Selected answers*
Problem set 6

Wireless Communications, 2nd Ed.

24.3/21.2[2](the second one) GSM channel correlation across a burst

A time slot in GSM has a length of 156.25 bit-times (577 μs). Of these, 8.25 bit-times are (in the normal case) a guard period. This makes the active burst length about 547 μs.

Isotropic uncorrelated scattering gives a Jakes doppler spectrum and a corresponding time correlation coefficient

$$\rho_t(\Delta t) = J_0\left(2\pi\nu_{\max}\Delta t\right), \quad (1)$$

where J_0 is the zeroth-order Bessel function of the first kind, $\nu_{\max}$ the maximal Doppler shift, and Δt the time delay. At 250 km/h and 1800 MHz carrier (GSM1800), the maximal Doppler shift becomes $\nu_{\max} = 417$ Hz.

The channel correlation coefficient between the **middle and end** (or start) of the burst, $\Delta t = 547/2\ \mu s$, is therefore $\rho_t(\Delta t) = 0.88$.

The channel correlation coefficient between the **start and end** of the burst, $\Delta t = 547\ \mu s$, is $\rho_t(\Delta t) = 0.55$.

The above means that by estimating the channel at the middle of the burst (as it is done in GSM), the estimate will be highly correlated to the channel across the whole burst. Had the GSM system been designed with channel estimation at the beginning of the burst, the channel estimate would be quite outdated by the end of the burst (low correlation to the actual channel) and channel equalization would have suffered a performance loss.

24.10/21.9 Comparison of standard GSM and EDGE modulation

To compare the SNR (E_b/N_0) required for GMSK and 8-PSK in GSM/EDGE, at a BER of 10^{-2}, we need BER graphs or closed form expressions/approximations. For 8-PSK we have an approximation

$$\mathrm{BER}_{\text{8-PSK}} \approx \frac{2}{3} Q\left(\sqrt{0.87\frac{E_b}{N_0}}\right), \quad (1)$$

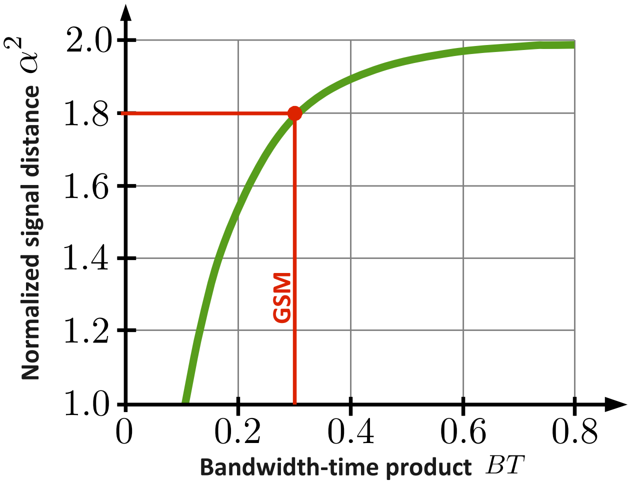
while for GMSK we need to find something similar. The BER for GMSK can be approximated as

$$\mathrm{BER}_{\mathrm{GMSK}} \approx Q\left(\sqrt{\alpha^2\frac{E_b}{N_0}}\right), \quad (2)$$

where α^2 is a normalized (squared) signal distance that depends on the bandwidth-time product of the GMSK modulation, which for GSM is $BT = 0.3$. Murota & Hirade published a graph in 1981 (re-drawn below) where it is possible to find the corresponding α^2:

* Note: Solutions provided here are less detailed than the ones expected during the exam. Many steps are excluded.

2 When there are two exercise numbers, the first one is pointing to the most recent textbook (2[nd] Ed), while the second one is pointing to the previous edition of the textbook (1[st] Ed).



We see that in GSM we have $\alpha^2 \approx 1.8$ and the BER approximation becomes[3]

$$\mathrm{BER_{GMSK}} \approx Q\left(\sqrt{1.8\frac{E_b}{N_0}}\right). \tag{3}$$

Solving for E_b/N_0 when the BER is 10^{-2} gives that

- GMSK requires $E_b/N_0 \approx 4.8$ dB and
- 8-PSK required $E_b/N_0 \approx 7.3$ dB.

This shows that GMSK is a modulation more "robust" against noise than 8-PSK. On the other hand, 8-PSK has more bits per symbol and can carry higher data rates on the 200 kHz GSM channel.

29.1/24.1 WLAN (802.11a) spectral efficiency

The loss in spectral efficiency due to:

i. *Not all subcarriers carrying data*
Of the 64 subcarriers, only 52 are used and 4 of those contain pilot signals. Hence, only 48 of 64 subcarriers are used for data transmission. Hence, the loss is $(64-48)/64 = 25\%$.

ii. *Cyclic prefix*
The duration of an OFDM symbol is 4 μs, including a cyclic-prefix of length 0.8 μs, which is only used to "absorb" ISI from the previous symbol. Hence, the loss is $0.8/4 = 20\%$.

iii. *Training sequence and signaling field (if 16 OFDM symbols [containing data] are transmitted)*
At the beginning of the transmission, 2 training sequence fields of 8 μs each are transmitted, for the purpose of synchronization and channel estimation. plus a 4 μs OFDM symbol with signaling information. This constitutes a "startup" of 2 x 8 + 4 = 20 μs, before the actual data transmission of 16 OFDM symbols of duration 4 μs (total 64 μs) takes place. Hence, the loss is $20/(64+20) = 24\%$.

The total loss in spectral efficiency, if all three effects are combined, is about 54%. It should, however, be noted that a lot of overhead is usually required in all wireless systems only to synchronize the transmitter and receiver sides.

3 If we don't have any filtering of the phase ($BT \to \infty$), GMSK becomes MSK ($\alpha^2 = 2$ in the graph) and performance becomes the same as QPSK. For this comparison that would have been an acceptable approximation too, with less than 0.5 dB error.

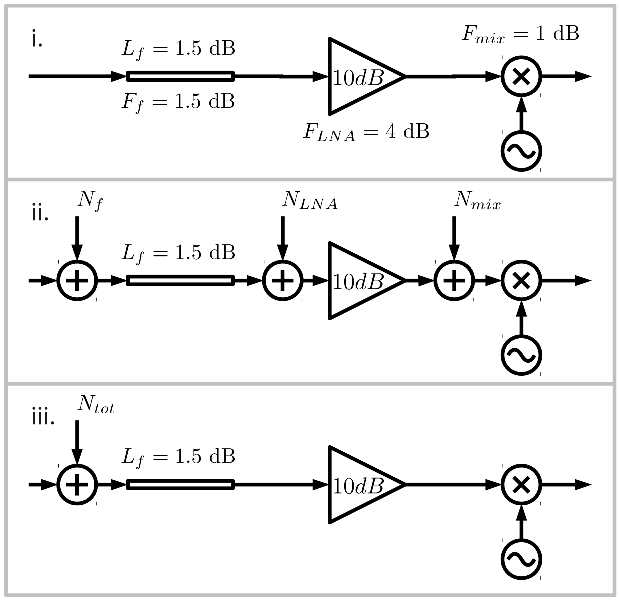
29.2/24.2

The maximum data-rates of 802.11, 802.11b, and 802.11a are 2, 11, and 54 Mbit/sec, respectively.

Some oldies

3.1 Total noise figure

To calculate the total noise figure of the described receiver system, we do the calculations in three steps.



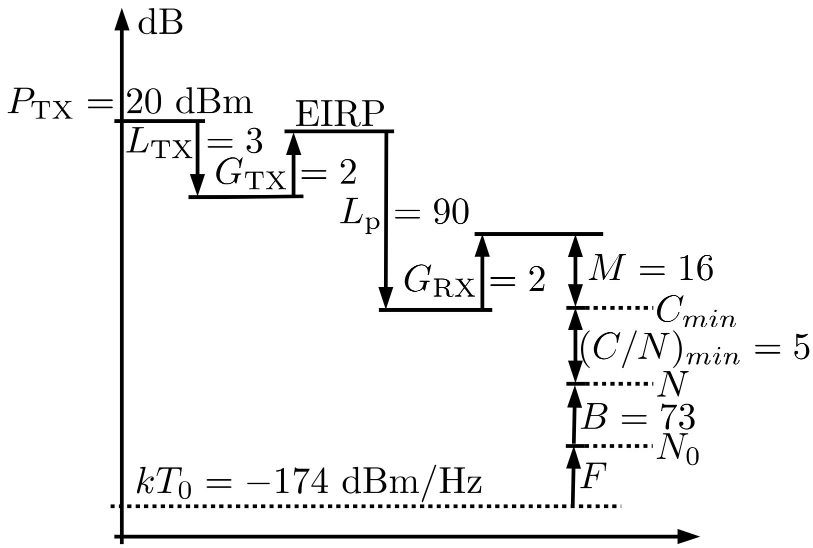
i. First we prepare for the calculations by converting to non-dB $L_f = F_f = 1.41$, $F_{LNA} = 2.51$, $F_{mix} = 1.26$, and the LNA gain $G = 10$.

ii. Then we calculate the noise power spectral densities of the noise sources $N_f = k(F_f - 1)T_0 = 0.41kT_0$, $N_{LNA} = 1.51kT_0$, and $N_{mix} = 0.26kT_0$.

iii. Finally we move all noise sources in front of the entire chain $N_{tot} = N_f + N_{LNA}L_f + N_{mix}L_f/G = 2.58kT_0$ and recognize that the noise factor of the noise source is 2.58 and therefore the **noise factor/figure of the entire system** (it has an input, since no antenna is connected) is 3.58 or **5.54 dB**.

3.4 WLAN admissible RF noise

Using the given WLAN system parameters, we can make the following link budget:

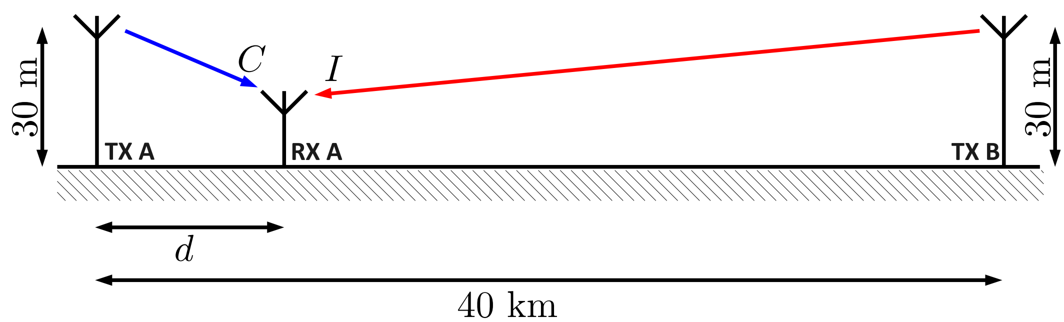


The link budget gives us the relationship

$$P_{TX} - L_{TX} + G_{TX} - L_p + G_{RX} - M - (C/N)_{min} - B - F = kT_0, \qquad (1)$$

which, with the numerical values substituted, leads to an **admissible receiver RF noise figure** of $F = 11$ dB.

5.11 Interference limited system with large-scale fading



The geometry of the system we are studying is shown above. Many unnecessary parameters are given in the problem formulation. The only parameters we need (except for the distance 40 km) are the propagation exponent $\eta = 3.6$, the log-normal fading $\sigma_F = 9$ dB, the propagation exponent $\eta = 3.6$, and the required $(C/I)_{min} = 7$ dB. Antenna heights and the carrier frequency are not necessary and, since both transmitters have equal antennas, the antenna types/gains are also irrelevant.

a) The fading margin needed to provide 99% accessibility, i.e. 99% propbability that $(C/I) \geq (C/I)_{min})$, is given by

$$Q\left(\frac{M}{\sqrt{2}\times 9}\right) = 0.01 \qquad (1)$$

where $\sqrt{2} \times 9 = \sqrt{9^2 + 9^2}$ adding two independent Gaussian variables[4], each with standard deviation 9 dB. This gives a **required fading margin** $M = 29.6$ dB.

b) The (C/I) we can tolerate is determined by the minimal value $(C/I)_{min}$ and the margin M

4 $(C/I) = C - I$ in the dB domain. Hence we add two Gaussian variables when calculating (C/I).

we need to protect against fading. We therefore need to fulfill

$$\left(\frac{C}{I}\right) = \left(\frac{d}{40-d}\right)^{-\eta} \geq \left(\frac{C}{I}\right)_{min} + M\,. \tag{2}$$

With the numerical values inserted we get a **maximal distance** at which RX A can be from TX A, as

$$d \leq d_{max} = 3.51 \text{ km}. \tag{3}$$

c) If the distance between TX A and TX B is reduced by a factor two, to 20 km, the maximal distance d_{max} will also be reduced by a factor two, to 1.75 km. This is easily seen by observing that

$$\frac{d}{40-d} = \frac{d/2}{(40-d)/2} = \frac{d/2}{20-d/2} \tag{4}$$

in expression (2).