Lecture no: 10

Multi-carrier
and
Multiple antennas

Ove Edfors, Department of Electrical and Information Technology
Ove.Edfors@eit.lth.se

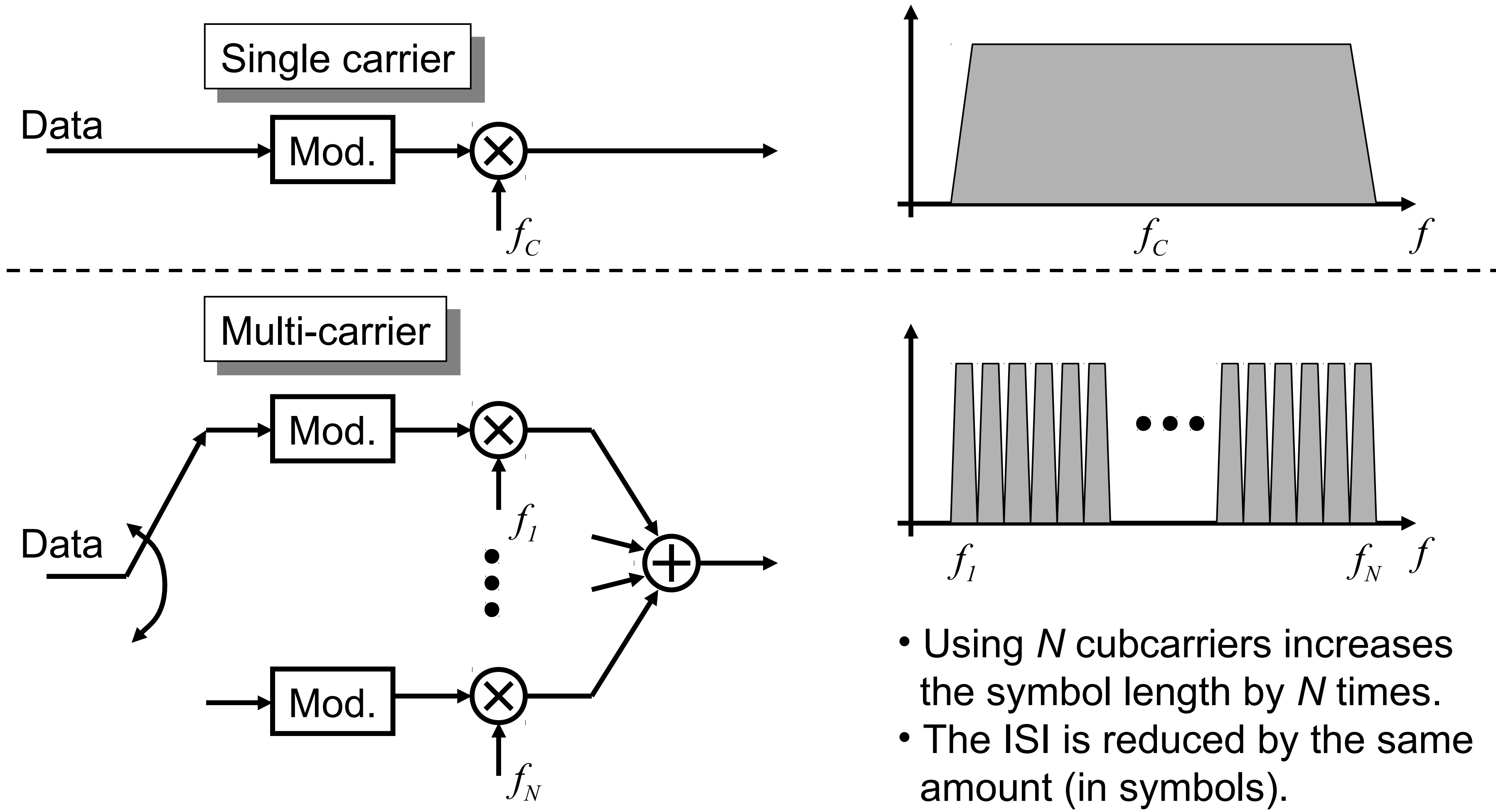
Contents

- Multicarrier systems
  - History of multicarrier
  - Modulation/demodulation
  - Equalization
  - Performance

- Multiple antenna systems
  - Different configuratuons
  - Diversity gains
  - Datarates using MIMO (capacity)

Multi-carrier
or
OFDM – orthogonal frequency-division multiplexing

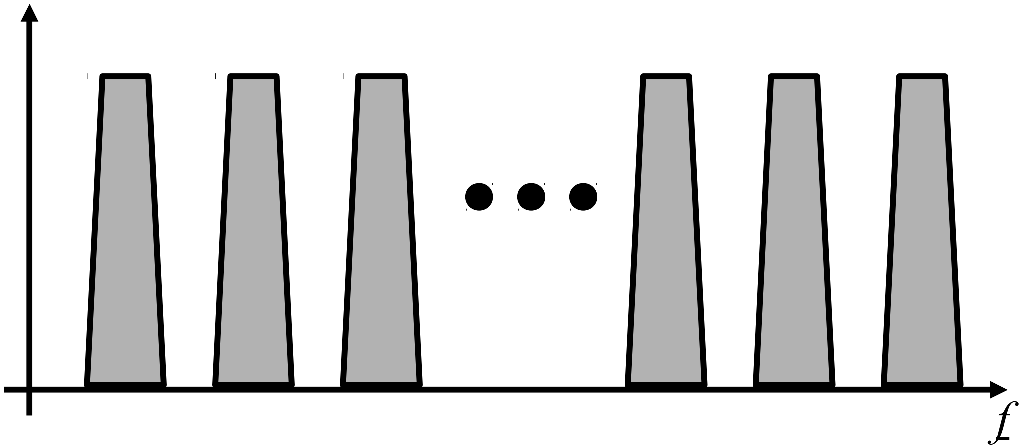
Single/multi-carrier



- Using N cubcarriers increases the symbol length by N times.
- The ISI is reduced by the same amount (in symbols).

History and evolution [1]

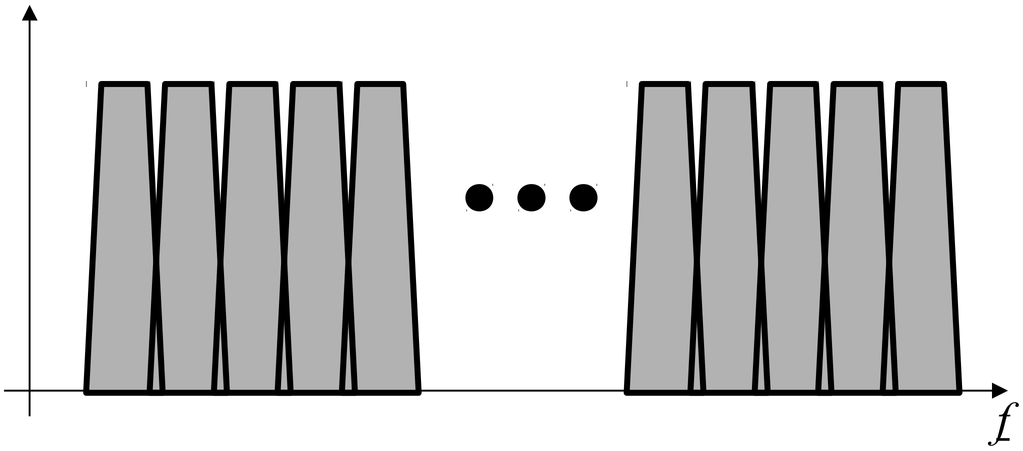
1950's: Few subcarriers, with non-overlapping spectra



- Military systems, e.g. the Kineplex-modem

History and evolution [2]

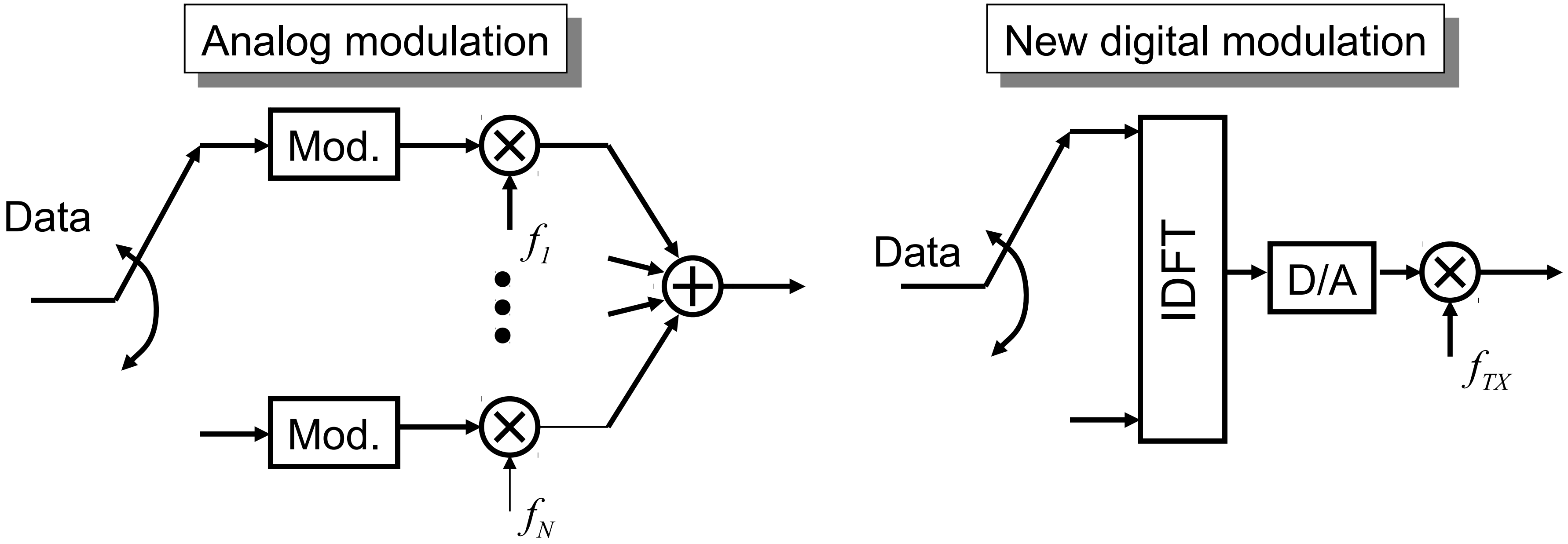
1960's: Subcarriers with overlapping spectra



Increased subchannel density and increased data rate.

History and evolution [3]

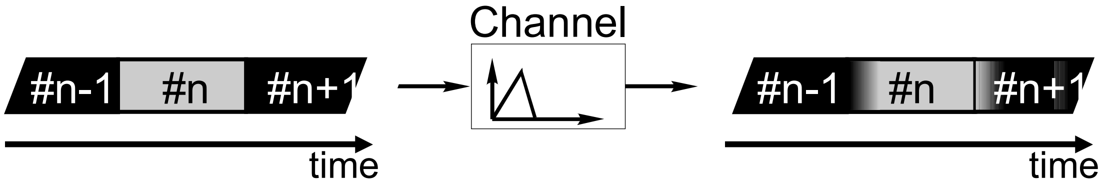
1970's: Digital modulation of subcarriers



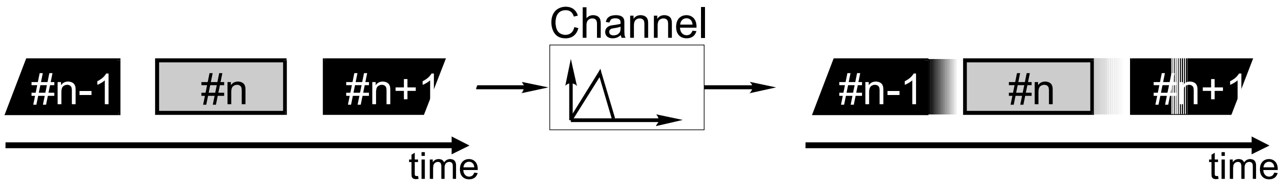
History and evolution [4]

1980's: Improved digital circuits increses interest

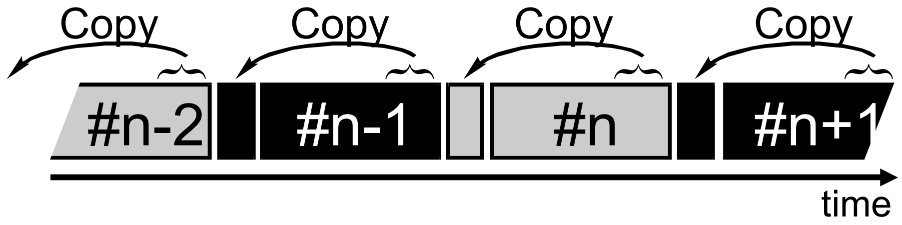
No guard interval => Interference between both subcarriers and symbols



Guard interval => No interference between symbols



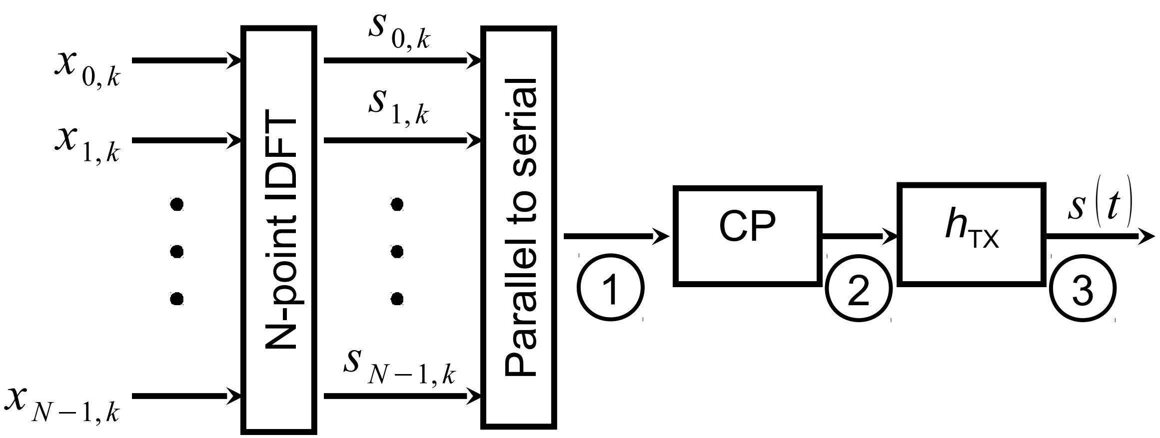
Cyclic prefix => No interference between neither subcarriers nor symbols



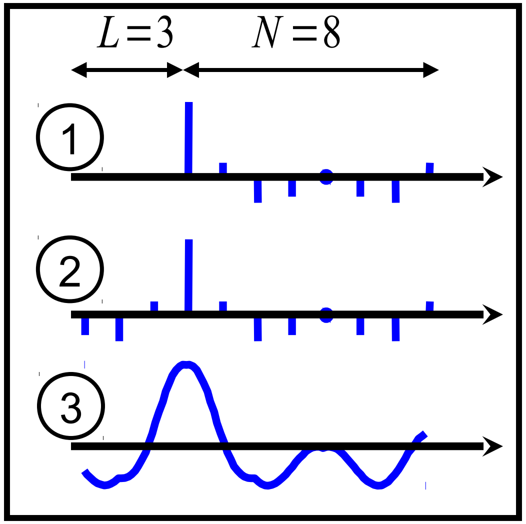
History and evolution [5]

- **1990's**: Commercial applications appear
  - Increased interest for OFDM in wireless applications
  - First applications in broadcasting (Audio/Video)
  - One of the candidates for UMTS (Beta proposal)
  - Applied in wireless LANs
- **2000's**: One of the really hot technologies
  - 54 Mbps and beyond WLANs (based on OFDM) hit the mass market (IEEE802.11g/n)
  - OFDM is the technology used when improving and moving beyond current 3G systems (LTE – long term evolution)

Transmitters and receivers
An N-subcarrier transmitter



k – symbol
m – sample
n – subcarrier
L – CP length
T_{samp} – sampling period
h_{TX} – TX filter

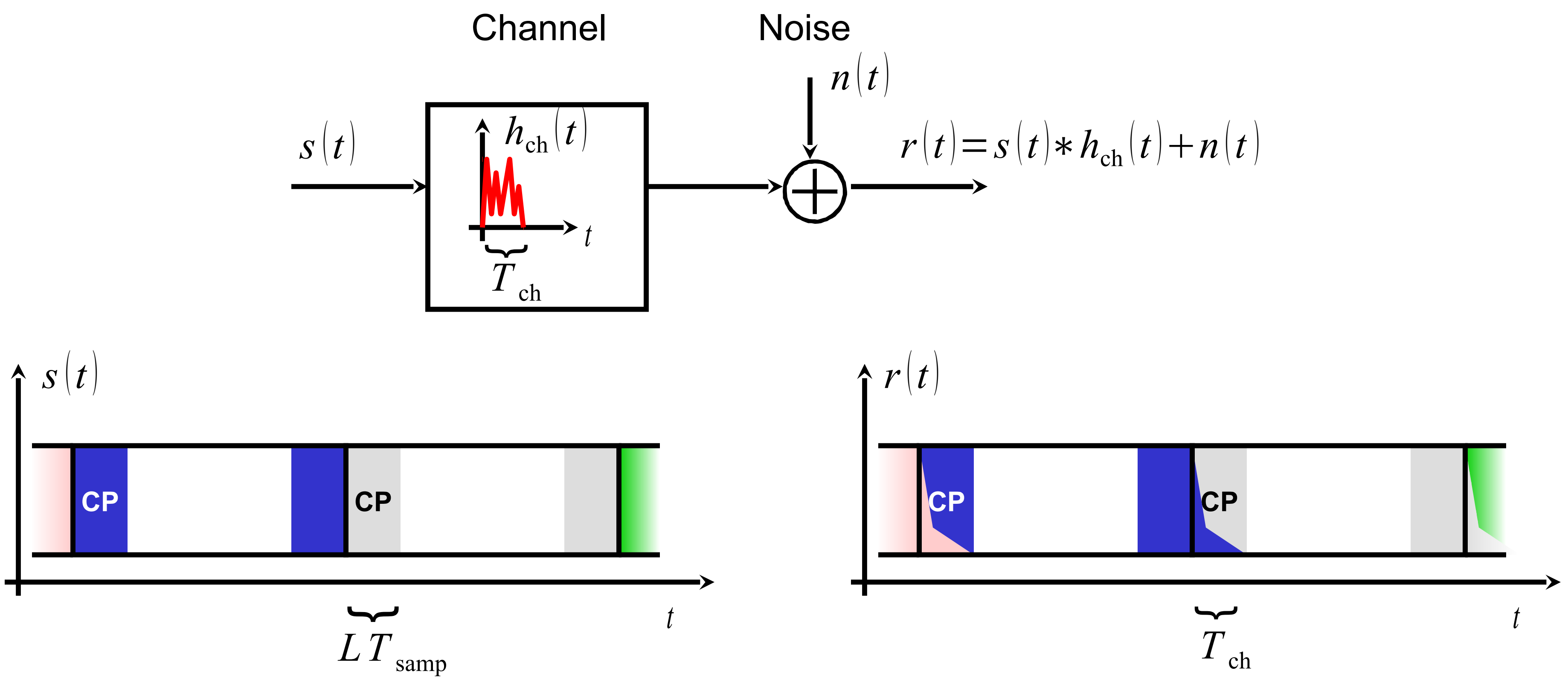


N-point IDFT: $s_{m,k} = \frac{1}{N}\sum_{n=0}^{N-1} x_{n,k}\exp\left(j2\pi\frac{mn}{N}\right)$ for $0 \le m \le N-1$

Adding CP: $s_{m,k} = s_{N+m,k}$ for $-L \le m \le -1$

TX filtering: $s(t) = h_{TX}(t) * \left(\sum_{k}\sum_{m=-L}^{N-1} s_{m,k}\,\delta\left(t-\left(k(N+L)+m\right)T_{samp}\right)\right)$

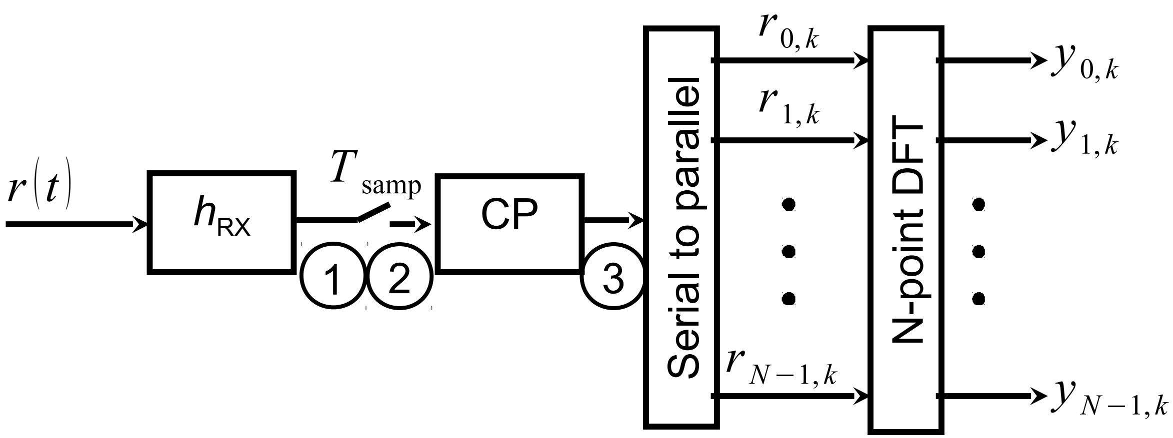
Transmitters and receivers … through the channel …



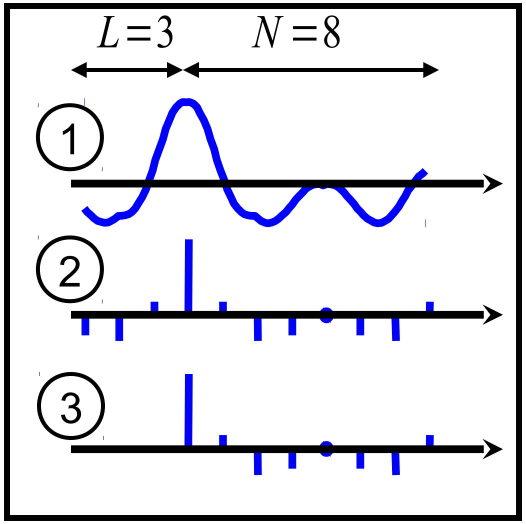
As long as the CP is longer than the delay spread of the channel, $LT_{samp} > T_{ch}$, it will absorb the ISI.

By removing the CP in the receiver, the transmission becomes ISI free.

Transmitters and receivers
N-subcarrier receiver



q – symbol
p – sample
n – subcarrier
L – CP length
T_{samp} – sampling period
h_{RX} – RX filter



RX filtering: $\tilde{z}(t) = h_{RX}(t) * r(t)$

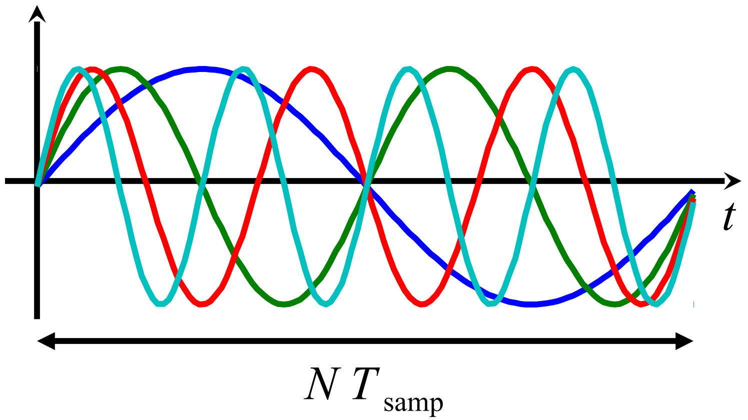
Sampling: $z_k = \tilde{z}(k\,T_{samp})$

Removing CP: $r_{p,q} = z_{q(N+L)+p}$ for $0 \leq p \leq N-1$

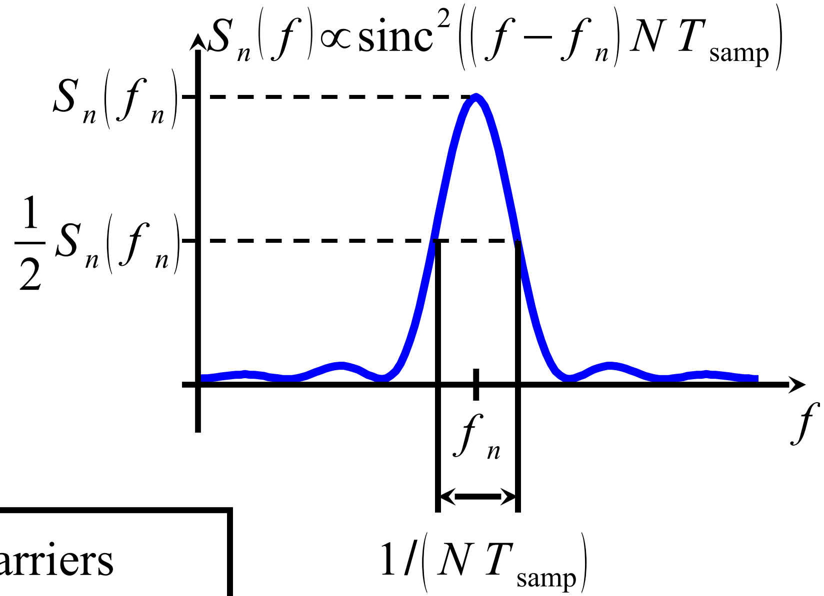
N-point DFT: $y_{n,q} = \sum_{p=0}^{N-1} r_{p,q} \exp\left(-j2\pi \frac{np}{N}\right)$ for $0 \leq n \leq N-1$

Transmitters and receivers Modulation spectrum [1]

Transmitted OFDM symbol decomposed into different subcarriers (ideal case, 4 subcarriers shown, no CP)



Power spectrum of one subcarrier transmitted at f_n Hz.

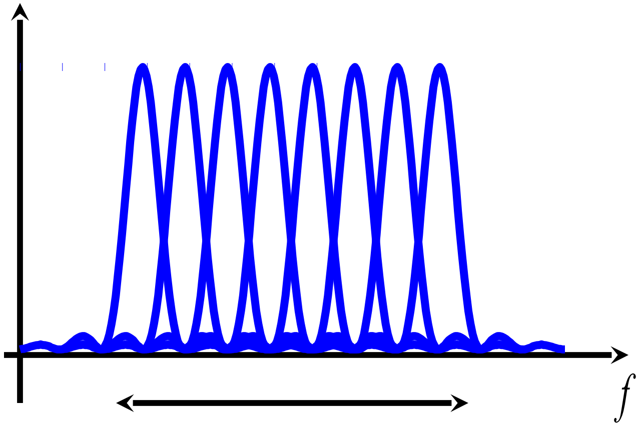


N	-	Subcarriers
T_{samp}	-	Sampling period

$$\mathrm{sinc}(x) = \frac{\sin \pi x}{\pi x}$$

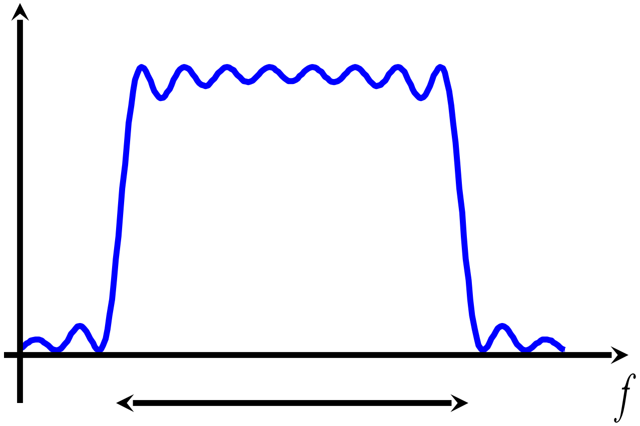
Transmitters and receivers Modulation spectrum [2]

The distance between each subcarrier becomes $1/(N\,T_{\text{samp}})$ which is the same as the 3 dB bandwidth of the individual subcarriers. Using all *N* subcarriers (8 in this case) we get:



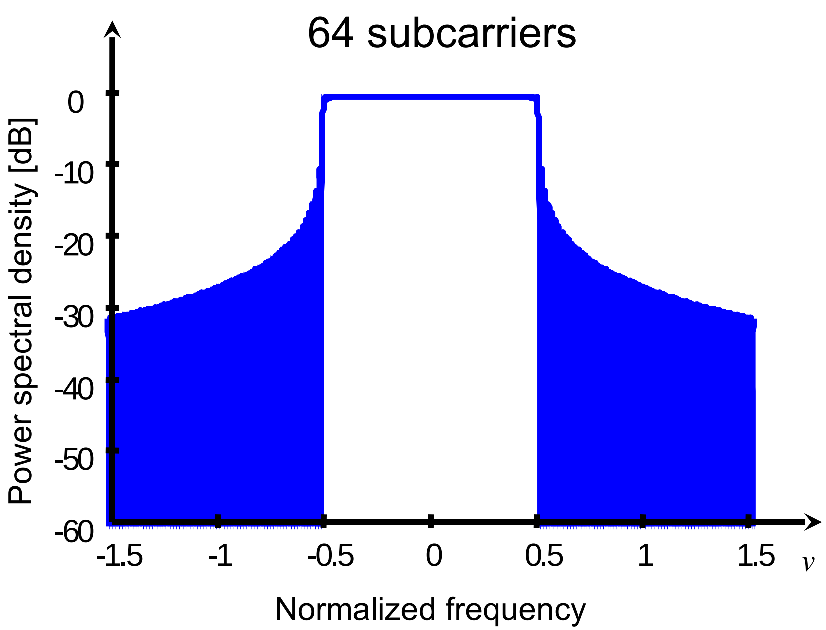
$$B = N \times 1/(N\,T_{\text{samp}}) = 1/T_{\text{samp}}$$

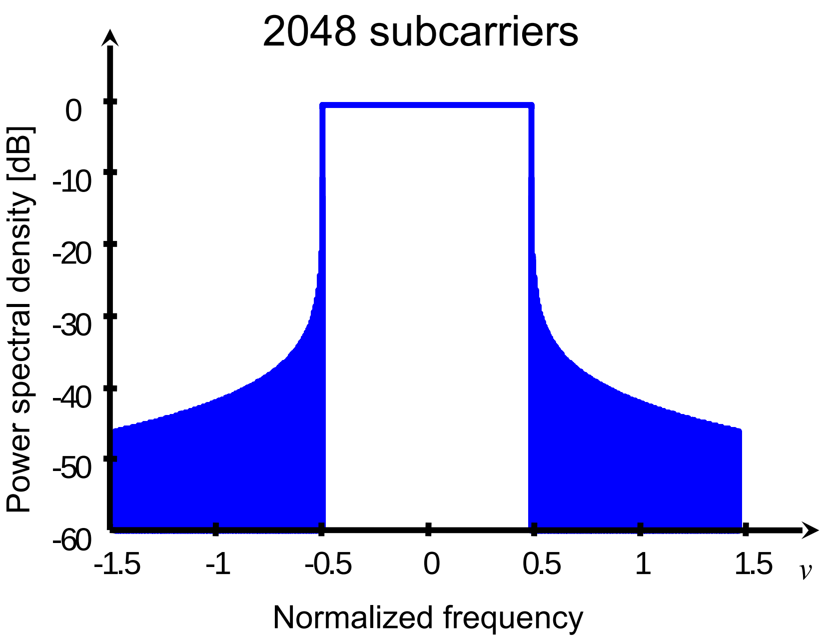
The total modulation spectrum is a sum of the individual subcarrier spectra (assuming independent data on them).



$$B = 1/T_{\text{samp}}$$

Transmitters and receivers
Modulation spectrum [3]



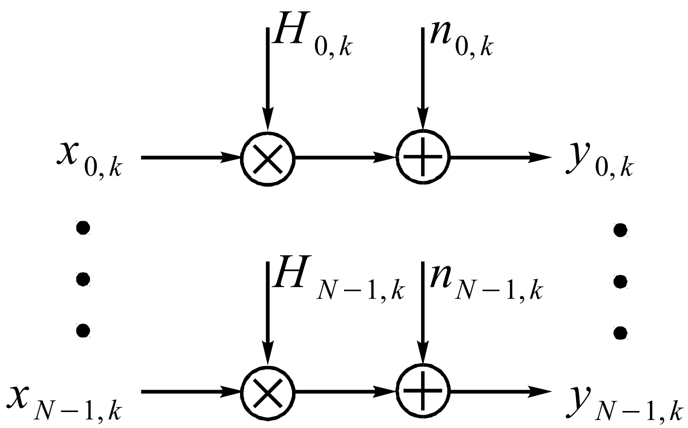


Normalized freq.:

$$\nu = T_{\text{samp}} f = f / B$$

Transmitters and receivers
Simplified model

Simplified model under ideal conditions
(no fading and sufficient CP)



Total filter in the signal path:

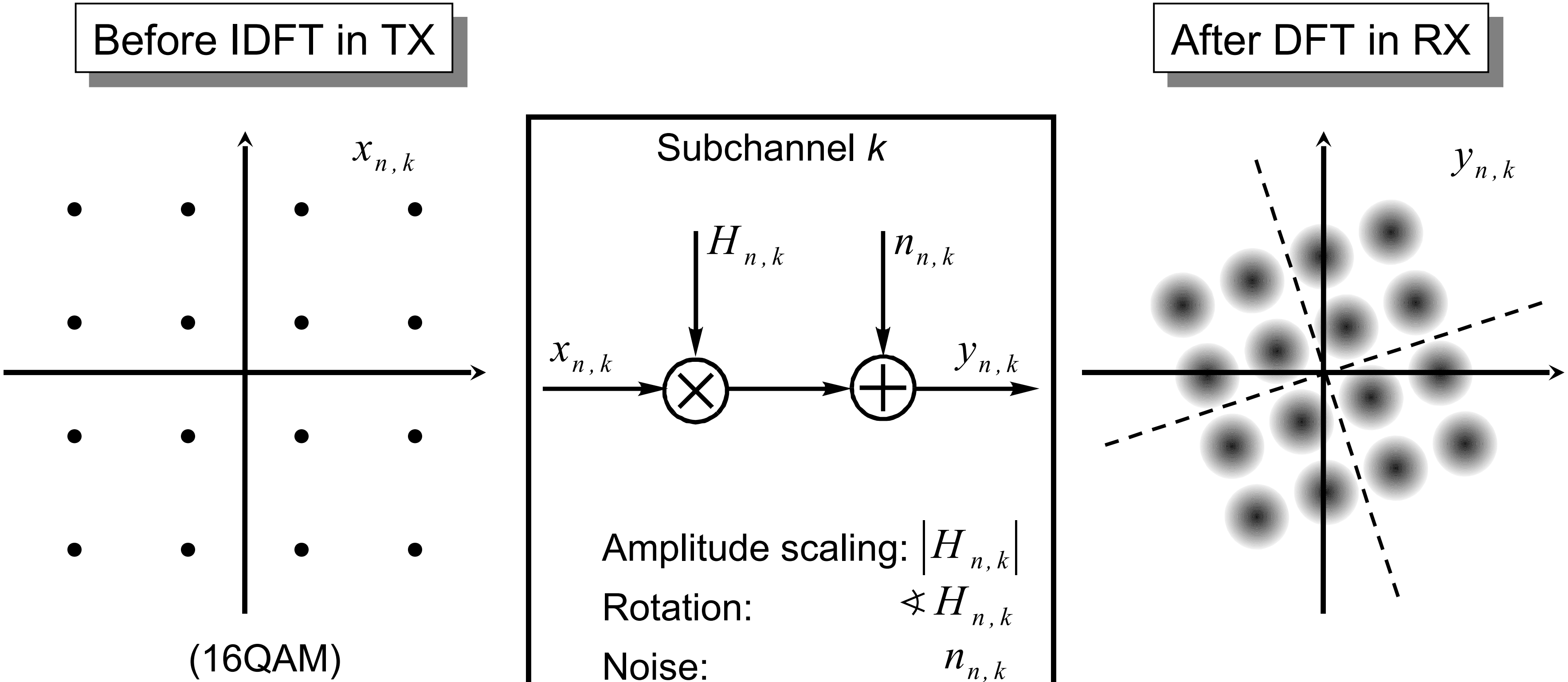
$$h_{\text{tot}}(t) = h_{\text{TX}}(t) * h_{\text{ch}}(t) * h_{\text{RX}}(t)$$

$$H_{\text{tot}}(f) = H_{\text{TX}}(f) \times H_{\text{ch}}(f) \times H_{\text{RX}}(f)$$

Given that subcarrier *n* is transmitted at frequency f_n the attenuations become:

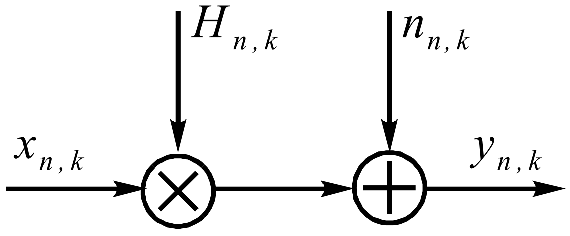
$$H_{n,k} = H_{\text{tot}}(f_n)$$

Transmitters and receivers
Focus on one subchannel



- Simple equalization of each subchannel: Back-rotate and scale

Coded OFDM (CODFM) Uncoded performance



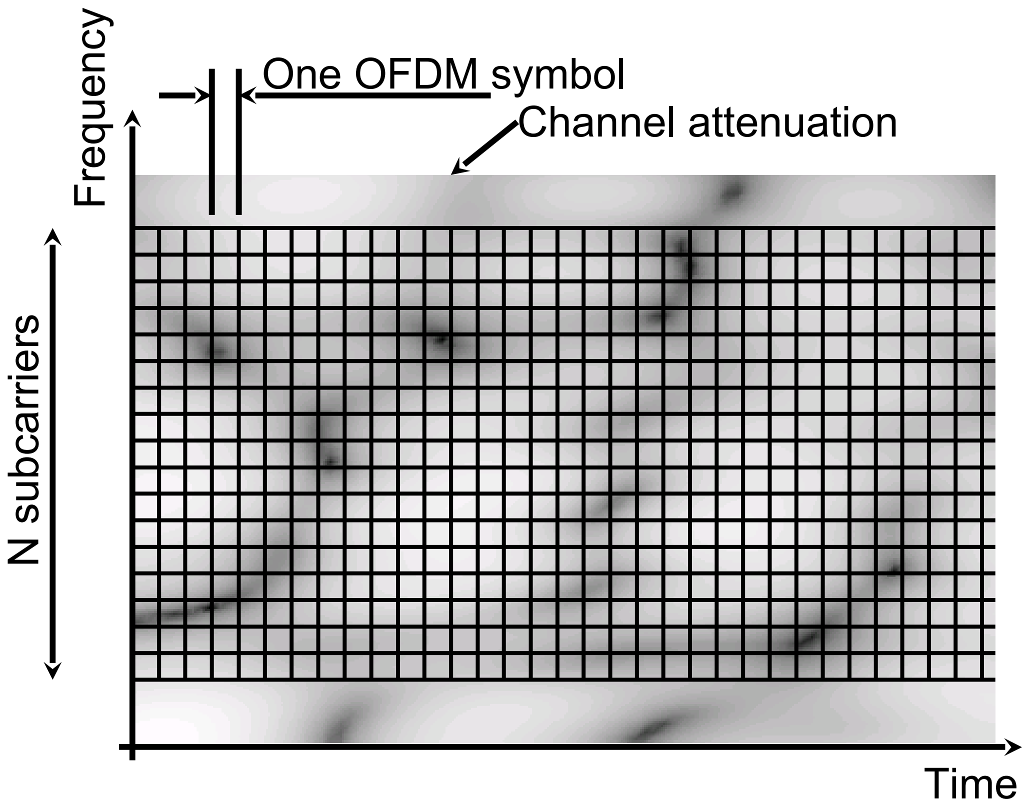
PROBLEM:

- Only one fading tap per subchannel => NO DIVERSITY => POOR PERFORMANCE
- The diversity is in there ... but additional techniques are needed to exploit it!

SOLUTION:

- Spreading the information (data) across several subcarriers or OFDM symbols
- This can be done using interleaving and coding => **Coded OFDM (CODFM)**

Coded OFDM (CODFM)
Channel correlation

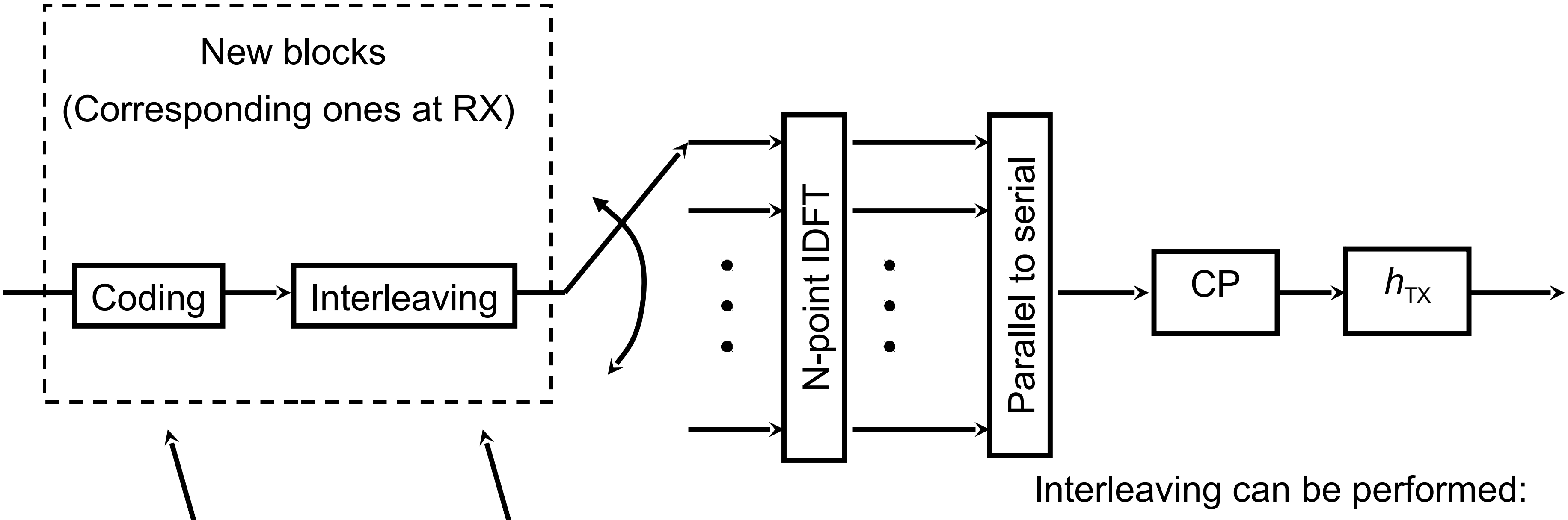


Channel attenuations are correlated in the time/frequency grid.

If we spread each bit of information over several well separated points in the OFDM time/frequency grid, the same ”bit” is is received over several ”one tap” fading channels.

Combining these in the receiver, we obtain diversity.

Coded OFDM (CODFM)
Coding and interleaving



The code spreads the information across several code symbols.

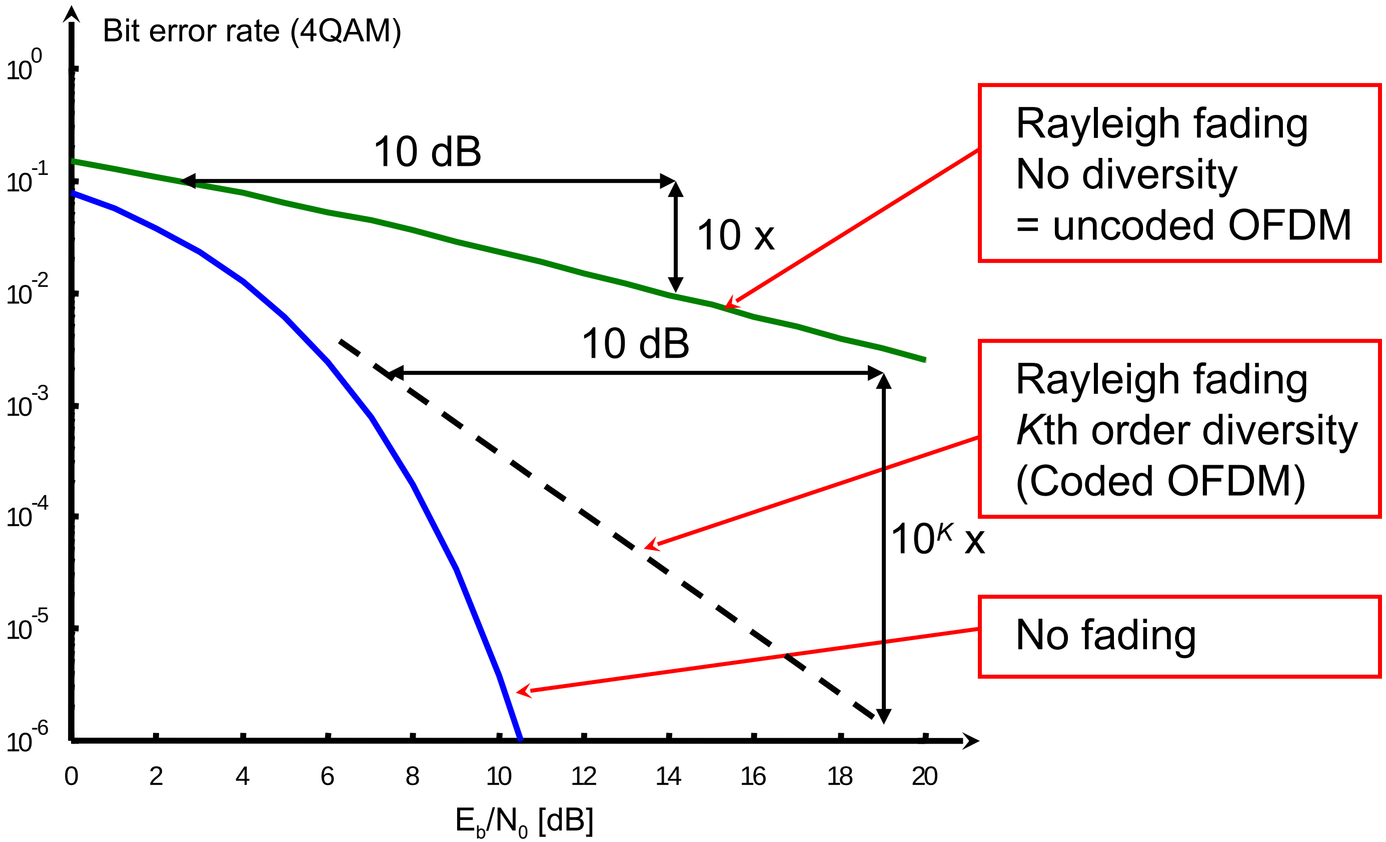
The interleaver reorders the code symbols so that neighbouring code symbols are ”well” separated in frequency and/or time during transmission.

Interleaving can be performed:

- across subcarriers in an OFDM symbol (small delay)
- in time over several OFDM symbols (longer delay)
- or in a combination of the above.

Coded OFDM (CODFM) Diversity

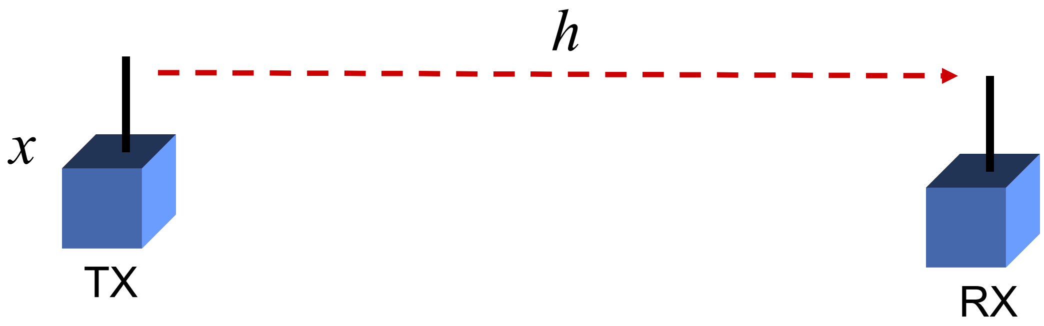
The better the coding and interleaving scheme, the larger the obtained diversity order.



Multiple antenna systems
or
MIMO – multiple input/multiple output

System model [2]

A simple model: Superposition of received waves
[Movement -> fading]



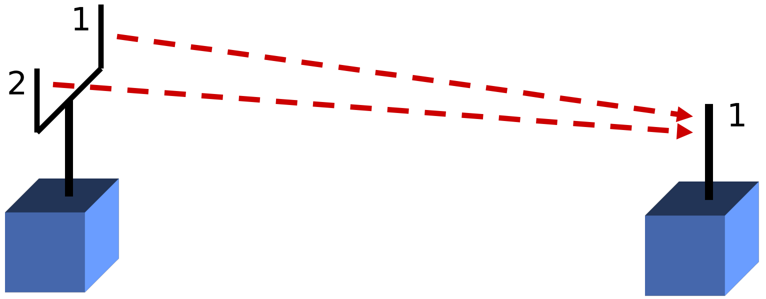
No diversity (SISO):

$$y = hx + n$$

Fading -> Poor performance

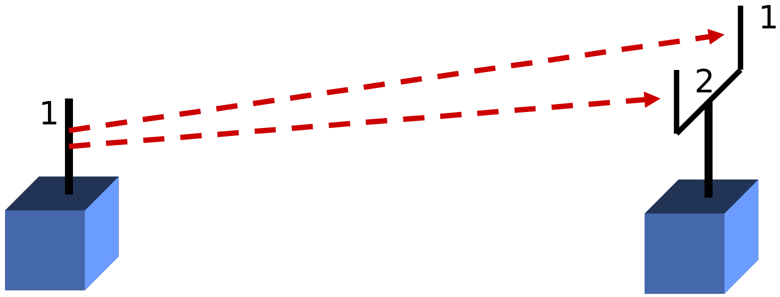
System model [3]

An improvement: Antenna diversity



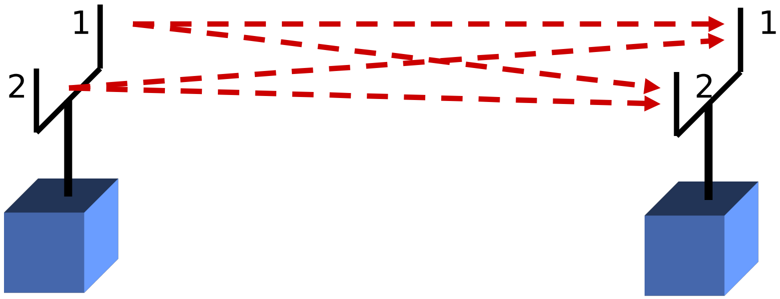
TX diversity (MISO):

$$y_1 = \begin{bmatrix} h_{1,1} & h_{1,2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n_1$$



RX diversity (SIMO):

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{1,1} \\ h_{2,1} \end{bmatrix} x_1 + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

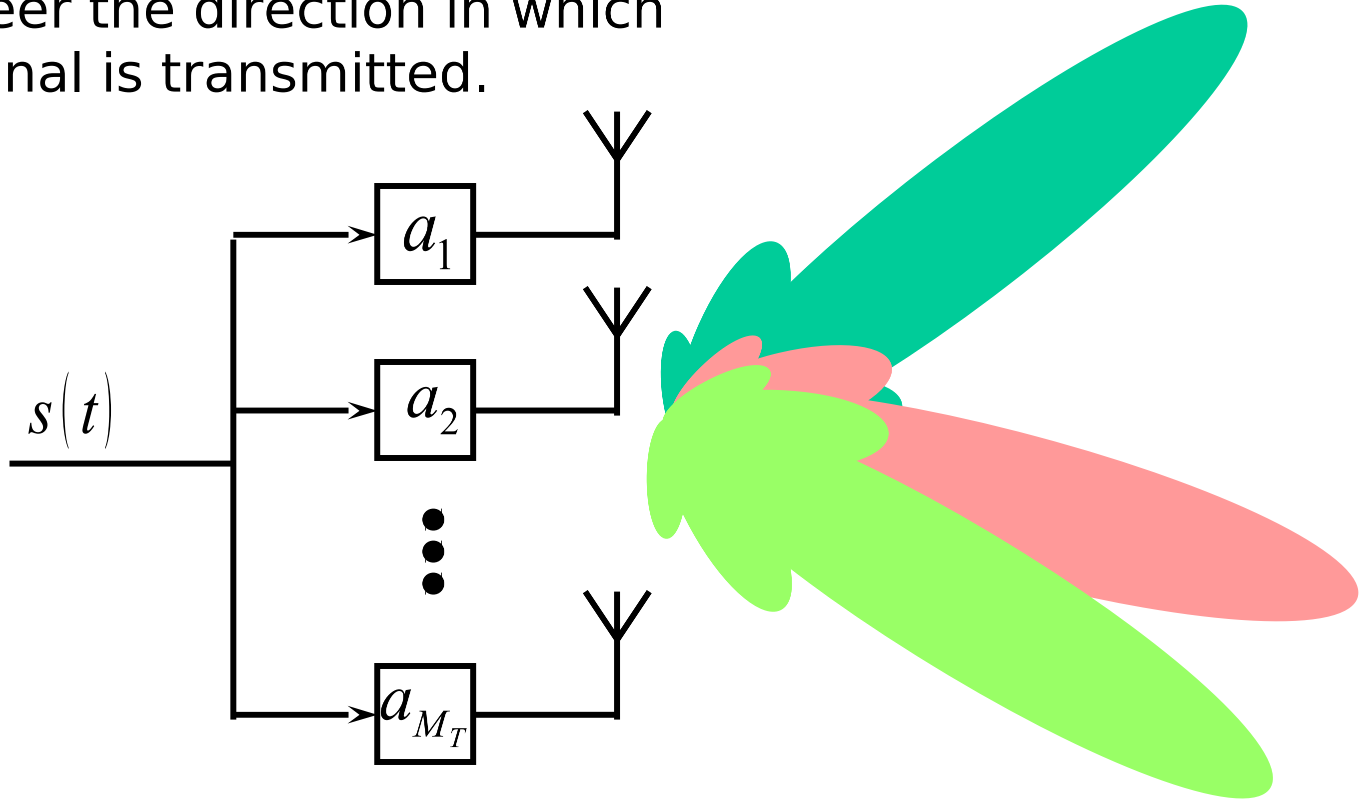


TX&RX diversity (MIMO):

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

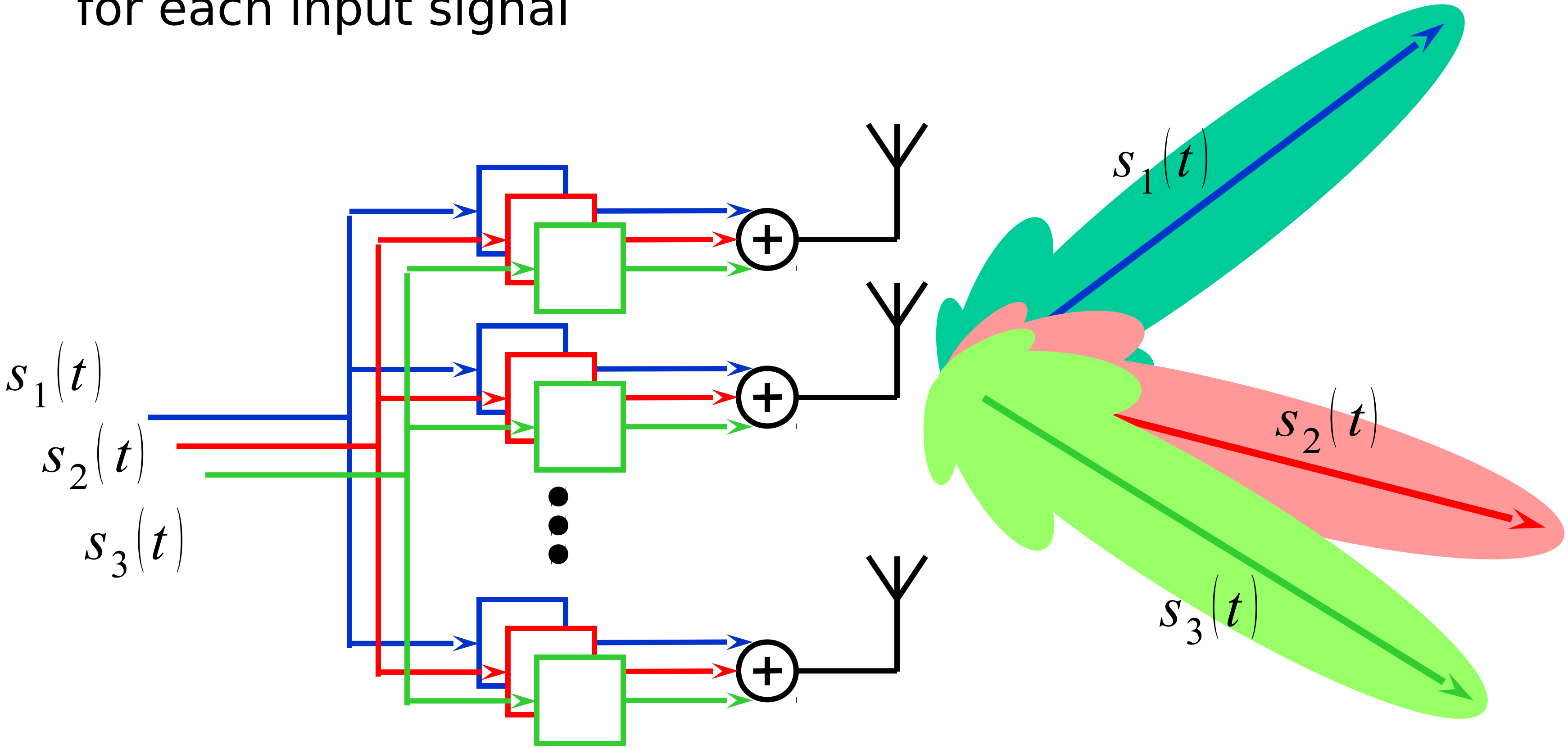
Lobe-forming at transmitter

The lobe forming coefficients can steer the direction in which the signal is transmitted.

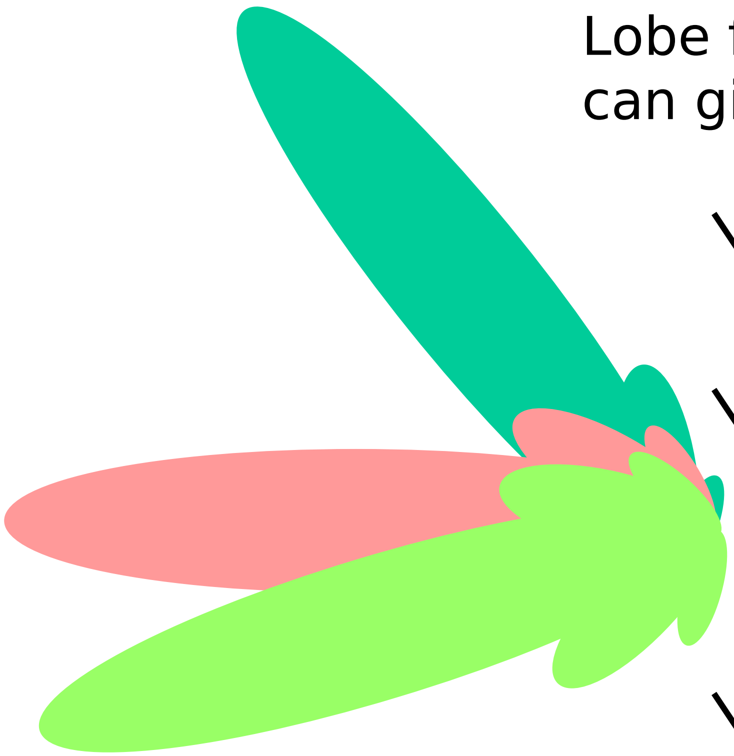


Several input signals

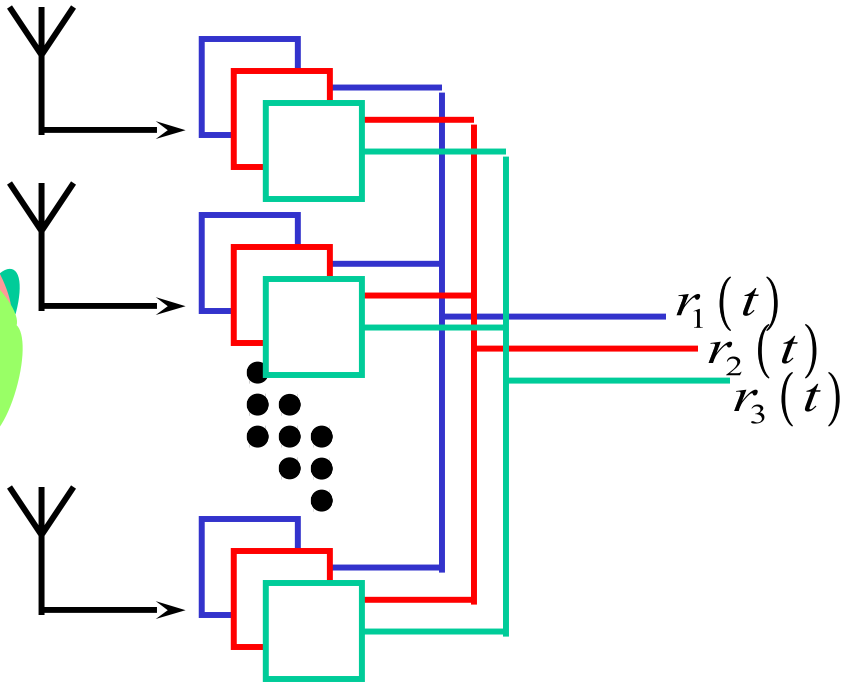
One set of lobe forming coefficients
for each input signal



Several output signals

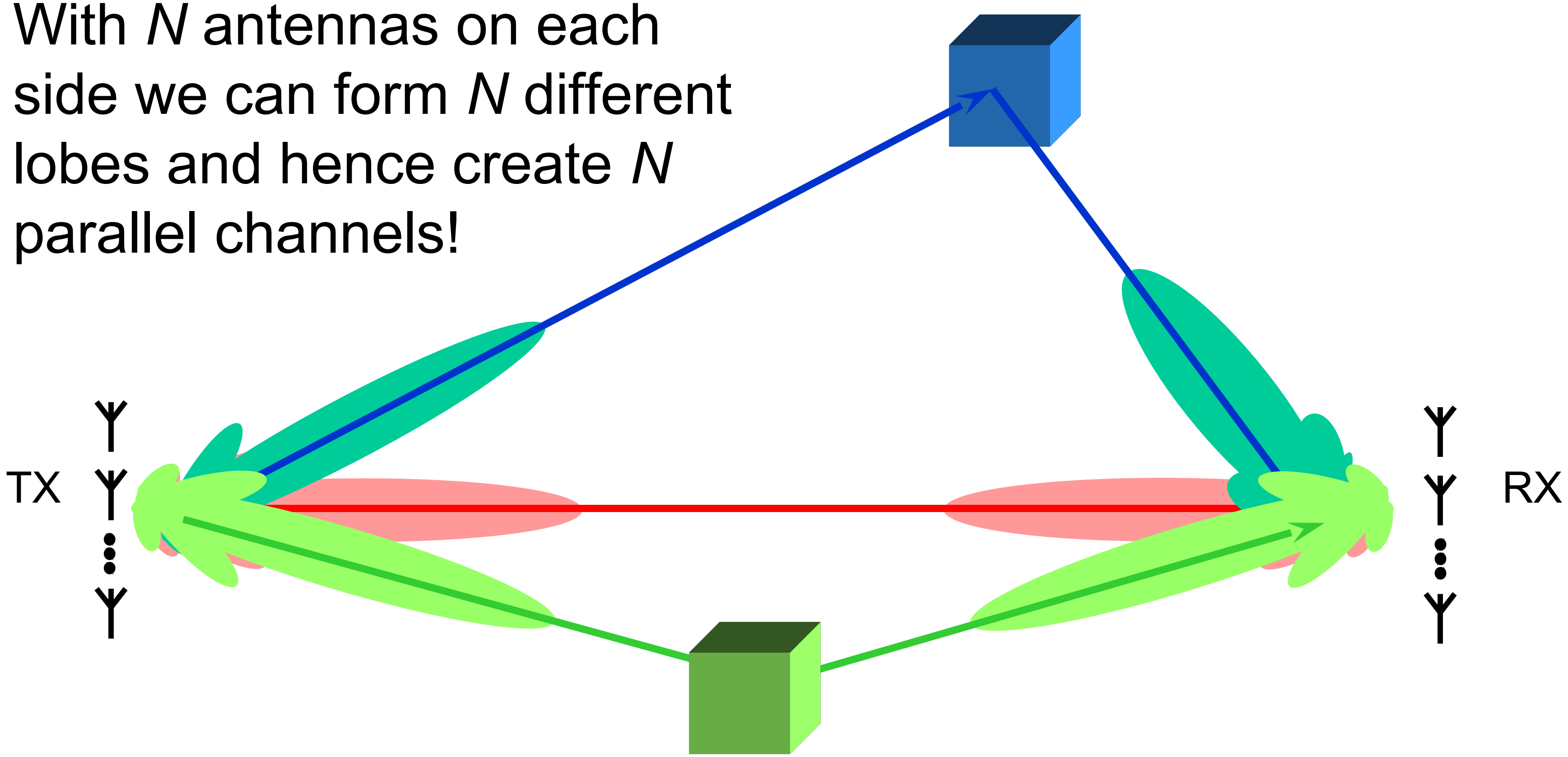


Lobe forming on the receiver side can give several output signals.



Multiple antennas at both ends

With *N* antennas on each side we can form *N* different lobes and hence create *N* parallel channels!



Note that the three channels are separated spatially and can therefore use the same bandwidth! We have ”trippled” the channel capacity.

A general (narrow-band) model

The ”general” case with M_T TX antennas and M_R RX antennas:

$$\mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_{M_R} \end{bmatrix} = \begin{bmatrix} h_{1,1} & h_{1,2} & \cdots & h_{1,M_T} \\ h_{2,1} & h_{2,2} & \cdots & h_{2,M_T} \\ \vdots & \vdots & \ddots & \vdots \\ h_{M_R,1} & h_{M_R,2} & \cdots & h_{M_R,M_T} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{M_T} \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_{M_R} \end{bmatrix} = \mathbf{Hx} + \mathbf{n}$$

Some fundamental questions:

- How do we model the channel matrix **H**?
- How do we model the noise (interference) **n**?

We will see that these have a large impact on what we can obtain.

What started the interest in MIMO?

J.H. Winters. **On the Capacity of Radio Communication Systems with Diversity in Rayleigh Fading Environment**. IEEE JSAC, vol. SAC-5, no. 5, June 1987.

Model

Equal number of RX and TX antennas, $M_T = M_R = M$.

H Independent Rayleigh fading. [i.i.d. complex Gaussian variables].

n I.i.d complex Gaussian variables.

Findings

Linear processing *at receiver:*
Up to ***M* /2 channels**, each with the same data rate as a single channel.

Non-linear processing *at receiver:*
Up to ***M* channels**, each with the same data rate as a single channel.

Capacity – No fading & AWGN [1]

Singular value decomposition of the (fixed) channel **H**:

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} = \mathbf{Q}_1 \boldsymbol{\Sigma} \mathbf{Q}_2^H \mathbf{x} + \mathbf{n}$$

where $\mathbf{Q}_1$ (M_RxM_R) and $\mathbf{Q}_2$ (M_TxM_T) are unitary matrices and $\boldsymbol{\Sigma}$ (M_RxM_T) is a matrix containing the singular values on its diagonal.

Multiply by $\mathbf{Q}_1^H$ from left:

$$\underbrace{\mathbf{Q}_1^H \mathbf{y}}_{\tilde{\mathbf{y}}} = \boldsymbol{\Sigma} \underbrace{\mathbf{Q}_2^H \mathbf{x}}_{\tilde{\mathbf{x}}} + \underbrace{\mathbf{Q}_1^H \mathbf{n}}_{\tilde{\mathbf{n}}} \quad \Rightarrow \quad \tilde{\mathbf{y}} = \boldsymbol{\Sigma} \tilde{\mathbf{x}} + \tilde{\mathbf{n}}$$

Only "rotations" of **y**, **x** and **n**.

All-zero, exept diagonal.

Capacity – No fading & AWGN [2]

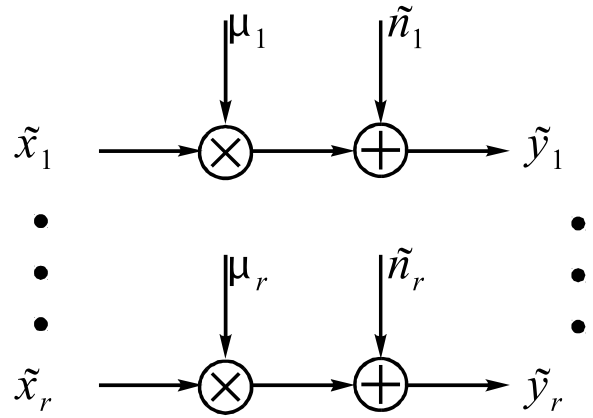
What have we obtained?

Parallel independent channels:

$$\tilde{\mathbf{y}} = \begin{bmatrix} \mu_1 & & & & \\ & \ddots & & & \\ & & \mu_r & & \\ & & & 0 & \\ & & & & \ddots \end{bmatrix} \tilde{\mathbf{x}} + \tilde{\mathbf{n}}$$

Number of non-zero singular values r = rank(**H**).

Shannon's ”standard case”:



(+ channels with $\mu_k = 0$)

Capacity – No fading & AWGN [3]

Shannon: The total capacity of parallel independent channels is the sum of their individual capacities.

$$C_k = \log_2\left(1 + \mathrm{SNR}_k\right)$$

$$C = \sum_k C_k = \sum_k \log_2\left(1 + \mathrm{SNR}_k\right)$$

Equal power distribution (channel not known at TX):

Constant dep. on e.g. TX power and noise.

$$C = \sum_k C_k = \sum_k \log_2\left(1 + \alpha \mu_k^2\right) = \boxed{\log_2 \prod_{k=1}^{r} \left(1 + \alpha \mu_k^2\right)}$$

Capacity – No fading & AWGN [4]

A neat trick:

$$\det\left(\mathbf{I}_{M_R}+\alpha\,\mathbf{H}\mathbf{H}^H\right)=\det\left(\underbrace{\mathbf{Q}_1\mathbf{Q}_1^H}_{\mathbf{I}_{M_R}}+\alpha\,\underbrace{\mathbf{Q}_1\boldsymbol{\Sigma}\mathbf{Q}_2^H}_{\mathbf{H}}\,\underbrace{\mathbf{Q}_2\boldsymbol{\Sigma}^H\mathbf{Q}_1^H}_{\mathbf{H}^H}\right)$$

$$=\det\mathbf{Q}_1\left(\boldsymbol{I}_{M_\mathrm{R}}+\alpha\,\boldsymbol{\Sigma}\,\mathbf{Q}_2^H\mathbf{Q}_2\,\boldsymbol{\Sigma}^H\right)\mathbf{Q}_1^H$$

$$=\det\left(\boldsymbol{I}_{M_\mathrm{R}}+\alpha\,\boldsymbol{\Sigma}\,\boldsymbol{\Sigma}^H\right)$$

$$=\det\begin{bmatrix}1+\alpha\mu_1^2 & & & & & \\ & \ddots & & & & \\ & & 1+\alpha\mu_r^2 & & & \\ & & & 1 & & \\ & & & & \ddots & \\ & & & & & 1\end{bmatrix}=\prod_{k=1}^{r}\left(1+\alpha\mu_k^2\right)$$

Capacity – No fading & AWGN [5]

CONCLUSION:

$$C = \log_2 \prod_{k=1}^{r} \left(1 + \alpha \mu_k^2\right) = \log_2 \det\left(\mathbf{I}_{M_R} + \alpha \mathbf{H}\mathbf{H}^H\right) \text{ [bit/sec/Hz]}$$

Normalization: ρ - SNR at each receiver branch

$$C = \log_2 \det\left(\mathbf{I}_{M_R} + \frac{\rho}{M_T} \mathbf{H}\mathbf{H}^H\right)$$

This leads to the fact that we can increase data rate by increasing the number of antennas, without using more transmit power.

This relation is also derived in *e.g*
G.J. Foschini and M.J. Gans. **On Limits of Wireless Communications in a Fading Environment when Using Multiple Antennas.** Wireless Personal Communications, no 6, pp. 311-335, 1998.

Summary

- Multi-carrier technology (OFDM) **reduces** the effect of **intersymbol interference** (as compared to single carrier).
- Only **simple equalization** is required in an OFDM receiver.
- Modulation/demodulation can be done using **Fast Fourier Transforms** (FFTs).

- Multiple antenna systems increase our ability to obtain **diversity gains**.
- With MIMO systems we can **increase the datarate** by using more antennas, **without increasing transmit power or bandwidth**.