Selected answers*
Problem set 5

Wireless Communications, 2nd Ed.

17.1 Cellular system capacity

a) The busy-hour traffic generated by each subscriber is $2/60 = 1/30$ Erlang.With 250 duplex channels, $D/R \leq 7$, and blocking limited to 3%, we get a cluster size $N_{cl} = 19$ and 13 trunk channels per cell, and each cell can therefore offer about 9 Erlang of traffic.

   i. This is enough to support $9/(1/30) = 270$ **subscribers per cell**.

   ii. In a cell of area $\pi 2^2 = 12.6$ km^2, we get **system capacity** of 0.72 Erlang/km^2.

b) Reducing the requirement on the re-use ratio to $D/R \geq 4$, we get a cluster size $N_{cl} = 7$. This, together with the new lower number of duplex channels, 125, gives us 17 trunk channels per cell. With the same requirement on blocking, each cell can now offer about 11 Erlang. This results in a **system capacity** of 0.87 Erlang/km^2.

c) Reducing the cell radius to 1 km, each cell becomes four times smaller and the **system capacity** increases accordingly, to about 3.5 Erlang/km^2.

17.4 TDMA guard interval

a) The travel-time of a radio signal, between the cell center and the cell edge, is $3000/3 \times 10^8 = 10^{-5} = 10\ \mu s$. This delay difference, plus the largest channel delay spread of $10\ \mu s$, gives us the required **guard time interval** $20\ \mu s$.

b) In GSM the guard interval can be shortened by applying "**timing advance**", where terminals far from the BS start their transmission a bit earlier, to compensate for the longer travel-time of their signals.

17.5 Cellular system C/I

a) We want to bound the (C/I) for the down-link (BS → MS) of our system from below. To do this, we select the worst case scenario, with the smallest C and the largest I.
The worst case for C is when the MS is at the edge of its cell, at distance R from the BS, and $C \propto R^{-\eta}$. Similarly, we choose the worst case interference, which comes from the BSs in the six interfering co-channel cells. All distances from the MS to the co-channel BSs are larger than or equal to $D - R$. By assuming this shortest distance to all six interferers, we get a worst case interference $I \propto 6(D-R)^{-\eta}$. We can now bound (C/I) in the down-link from below as

$$\left(\frac{C}{I}\right) > \frac{1}{6}\left(\frac{R}{D-R}\right)^{-\eta}. \qquad (1)$$

Note: This bound can be made tighter, by considering the fact that some of the co-channel BSs

* Note: Solutions provided here are less detailed than the ones expected during the exam. Many steps are excluded.

will be further away than $D - R$. This is, however, beyond the problem statement.

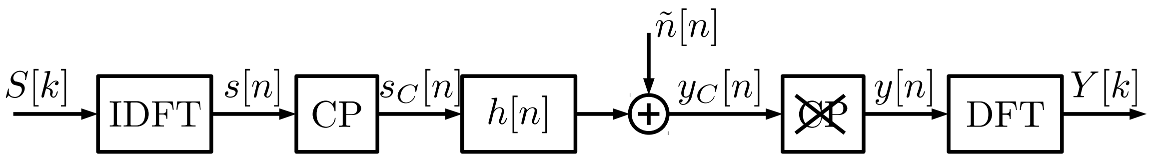
b) With the worst-case up-link scenario depicted in Fig. 30.11, we get a lower bound on the up-link (C/I) as

$$\left(\frac{C}{I}\right) > \frac{1}{6}\left(\frac{R}{D-R}\right)^{-\eta}. \tag{2}$$

This bound is the same as the one for the down-link in a). Hence, the same bound can be used for both up- and down-link.

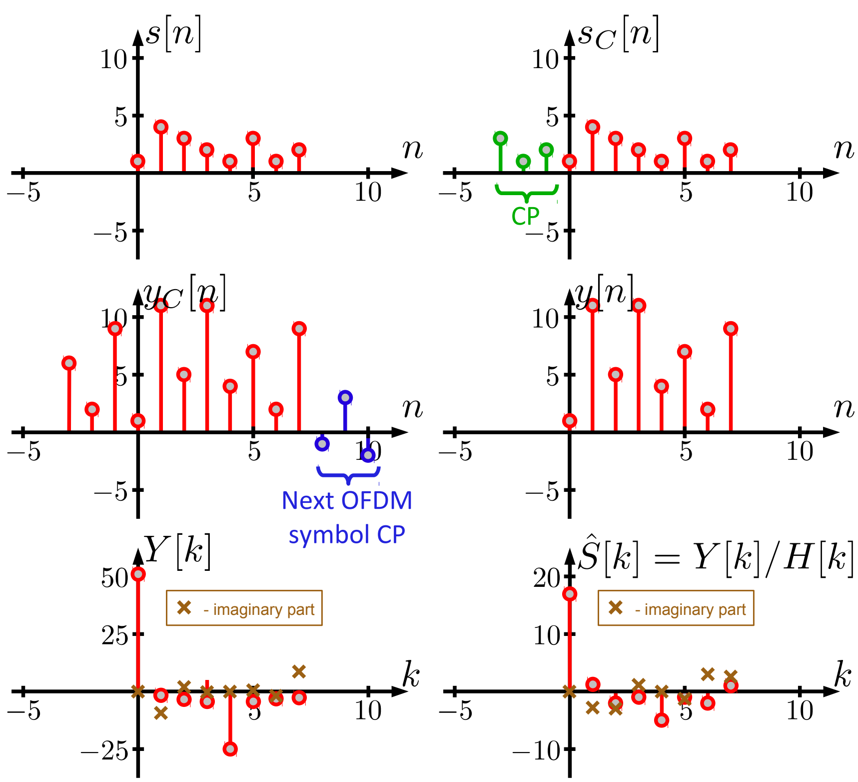
19.1 OFDM system

a) The base-band OFDM system has a block diagram (*capital letter* and *index* k indicate *frequency domain*) :



b) Since the channel contains four taps, the CP has to be of length $L = 3$ samples.

c) The signal vectors i) to vi) become:



and the last one, signal vii) $\hat{s}[n]$, is equal to the first plot, $s[n]$, since we did not have any noise in the calculations, i.e. $\tilde{n}[n] = 0$.

19.2 OFDM distorsion

With many OFDM subcarriers, each basband sample (c.f. $s[k]$ in 19.1) is a sum of many independent contributions (data/constellation points "mixed" by the IDFT). The baseband samples therefore have a Rayleigh distributed amplitude. We can basically use the same calculations as we do when calculating the required fading margin against Rayleigh fading, with the difference that here we want to calculate the probability that we are ABOVE a certain threshold, rather than below it. Using his, we obtain:

a) $A_0 = \sqrt{-\ln(0.1)} = 1.52$ for a 10% cutoff probability,

b) $A_0 = \sqrt{-\ln(0.01)} = 2.15$ for a 1% cutoff probability, and

c) $A_0 = \sqrt{-\ln(0.001)} = 2.63$ for a 0.1% cutoff probability.