(t) = s_I(t) + j\,s_Q(t) = A(t)e^{j\phi(t)}$$

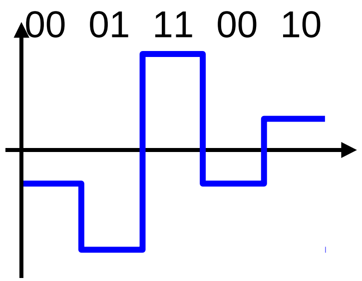
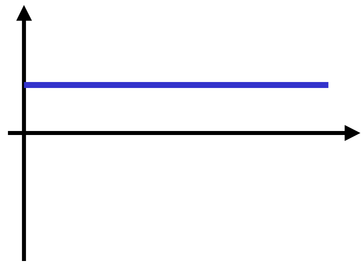
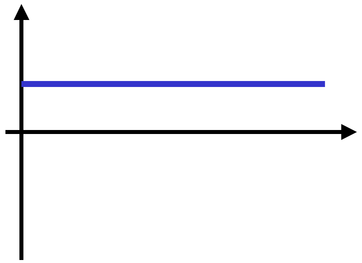
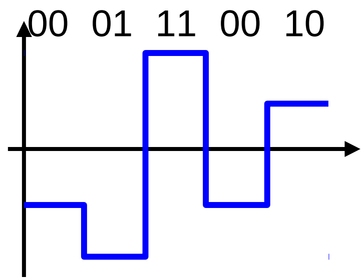
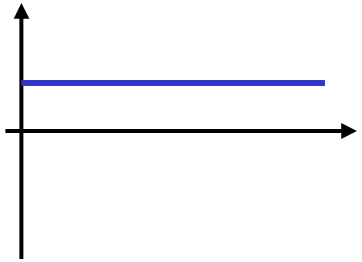
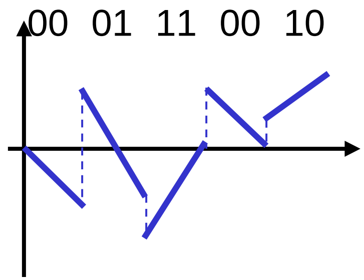
Transmitted radio signal

$$\begin{aligned} s(t) &= \mathrm{Re}\left\{\tilde{s}(t)e^{j2\pi f_c t}\right\} \\ &= \mathrm{Re}\left\{A(t)e^{j\varphi(t)}e^{j2\pi f_c t}\right\} \\ &= \mathrm{Re}\left\{A(t)e^{j(2\pi f_c t + \varphi(t))}\right\} \\ &= A(t)\cos(2\pi f_c t + \varphi(t)) \end{aligned}$$

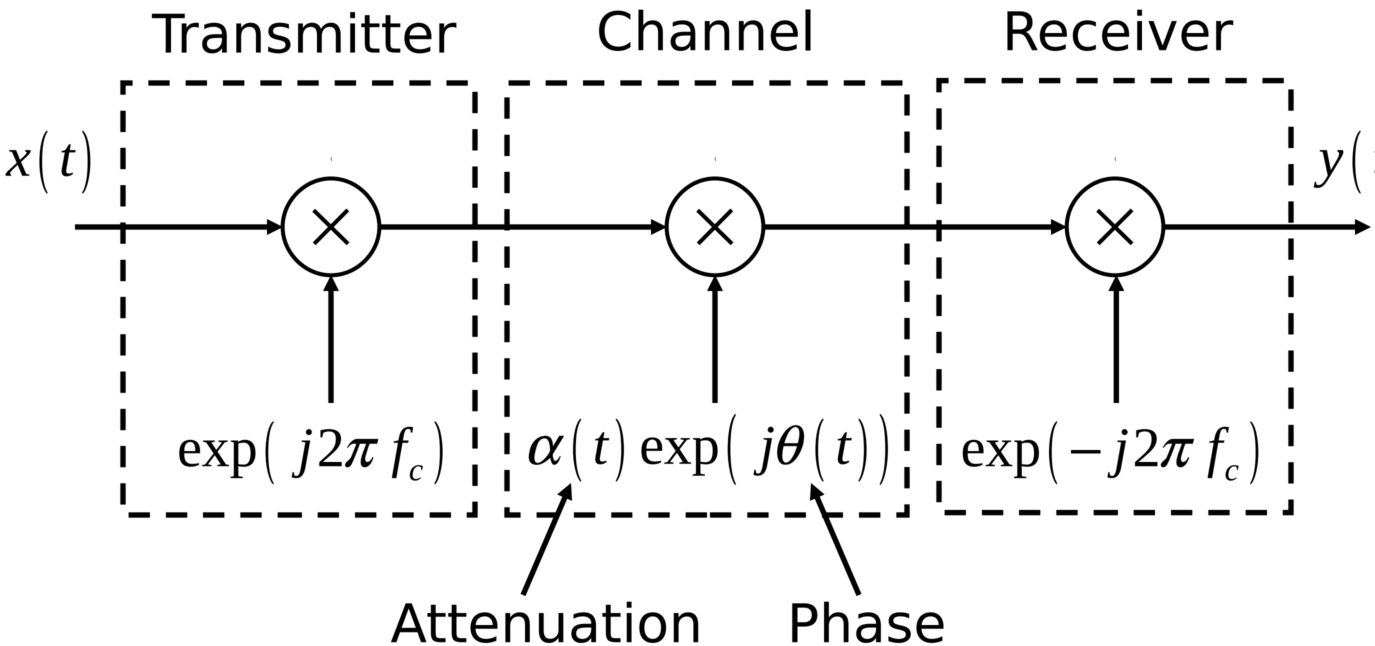
By manipulating the amplitude *A*(t) and the phase *Φ*(t) of the complex envelope (phasor), we can create any type of modulation/radio signal.

Example: Amplitude, phase and frequency modulation

$$s(t) = A(t)\cos\left(2\pi f_c t + \phi(t)\right)$$

	$A(t)$	$\phi(t)$	Comment:
4ASK			- Amplitude carries information - Phase constant (arbitrary)
4PSK			- Amplitude constant (arbitrary) - Phase carries information
4FSK			- Amplitude constant (arbitrary) - Phase slope (frequency) carries information

A narrowband system described in complex notation (noise free)



In: $x(t) = A(t)\exp(j\phi(t))$

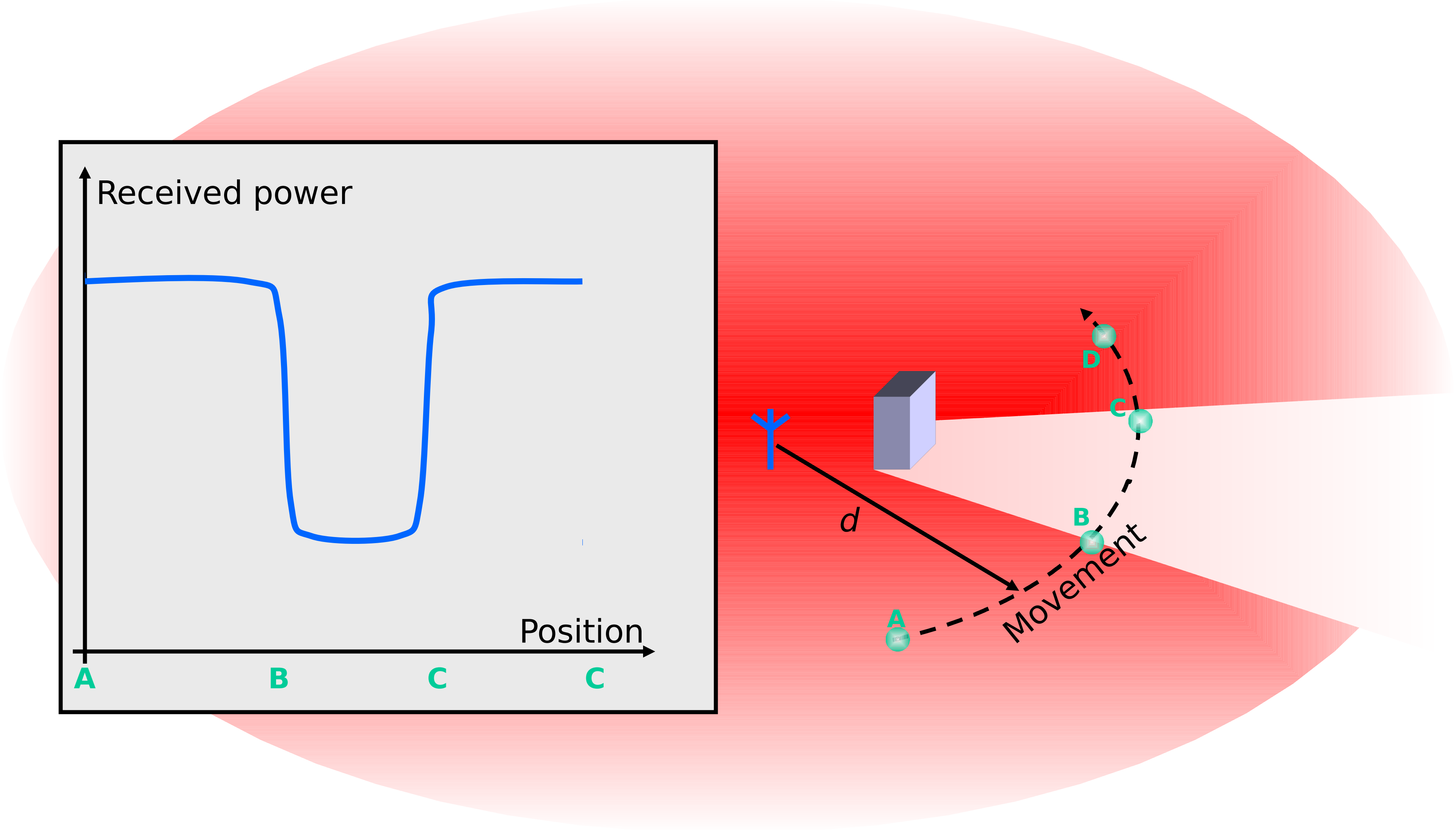
Out: $y(t) = A(t)\exp(j\phi(t))\,\exp(j2\pi f_c t)\,\alpha(t)\exp(j\theta(t))\,\exp(-j2\pi f_c t)$

$= A(t)\alpha(t)\exp\big(j(\phi(t)+\theta(t))\big)$

It is the behaviour of the channel attenuation and phase we are going to model.

LARGE-SCALE FADING

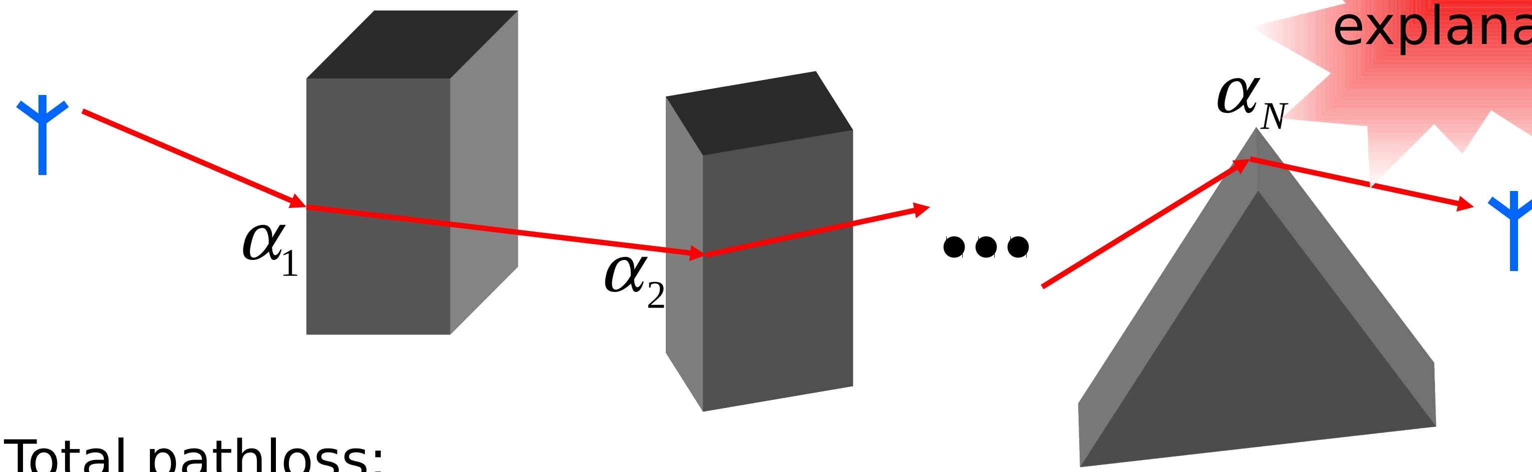
Large-scale fading
Basic principle



Large-scale fading
More than one shadowing object

Signal path in terrain with several diffraction points adding extra attenuation to the pathloss.

This is ONE explanation



Total pathloss:

$$L_{\text{tot}} = L(d) \times \alpha_1 \times \alpha_2 \times \cdots \times \alpha_N$$

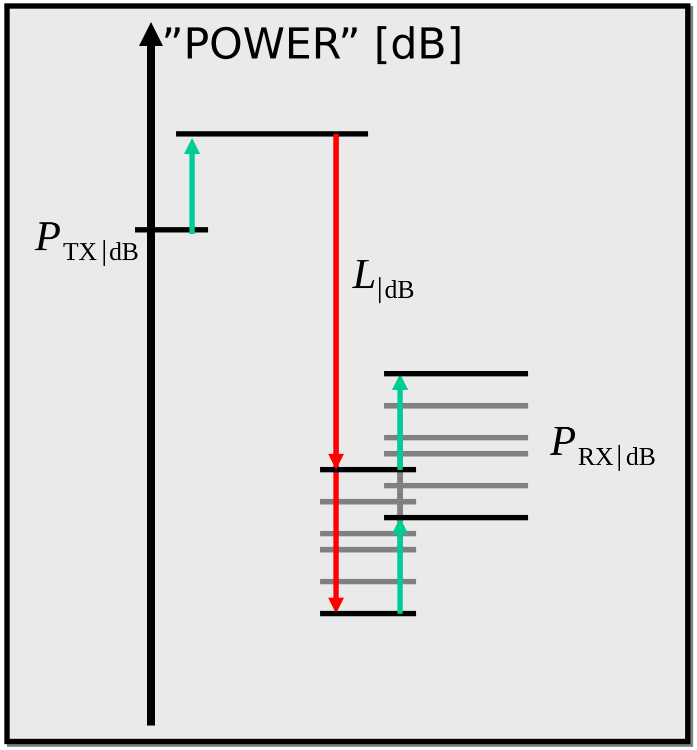
$$L_{\text{tot}|dB} = L(d)_{|dB} + \alpha_{1|dB} + \alpha_{2|dB} + \cdots + \alpha_{N|dB}$$

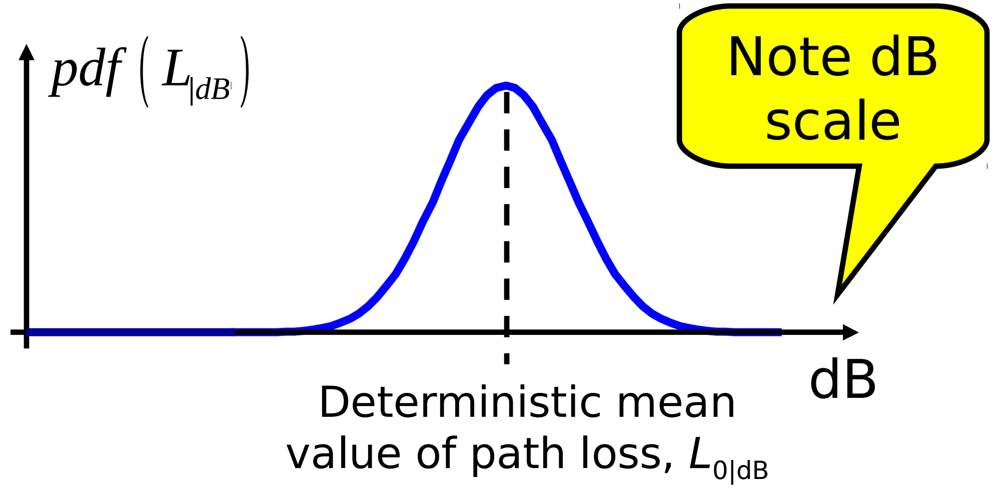
Deterministic

If these are considered random and independent, we should get a normal distribution in the dB domain.

Large-scale fading
Log-normal distribution

Measurements confirm that in many situations, the large-scale fading of the received signal strength has a normal distribution in the dB domain.





$$pdf\left(L_{|\mathrm{dB}}\right)=\frac{1}{\sqrt{2\pi}\,\sigma_{F|\mathrm{dB}}}\exp\left(-\frac{\left(L_{|\mathrm{dB}}-L_{0|\mathrm{dB}}\right)^2}{2\sigma_{F|\mathrm{dB}}^2}\right)$$

Standard deviation $\sigma_{F|\mathrm{dB}}\approx 4-10\,\mathrm{dB}$

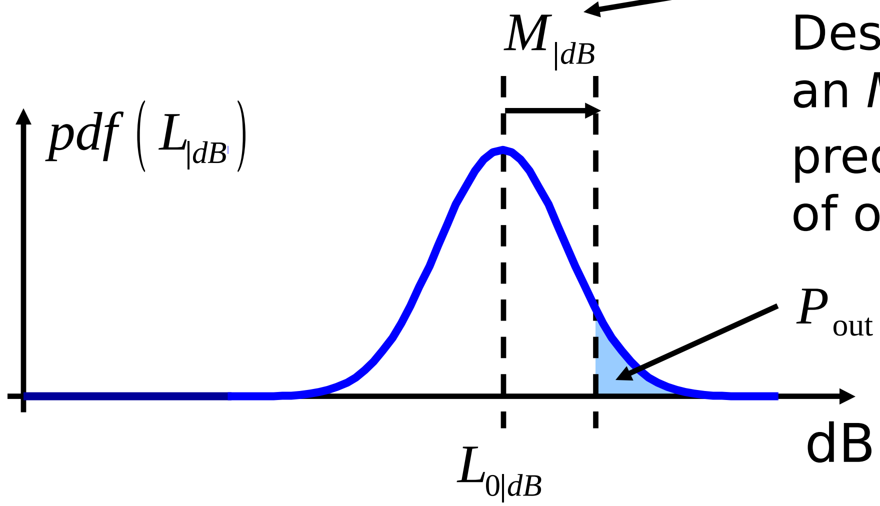
Large-scale fading
Fading margin

We know that the path loss will vary around the deterministic value predicted.

We need to design our system with a “margin” allowing us to handle higher path losses than the deterministic prediction. This margin is called a **fading margin**.

Increasing the fading margin decreases the **probability of outage**, which is the probability that our system receive a too low power to operate correctly.

Large-scale fading
Fading margin (cont.)



Fading margin
Designing the system to handle an $M_{|\mathrm{dB}}$ dB higher loss than predicted, lowers the probability of outage.

$$P_{\mathrm{out}} = \Pr\left\{L_{|\mathrm{dB}} > L_{0|\mathrm{dB}} + M_{|\mathrm{dB}}\right\} = Q\left(\frac{M_{|\mathrm{dB}}}{\sigma_{F|\mathrm{dB}}}\right)$$

The upper tail probability of a unit variance, zero-mean, Gaussian (normal) variable:

$$Q(y) = \int_y^\infty \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right) dx = \frac{1}{2}\,\mathrm{erfc}\left(\frac{y}{\sqrt{2}}\right)$$

The complementary error-function can be found in e.g. MATLAB

The Q(.)-function
Upper-tail probabilities

x	Q(x)
4.265	0.00001
4.107	0.00002
4.013	0.00003
3.944	0.00004
3.891	0.00005
3.846	0.00006
3.808	0.00007
3.775	0.00008
3.746	0.00009
3.719	0.00010
3.540	0.00020
3.432	0.00030
3.353	0.00040
3.291	0.00050
3.239	0.00060
3.195	0.00070
3.156	0.00080
3.121	0.00090

x	Q(x)
3.090	0.00100
2.878	0.00200
2.748	0.00300
2.652	0.00400
2.576	0.00500
2.512	0.00600
2.457	0.00700
2.409	0.00800
2.366	0.00900
2.326	0.01000
2.054	0.02000
1.881	0.03000
1.751	0.04000
1.645	0.05000
1.555	0.06000
1.476	0.07000
1.405	0.08000
1.341	0.09000

x	Q(x)
1.282	0.10000
0.842	0.20000
0.524	0.30000
0.253	0.40000
0.000	0.50000

Large-scale fading
A numeric example

How many dB fading margin, against $\sigma_{F|dB} = 7$ dB log-normal fading, do we need to obtain an outage probability of 0.5%?

$$P_{out} = Q\left(\frac{M_{|dB}}{\sigma_{F|dB}}\right) = 0.5\% = 0.005$$

Consulting the Q(.)-function table (or using a numeric software), we get

$$\frac{M_{|dB}}{\sigma_{F|dB}} = 2.576 \Rightarrow \frac{M_{|dB}}{7} = 2.576 \Rightarrow \boxed{M_{|dB} = 18}$$

SMALL-SCALE FADING

Small-scale fading
Illustration shown during Lecture 1

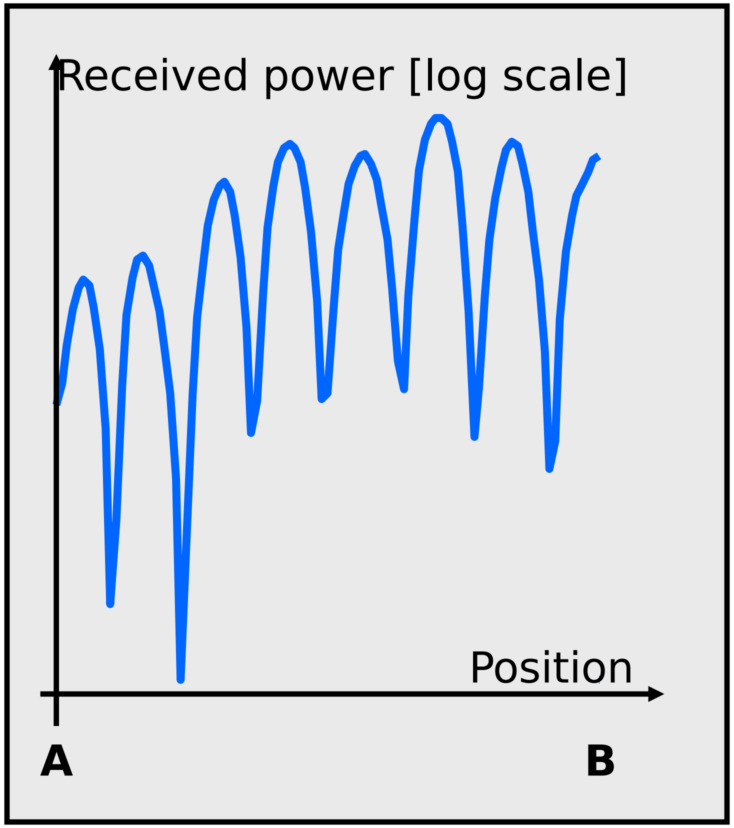
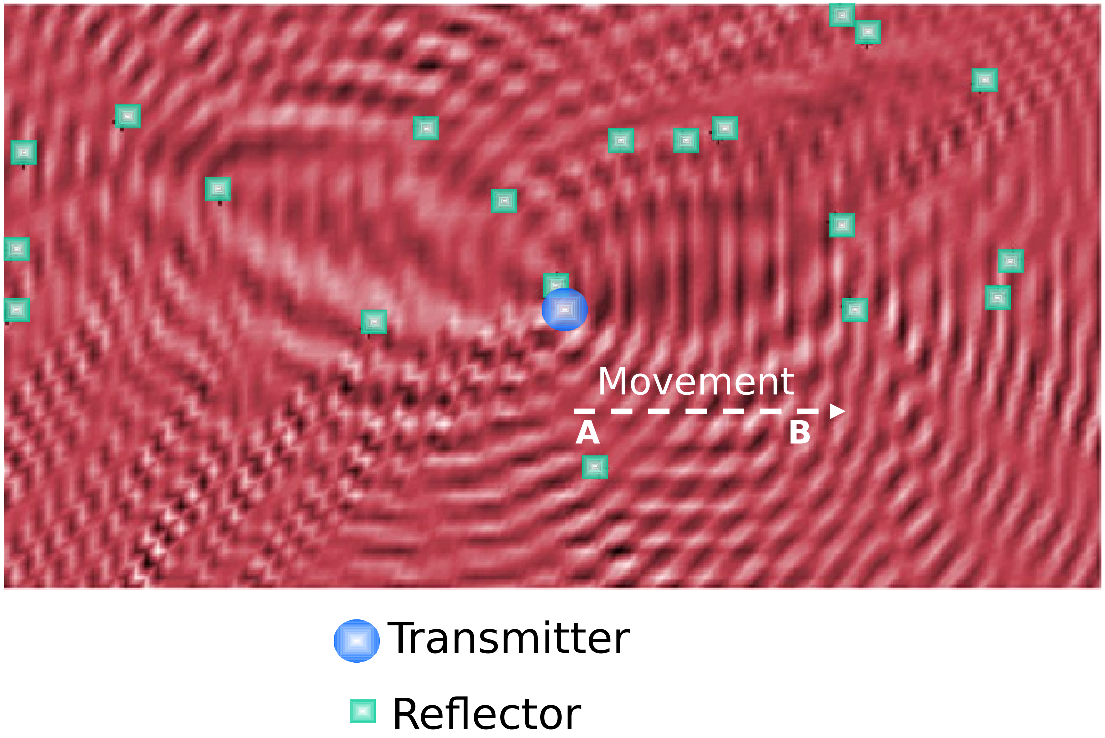
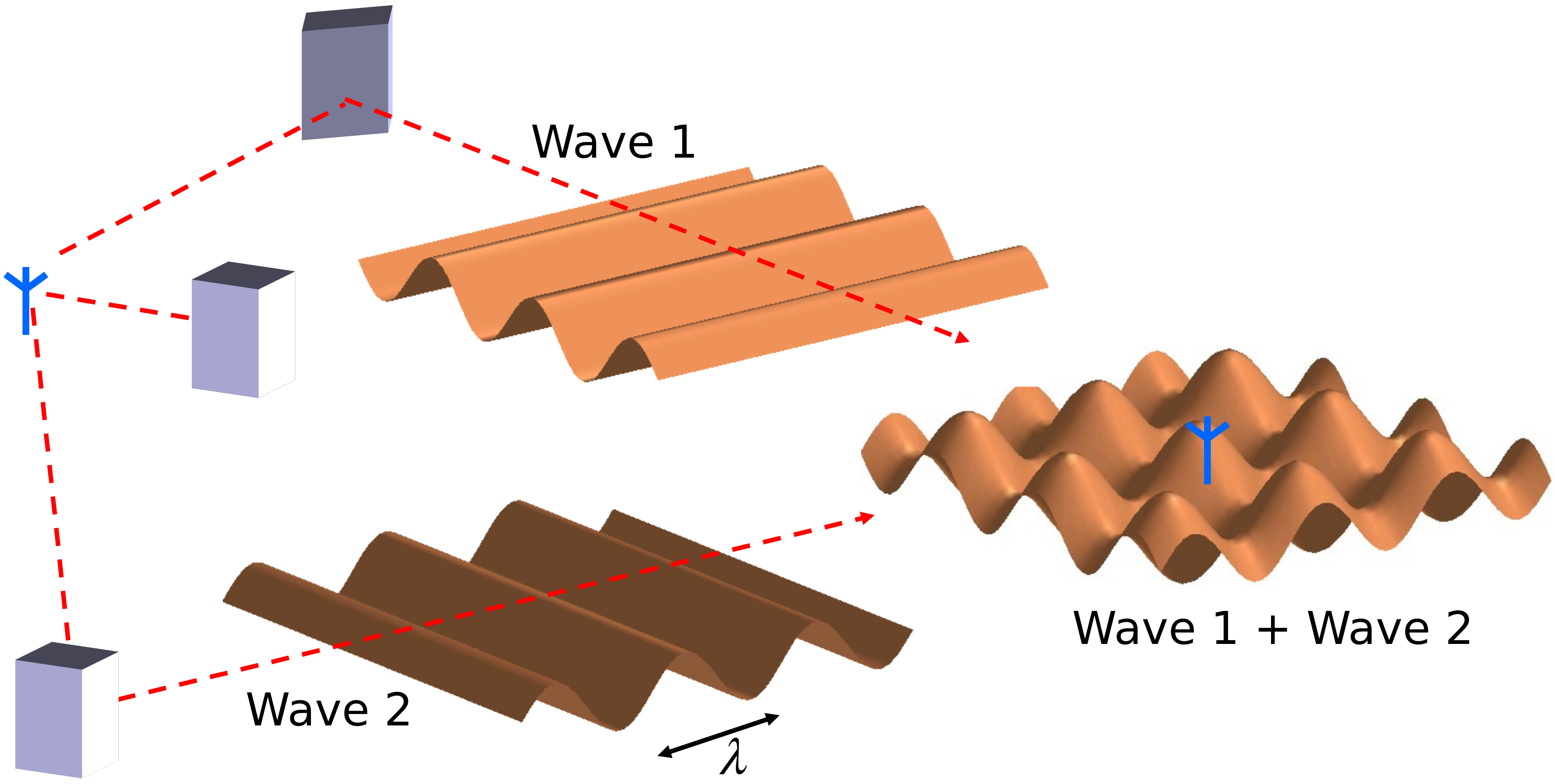


Illustration of interference pattern from above



Many reflectors ... let’s look at a simpler case!

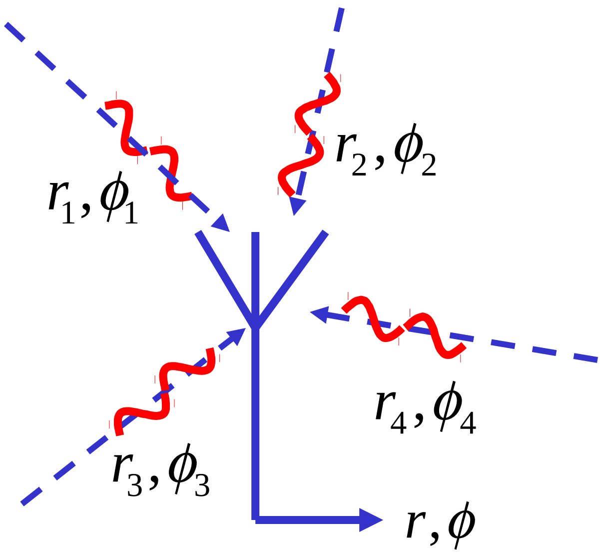
Small-scale fading
Two waves



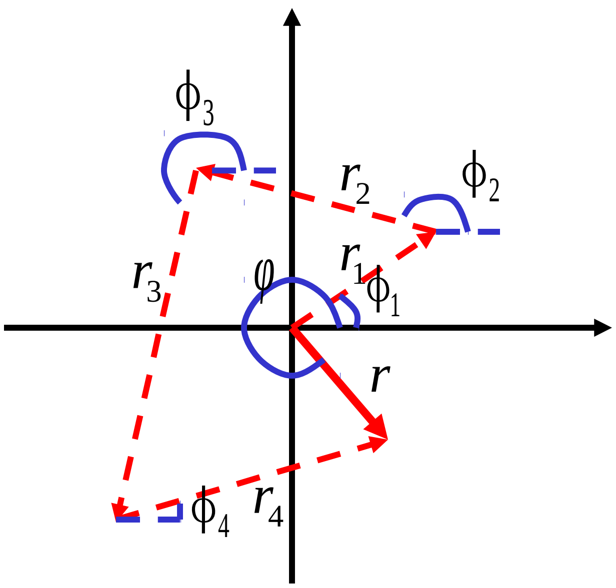
At least in this case, we can see that the interference pattern changes on the wavelength scale.

Small-scale fading
Many incoming waves

Many incoming waves with independent amplitudes and phases

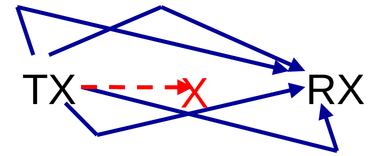


Add them up as phasors



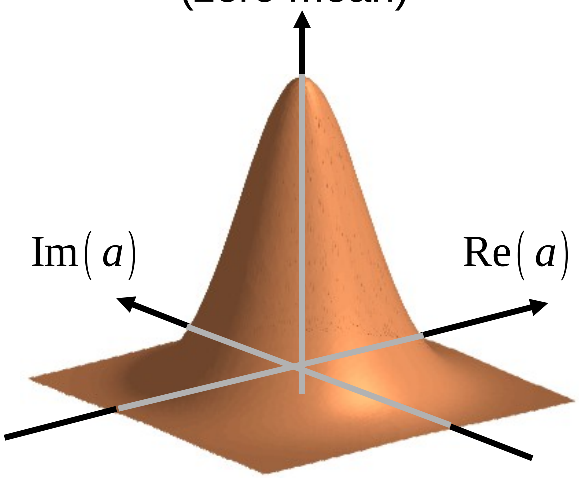
$$r\exp(j\phi) = r_1\exp(j\phi_1) + r_2\exp(j\phi_2) + r_3\exp(j\phi_3) + r_4\exp(j\phi_4)$$

Small-scale fading
Rayleigh fading



No dominant component
(no line-of-sight)

Tap distribution
2D Gaussian
(zero mean)

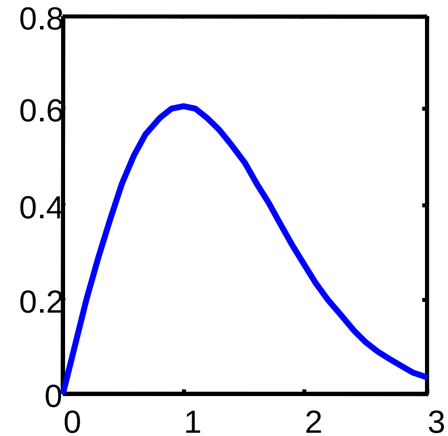


No line-of-sight component

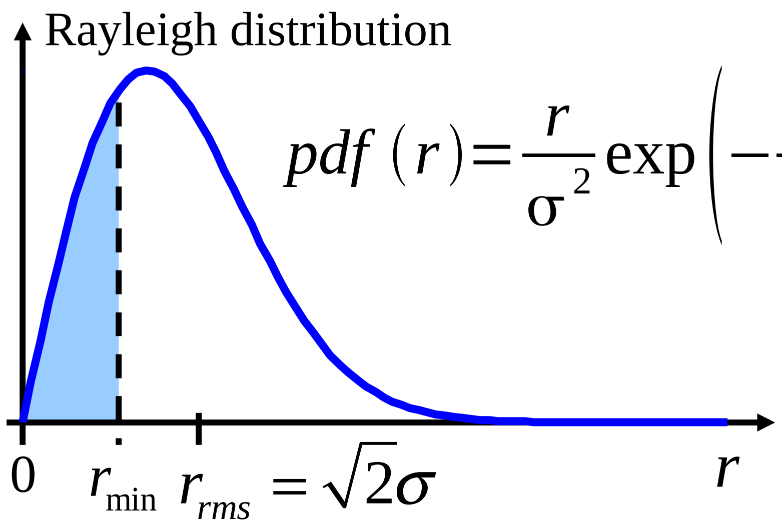
$$r = |a|$$

Amplitude distribution

Rayleigh



$$pdf(r) = \frac{r}{\sigma^2} \exp\left(-\frac{r^2}{2\sigma^2}\right)$$



$$pdf(r) = \frac{r}{\sigma^2} \exp\left(-\frac{r^2}{2\sigma^2}\right)$$

Fading margin

$$M = \frac{r_{\text{rms}}^2}{r_{\min}^2}$$

$$M_{|\text{db}} = 10 \log_{10}\left(\frac{r_{\text{rms}}^2}{r_{\min}^2}\right)$$

Probability that the amplitude r is below some threshold $r_{\min}$:

$$\Pr(r < r_{\min}) = \int_0^{r_{\min}} pdf(r)\, dr = 1 - \exp\left(-\frac{r_{\min}^2}{r_{\text{rms}}^2}\right)$$

Small-scale fading
A numeric example

How many dB fading margin, against Rayleigh fading, do we need to obtain an outage probability of 1%?

$$\Pr\left(r < r_{\min}\right) = 1 - \exp\left(-\frac{r_{\min}^2}{r_{\text{rms}}^2}\right) = 1\% = 0.01$$

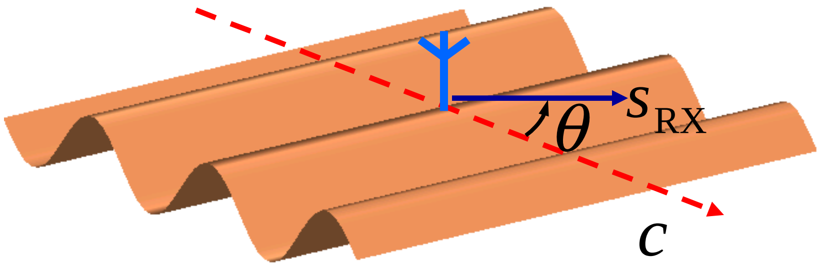
Some manipulation gives

$$1 - 0.01 = \exp\left(-\frac{r_{\min}^2}{r_{\text{rms}}^2}\right) \Rightarrow \ln(0.99) = -\frac{r_{\min}^2}{r_{\text{rms}}^2}$$

$$\Rightarrow \frac{r_{\min}^2}{r_{\text{rms}}^2} = -\ln(0.99) = 0.01 \quad \Rightarrow M = \frac{r_{\text{rms}}^2}{r_{\min}^2} = 1/0.01 = 100$$

$$\boxed{M_{|\text{dB}} = 20}$$

Small-scale fading
Doppler shifts



Receiving antenna moves with speed s_{RX} at an angle θ relative to the propagation direction of the incoming wave, which has frequency f_0.

[c = speed of light = 3×10^8 m/s]

Frequency of received signal:

$$f = f_0 + \nu$$

where the Doppler shift is

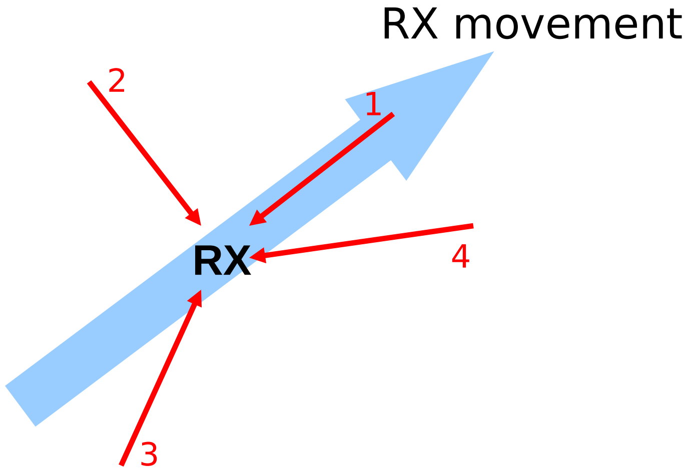
$$\nu = -f_0 \frac{s_{RX}}{c} \cos(\theta)$$

The maximal Doppler shift is

$$\nu_{max} = f_0 \frac{s_{RX}}{c}$$

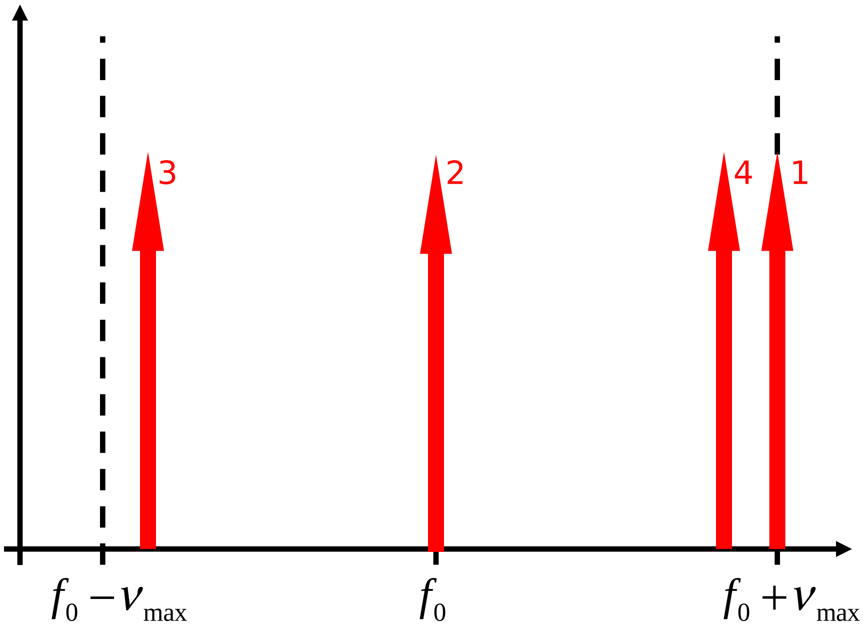
Small-scale fading
Doppler spectrum

Incoming waves from several directions
(relative to movement or RX)



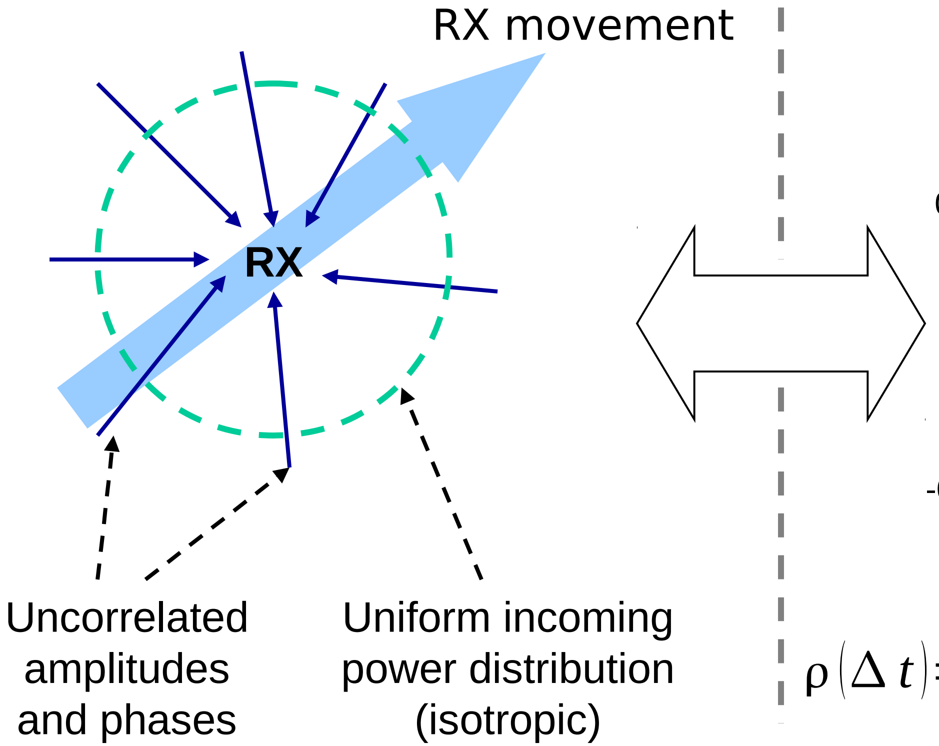
All waves of equal strength in this example, for simplicity.

Spectrum of received signal when a f_0 Hz signal is transmitted.

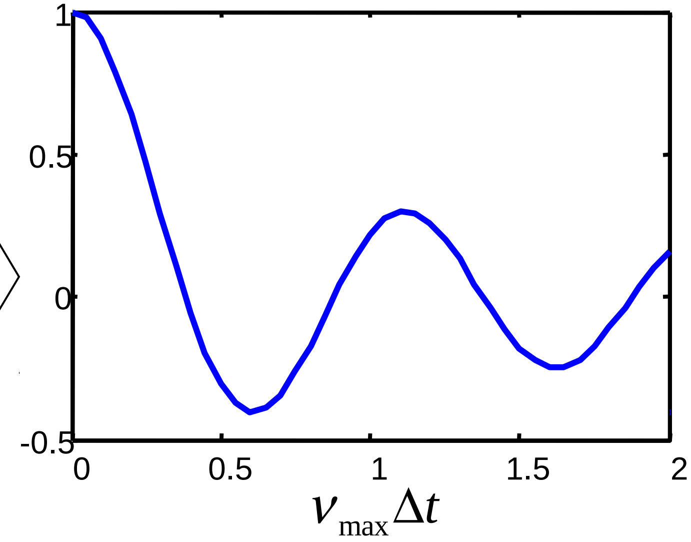


Small-scale fading
Doppler spectrum

Isotropic uncorrelated scattering



Time correlaion



$$\rho(\Delta t) = E\{a(t)a^*(t+\Delta t)\} \sim J_0(2\pi \nu_{max} \Delta t)$$

Small-scale fading
The Doppler spectrum

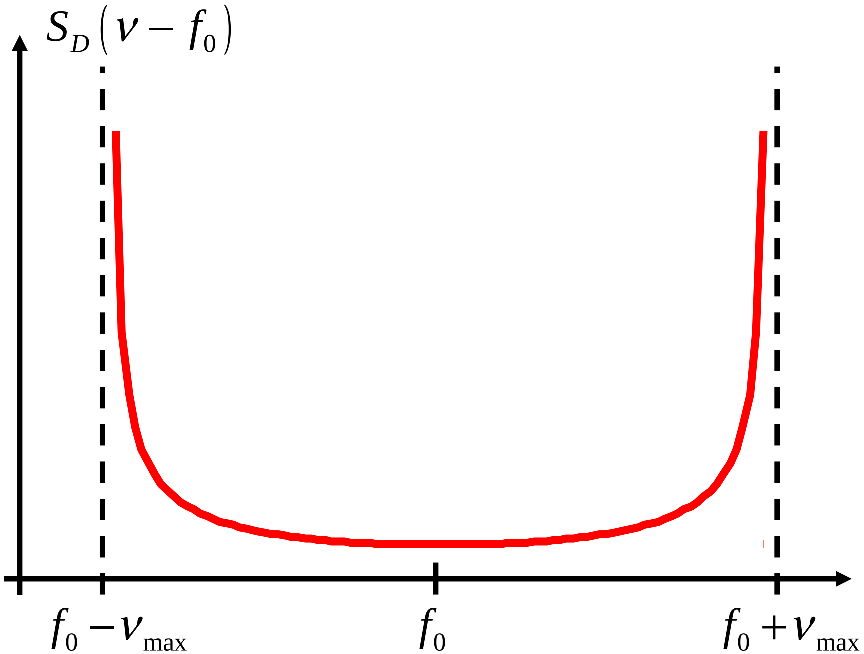
For the uncorrelated scattering with uniform angular distribution of incoming power (isotropic scattering), we obtain the Doppler spectrum by Fourier transformation of the time correlation of the signal:

$$S_{\mathrm{D}}(\nu) = \int \rho(\Delta t) e^{-j 2\pi \nu \Delta t} d\Delta t$$

$$\sim \frac{1}{\pi \sqrt{\nu_{\max}^2 - \nu^2}}$$

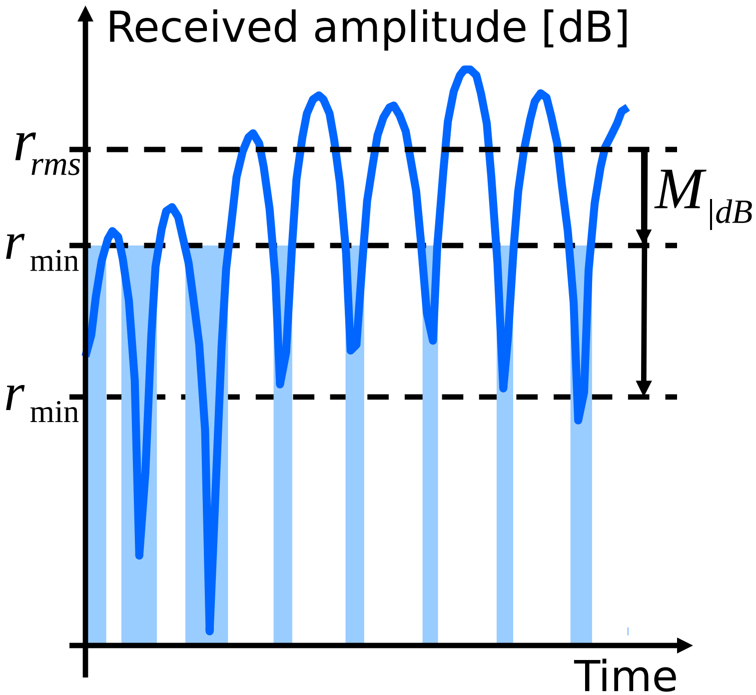
for $-\nu_{\max} < \nu < \nu_{\max}$

Doppler spectrum at center frequency f_0.



This is the "classical" Doppler spectrum, a.k.a. the **Jakes' doppler spectrum**.

Small-scale fading
Fading dips



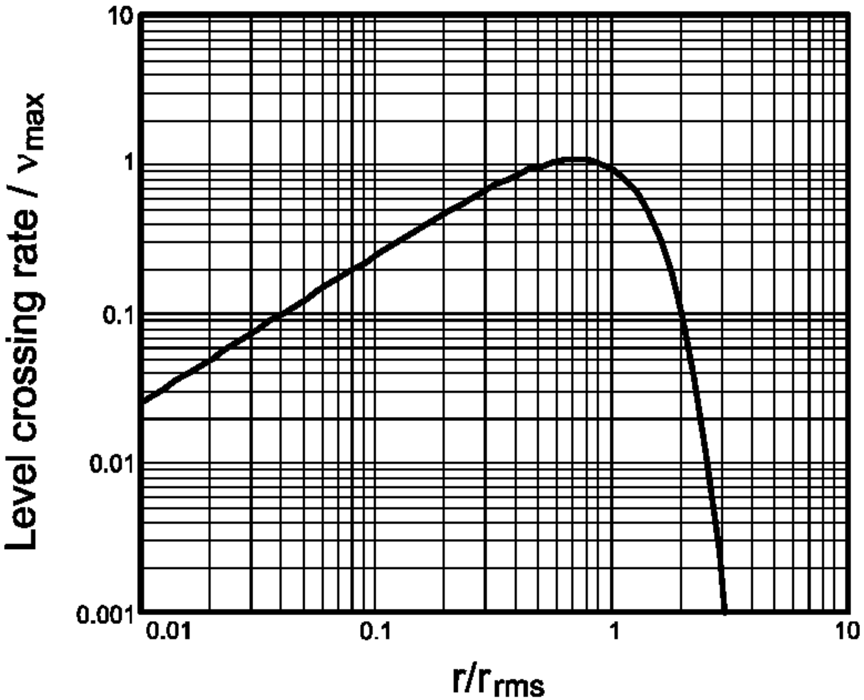
The larger the fading margin, the rarer the fading dips, and the shorter they are.

The length and the frequency of fading dips can be important for the functionality of a radio system.

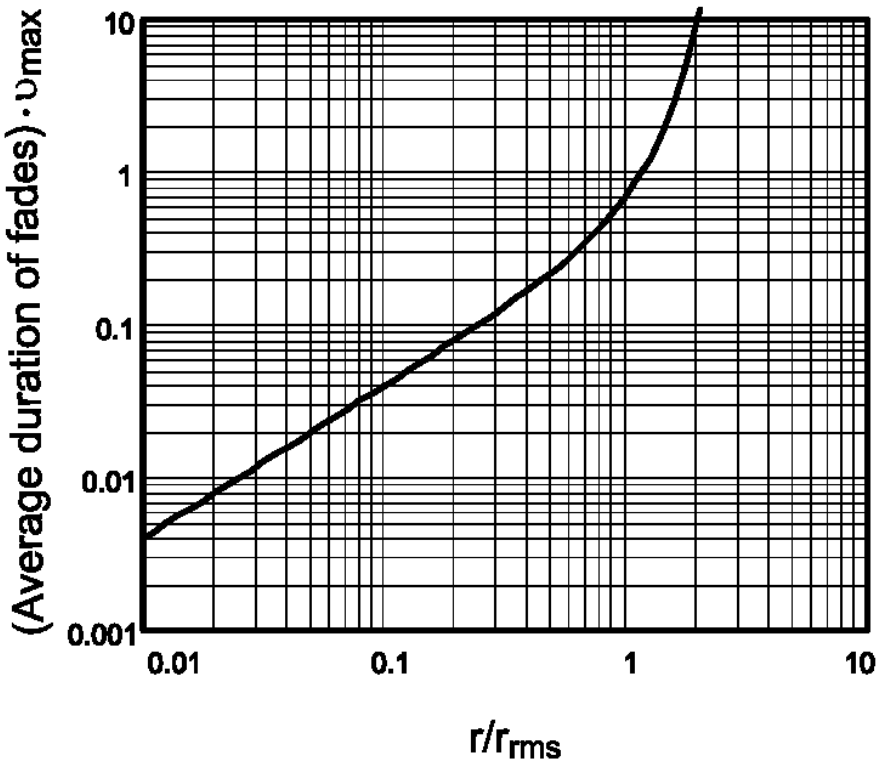
Can we quantify these?

Small-scale fading
Statistics of fading dips

Frequency of the fading dips
(normalized dips/second)



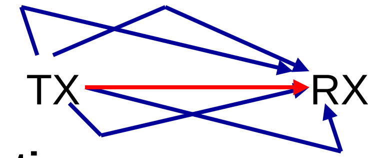
Length of fading dips
(normalized dip-length)



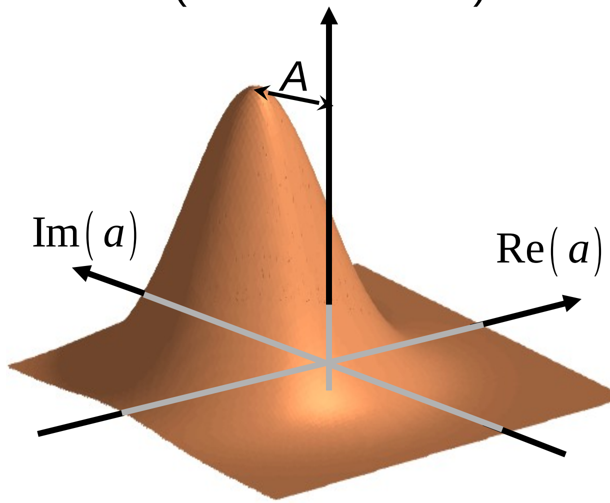
These curves are for Rayleigh fading and isotropic uncorrelated scattering (**Jakes’ doppler spectrum**).

Small-scale fading
Rice fading

A dominant component (line of sight)



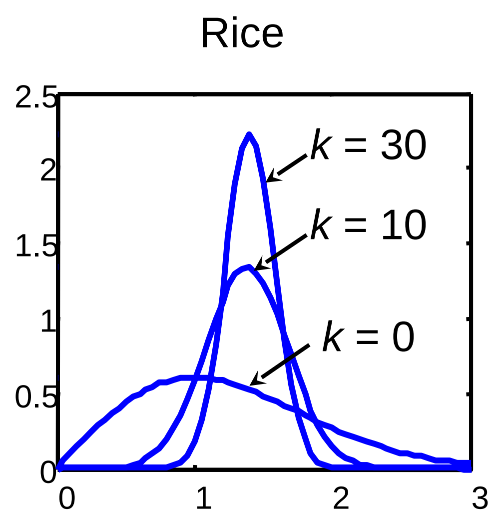
Tap distribution
2D Gaussian
(non-zero mean)



Line-of-sight (LOS) component with amplitude *A*.

$$r = |a|$$

Amplitude distribution



$$pdf(r) = \frac{r}{\sigma^2} \exp\left(-\frac{r^2 + A^2}{2\sigma^2}\right) \mathrm{I}_0\left(\frac{rA}{\sigma^2}\right)$$

$$k = \frac{\text{Power in LOS component}}{\text{Power in random components}} = \frac{A^2}{2\sigma^2}$$

COMBINING LARGE- AND SMALL-SCALE FADING

Large- and small-scale fading
Combining the two

We will start using Alternative 1

We have seen examples of how we can compute the required fading margins, due to large- and small-scale fading, given certain criteria (e.g. outage probability).

If we have both types of fading, how do we combine them into a ”total” fading margin?

There are basically two options:

1) Calculate the fading margins separately and add them up.

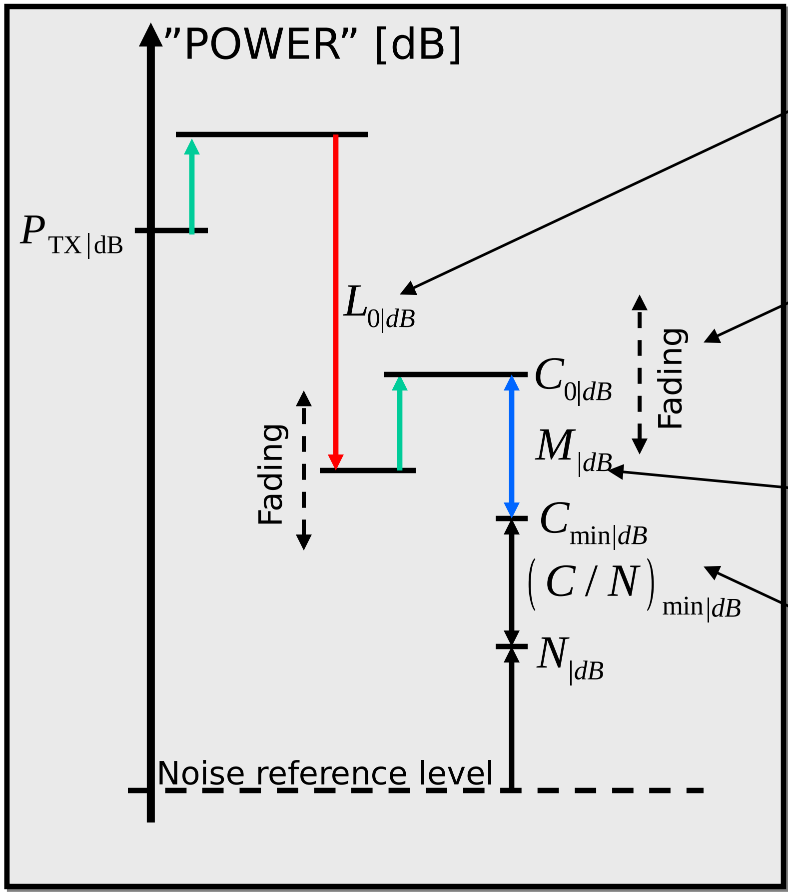
2) Derive the pdf (or cdf) of the total fading and calculate a single fading margin for both.

Alternative 1 is the simple solution, but it will overdimension the system a bit.

Alternative 2 is a much more complex operation.

NOISE- AND INTERFERENCE-LIMITED LINKS

Noise-limited system
Fading margin and the link budget



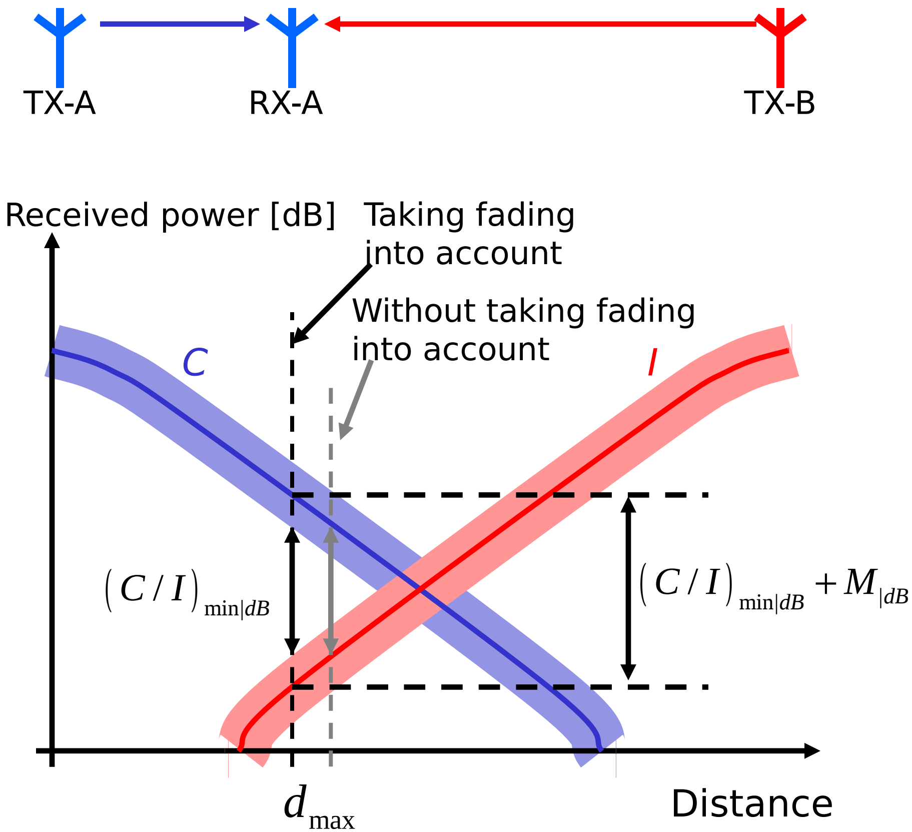
We use some propagation model to calculate a deterministic propagation loss.

Variations in the environment and movements will cause variations in the the propagation loss, which will influence the instantaneous received power.

To protect the receiver from too low received power, we add a fading margin.

Requirement for the receiver to operate properly.

Interference-limited system
Interference fadning margin



In interference limited systems, we are preliminary interested in how far from the transmitter we can be, without receiveing too much interference.

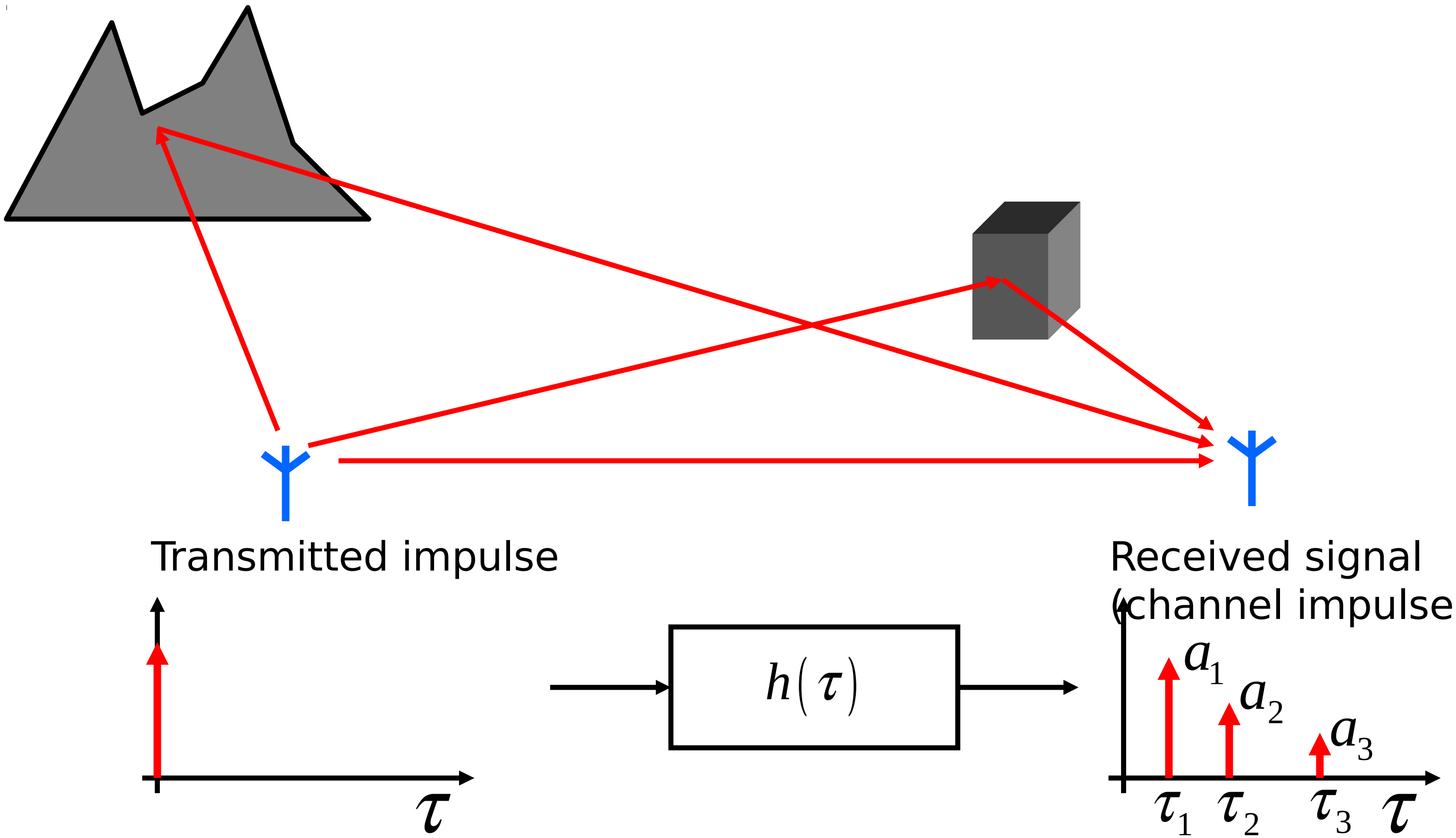
Depending on the system design and requirements on quality, our receiver can tolerate a certain $(C/I)_{min}$.

Assuming fading on the wanted and interfering signal we can calculate a fading margin $M_{|dB}$ required to fulfill som criterion on e.g. outage.

For independent log-normal fading, we can add the variances of the two fading characteristics and get a "total" lognormal fading with standard deviation: $\sigma_{tot|dB} = \sqrt{\sigma_{C|dB}^2 + \sigma_{I|dB}^2}$

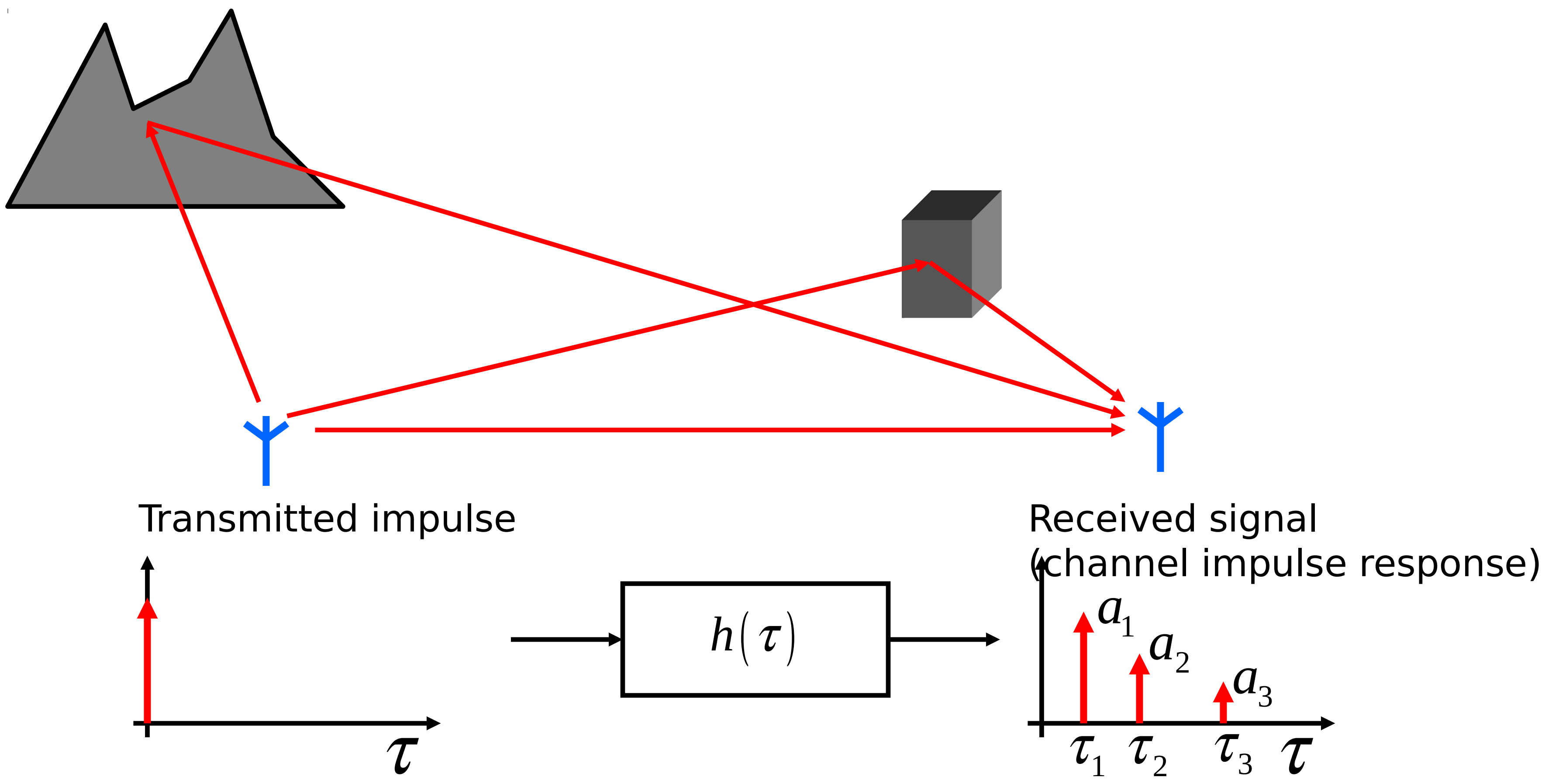
DELAY (TIME) DISPERSION

Delay (time) dispersion
A simple case



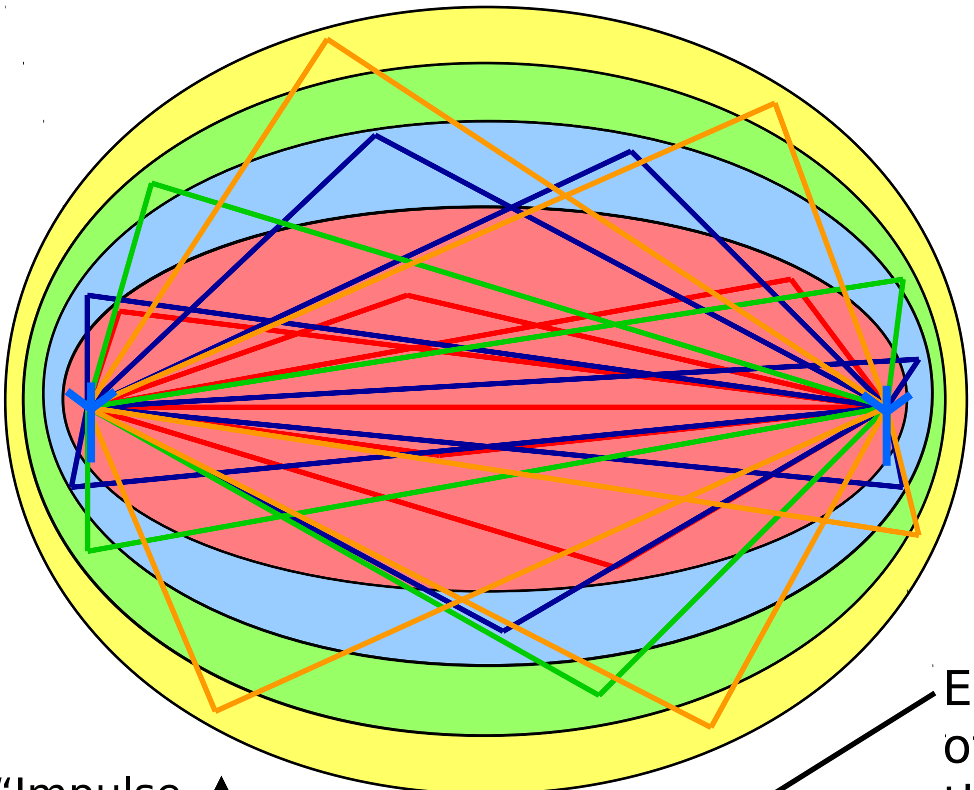
$$h(\tau) = a_1\delta(\tau - \tau_1) + a_2\delta(\tau - \tau_2) + a_3\delta(\tau - \tau_3)$$

Delay (time) dispersion
A simple case

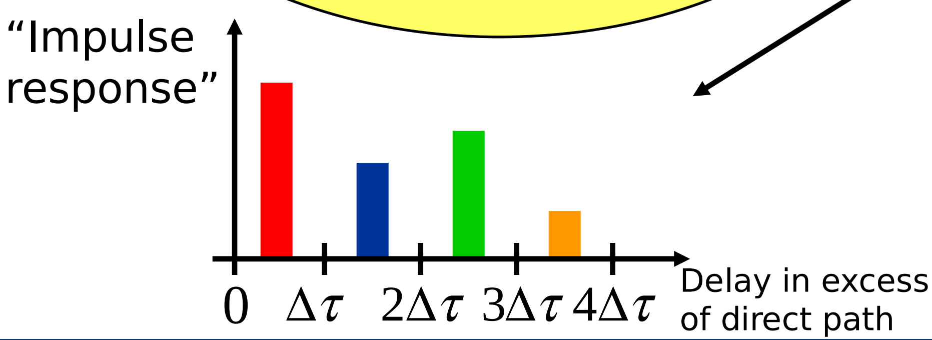


$$h(\tau) = a_1\delta(\tau - \tau_1) + a_2\delta(\tau - \tau_2) + a_3\delta(\tau - \tau_3)$$

Delay (time) dispersion
One reflection/path, many paths



Since each bin consists of contributions from several waves, each bin will fade if we introduce movement.

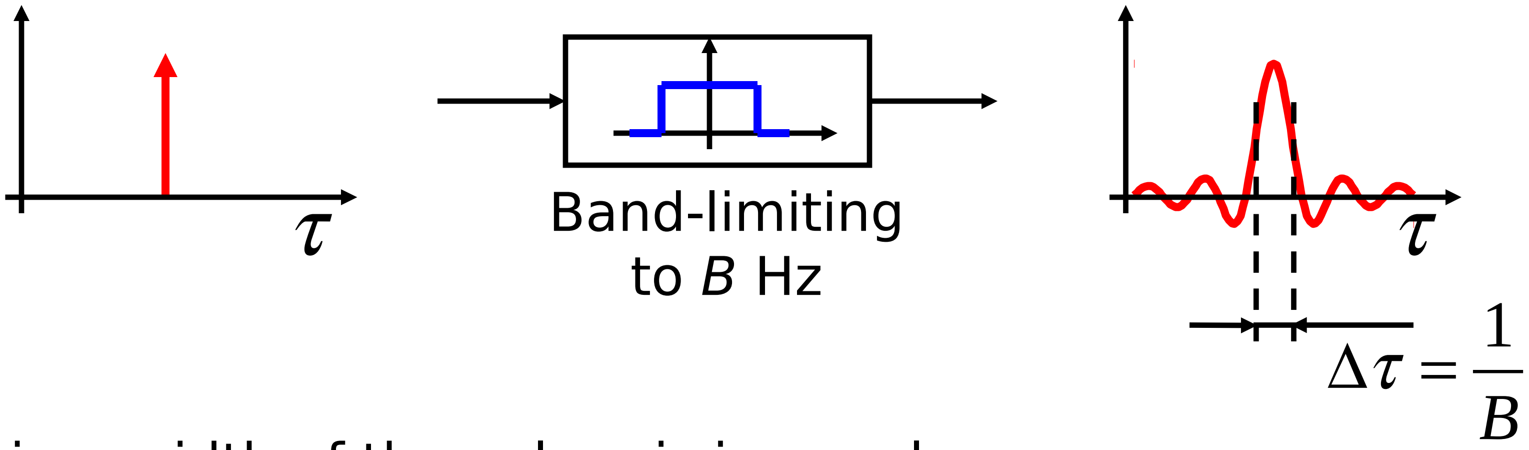


Each bin consists of incoming waves that are too close in time to resolve.

What do we mean by "too close in time"?

Delay (time) dispersion Bandwidth and time-resolution

Radio systems are band-limited, which makes our infinitely short impulses become waveforms with a certain width in time.

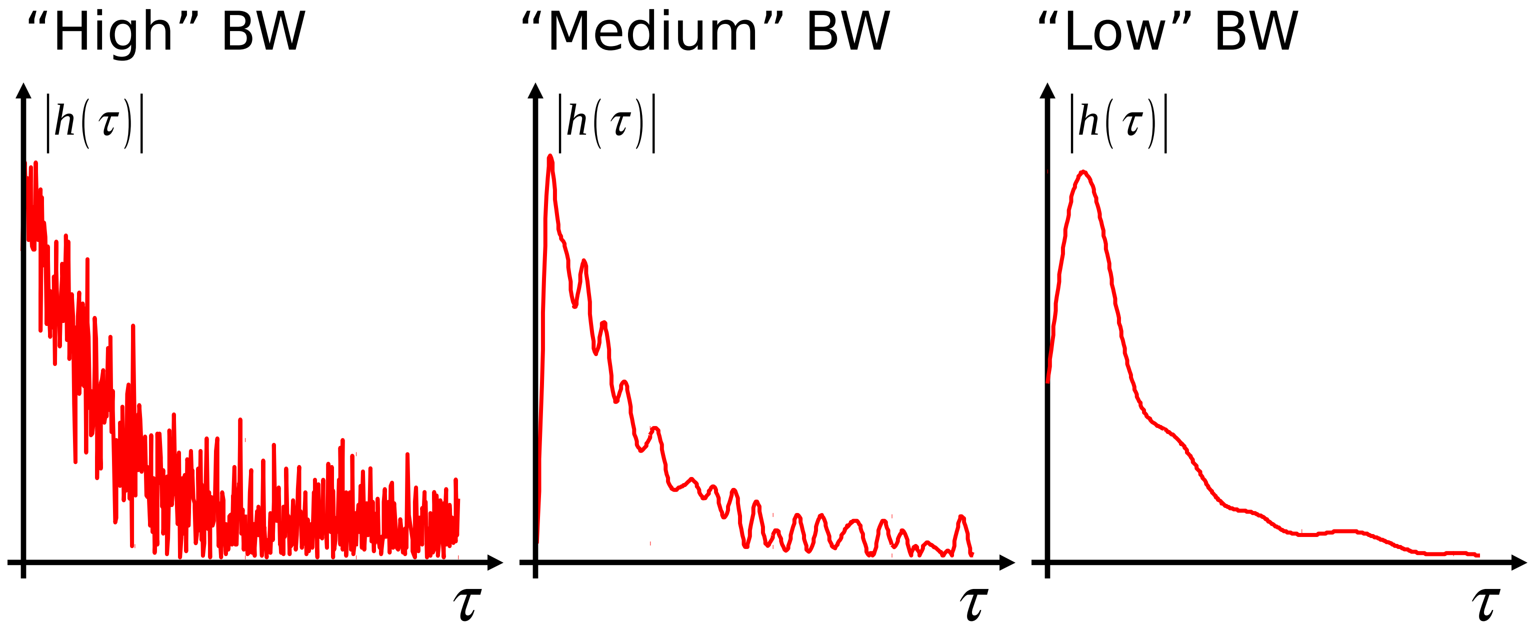


The time-width of the pulses is inversely proportional to the bandwidth.

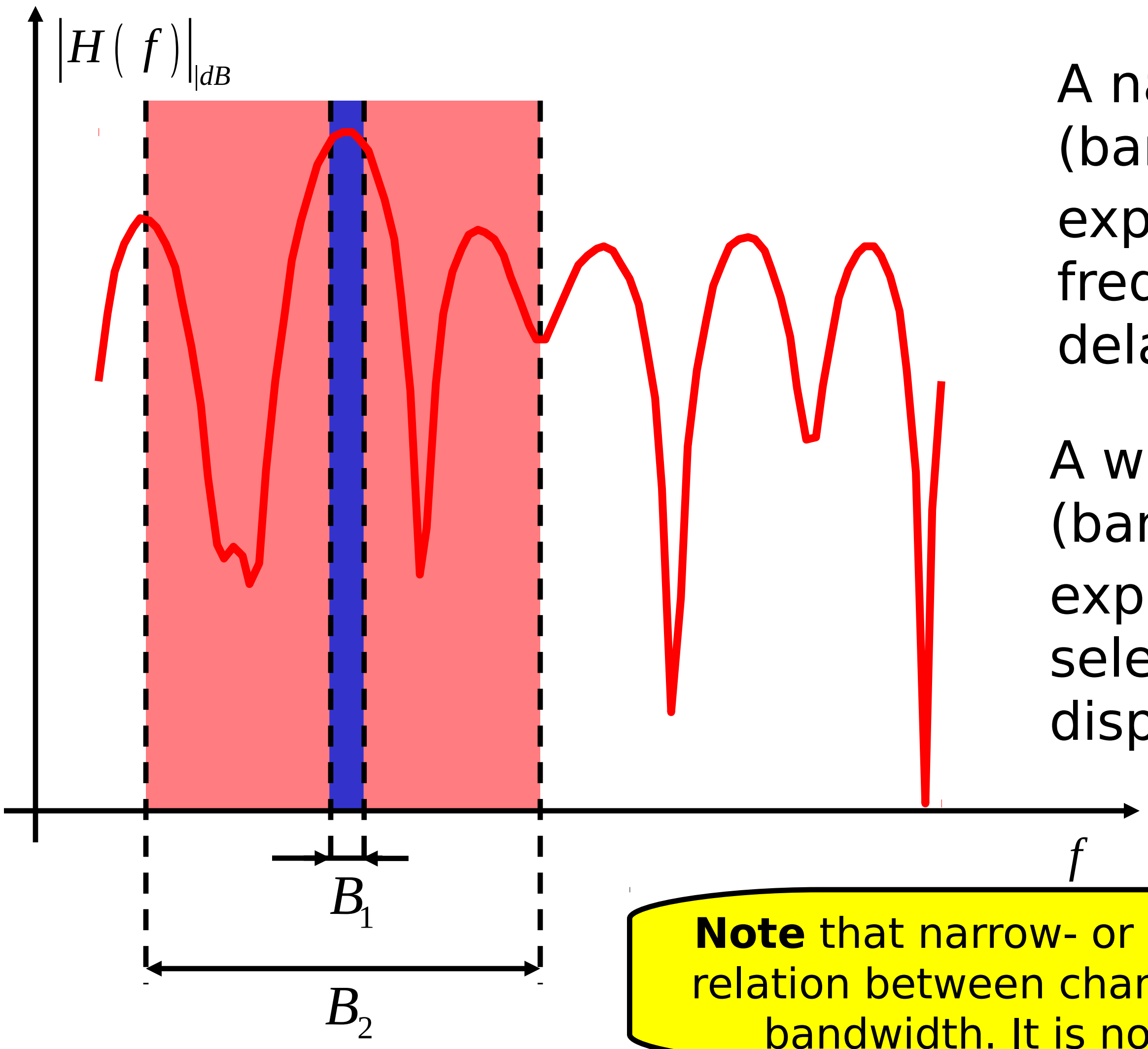
NARROW- VERSUS WIDE-BAND

Narrow- versus wide-band Channel impulse response

The same radio propagation environment is experienced differently, depending on the system bandwidth.



Narrow- versus wide-band Channel frequency response



A narrow-band system (bandwidth B_1) will not experience any significant frequency selectivity or delay dispersion.

A wide-band system (bandwidth B_2) will however experience both frequency selectivity and delay dispersion.

Note that narrow- or wide-band depends on the relation between channel properties and system bandwidth. It is not an absolute measure.

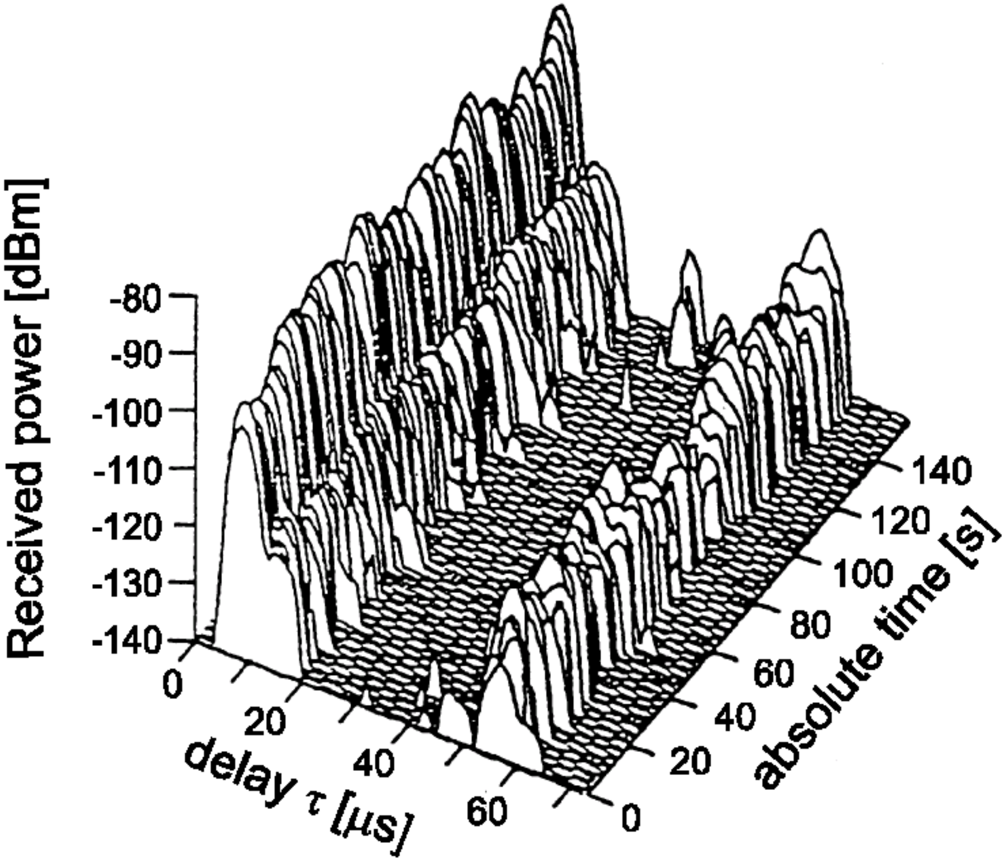
Narrow- versus wide-band
Let's not forget the time-dependence!

Measurement in hilly terrain at 900 MHz.

We need to take **absolute time** *t* into consideration, as the channel will change when things move.

The channel impulse response becomes:

$$h(t, \tau)$$



[Liebenow & Kuhlmann 1993]

Narrow- versus wide-band Doppler spectrum and delay

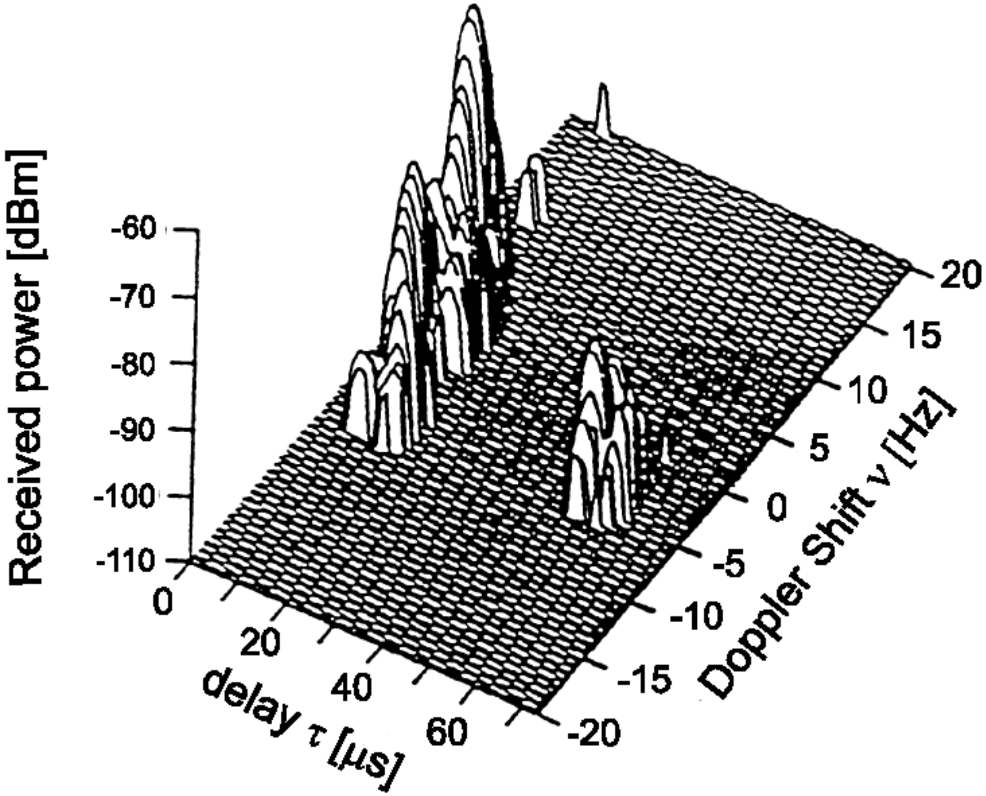
Since the channel at each delay τ is the result of different propagation paths, we can have different Doppler spectra for each delay.

This effect is shown by the scattering function:

$$P_S(\nu,\tau)$$

(received power as function of doppler shift and delay)

Measurement in hilly terrain at 900 MHz.



Summary

NARROW-BAND CHANNELS

- Complex notation with **amplitude**, **phase** and **complex envelope** (phasor).
- Large-scale fading with **log-normal distribution** and calculation of **fading margin**.
- Small-scale fading with **Rayleig** and **Rice** distribution, calculation of **fading margin**.
- Received signal **maximal Doppler shift, Doppler spectrum** and **time characteristics**.
- Fading margin in the **link-budget** of noise limited systems.
- Fading margin and **maximal distance** for interference limited systems.

WIDE-BAND CHANNELS

- Instead of one (time varying) channel coefficient, we have an entire **(time varying) impulse response**
- Channel **delay (time) dispersion** and **frequency selectivity**
- Doppler spectrum as function of delay, i.e. the **scattering function**

There is **MUCH MORE** to learn about this – which many of you have done in in the Channel Modeling course (ETIN10)!