Selected answers*
Problem set 3

Wireless Communications, 2nd Ed.

10.3 Concatenated amplifiers

The equivalent noise figures of two cascaded amplifiers is (1 followed by 2) [calculations in non-dB!]

$$F_{1,2} = F_1 + \frac{F_2 - 1}{G_1} \tag{1}$$

or (2 followed by 1)

$$F_{2,1} = F_2 + \frac{F_1 - 1}{G_2} \tag{2}$$

We want to prove that the amplifier with the smaller value on

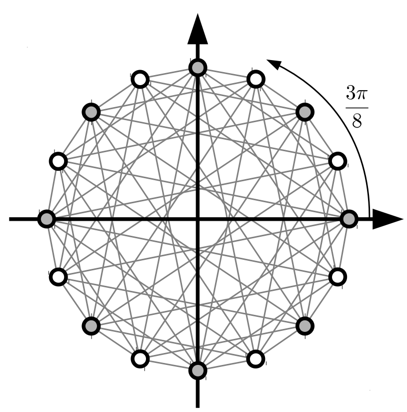
$$M = \frac{F - 1}{1 - 1/G} \tag{3}$$

should be placed first, to make the equivalent noise figure as small as possible.

The strategy to use here is to calculate (3) for both amplifiers, i.e. calculate M_1 and M_2. Then, show that when assuming that one is smaller than the other, e.g. $M_1 < M_2$, this assumption leads to the correct conclusion regarding which equivalent noise figure, (1) or (2), is the smaller.

11.3 EDGE modulation

The modulation in EDGE is based on rotating an 8PSK constellation by $3\pi/8$ radians for each transmitted symbol. This creates the following **transitions between constellation points**, assuming that the 8PSK is implemented with linear transitions between constellation points:



Calculating the "average ampliude" is not possible, unless we know the speed at which the linear transitions take place. It is, however, possible to find the dynamic range, namely the ratio between the largest and smallest amplitude (on the complex envelope). By assuming that the constellation points (max amplitude) are at radius 1, the **minimal amplitude** becomes $\sin(11.25°) \approx 0.195$.

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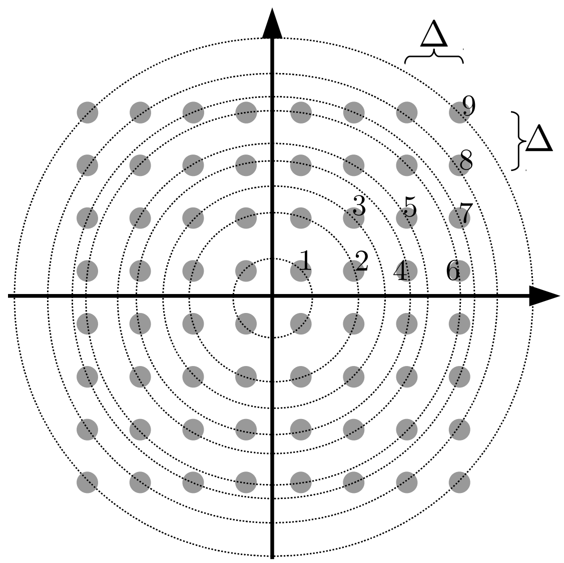
* Note: Solutions provided here are less detailed than the ones expected during the exam. Many steps are excluded.

Using a $\pi/4$ radian rotation between successive 8PSK constellations does not change the behaviour of the transmission, as compared to standard 8PSK, since the rotated constellation points are in the same positions before (they overlap) and the set of transitions remain the same.

11.7 Mean signal energy

When calculating the mean signal energy of a 64-QAM signal, it helps to draw circles for the different amplitudes at which the constellation points are. For 64-QAM there are 9 different amplitudes, as shown below.



Expressing these radii in terms of the distance Δ between constellation points, we get

Circle	Radius	Points
1	$\Delta/2 \times \sqrt{1^2+1^2} = \sqrt{1/2}\Delta$	4
2	$\Delta/2 \times \sqrt{3^2+1^2} = \sqrt{5/2}\Delta$	8
3	$\Delta/2 \times \sqrt{3^2+3^2} = \sqrt{9/2}\Delta$	4
4	$\Delta/2 \times \sqrt{5^2+1^2} = \sqrt{13/2}\Delta$	8
5	$\Delta/2 \times \sqrt{5^2+3^2} = \sqrt{17/2}\Delta$	8
6	$\Delta/2 \times \sqrt{7^2+1^2} = \sqrt{25/2}\Delta$	12
7	$\Delta/2 \times \sqrt{7^2+3^2} = \sqrt{29/2}\Delta$	8
8	$\Delta/2 \times \sqrt{7^2+5^2} = \sqrt{37/2}\Delta$	8
9	$\Delta/2 \times \sqrt{7^2+7^2} = \sqrt{49/2}\Delta$	4

Keeping in mind that energy is proportional to amplitude-squared, The **average symbol energy** can be expressed in terms of the constellation point (minimum) distance Δ as

$$\bar{E}_s = \frac{21}{2}\Delta^2. \quad (1)$$

11.8 Spectral efficiency

The MSK spectrum (base-band)

$$S_{\mathrm{MSK}}(f) = \frac{8T\left(\cos\left(4\pi fT\right)+1\right)}{\pi^2\left(1-16f^2T^2\right)^2} \tag{1}$$

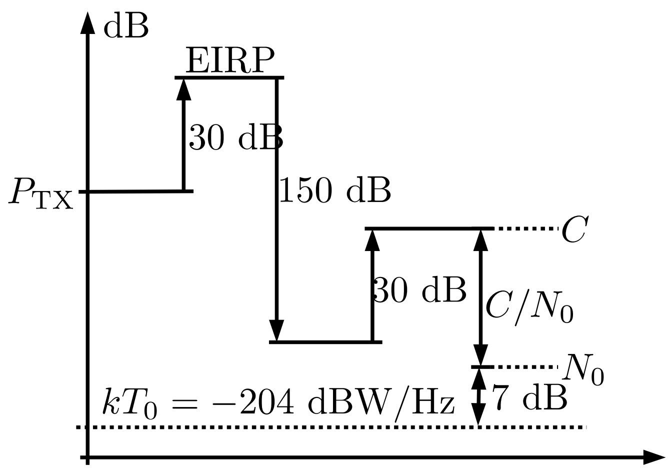
falls of rather slowly with frequency, as compared to the BPSK $\alpha = 0.35$ root-raised cosine spectrum

$$S_{0.35}(f) = \begin{cases} T, & |f| \leq \frac{0.65}{2T} \\ \frac{T}{2}\left[1+\cos\left(\frac{\pi T}{0.35}\left[|f|-\frac{0.65}{2T}\right]\right)\right], & \frac{0.65}{2T} < |f| \leq \frac{1.35}{2T}, \\ 0, & \text{otherwise} \end{cases} \tag{2}$$

which becomes identical to zero outside a bandwidth 1.35 times the bit rate. Since the requirement is so tough, that the energy outside the own bandwidth can be only -50 dBm when the transmitted signal is 20 W (43 dBm), the MSK signal will require a much too large bandwidth. Only the BPSK signal will require a reasonable bandwidth.

12.1 Point-to-point radio link

A quick link budget for the problem would look like:



which gives a transmit power

$$P_{TX} = C/N_0 - 204 + 7 - 30 + 150 - 30 = C/N_0 - 107 \text{ dBW}. \tag{1}$$

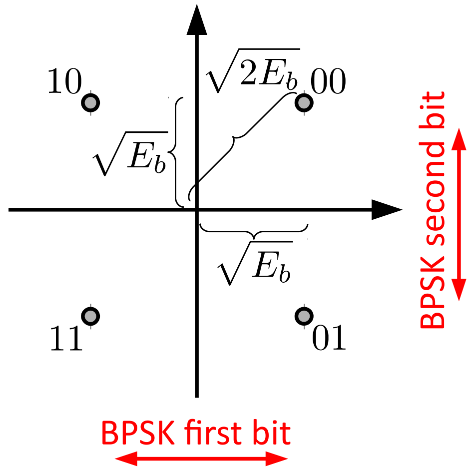
Given that the symbol rate is 20 Msymb/sec (73 dB[symb/sec]), we have

$$P_{TX} = E_s/N_0 + 73 - 107 = E_s/N_0 - 34 \text{ dBW} \tag{2}$$

where E_s/N_0 is given by modulation and the 10^{-5} BER requirement.

a) Using the four different (binary) modulation schemes, we get transmit powers P_{TX} as
Coherent BPSK → **-25.3 dBW**, Coherent FSK → **-22.3 dBW**,
Differential BPSK → **-24.6 dBW**, and Non-coherent FSK → **-21.6 dBW**

b) We can see QPSK as two independent BPSK signals:



Since all distances are the same as for BPSK and the I and Q channels are independent, we get the same BER as BPSK, i.e.

$$BER_{\text{QPSK}} = BER_{\text{BPSK}} = Q\left(\sqrt{\frac{2Eb}{N_0}}\right). \tag{3}$$

The SER cab be calculated as one minus the probability that both bits are correct, namely

$$SER_{\text{QPSK}} = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)\left(2 - Q\left(\sqrt{\frac{2E_b}{N_0}}\right)\right). \tag{4}$$

c) Since QPSK has the same BER as BPSK, in terms of E_b/N_0, the required power is the same [as in a)], but compensated by 3 dB since the data rate is twice as high, -25.3 + 3 = **-22.3 dBW**.

d) With differential QPSK, instead of coherent QPSK, at the same transmit power, the BER increases to about 6×10^{-4}.

12.2 Union bound

a) For Gray-coded QPSK, the **union bound** is given by (two single-bit errors and one double-bit error)

$$BER \leq \frac{1}{2}Q\left(\sqrt{\frac{2E_b}{N_0}}\right) + \frac{1}{2}Q\left(\sqrt{\frac{2E_b}{N_0}}\right) + \frac{2}{2}Q\left(\sqrt{\frac{4E_b}{N_0}}\right) \tag{1}$$

b) The difference between the bound and the exact expression is

$$Q\left(\sqrt{\frac{4E_b}{N_0}}\right), \tag{2}$$

which is less than 10^{-5} when $E_b/N_0 > 6.6\ \text{dB}$.

[12.7] Error-free *M*-ary orthogonal modulation

In M-ary modulation, each symbol carries a symbol energy $E_s = \log_2(M)E_b$ and, for the orthogonal case, the distance between all pairs of symbols is $d = \sqrt{2E_s} = \sqrt{2\log_2(M)Eb}$.

With coherent detection, the union bound on SER becomes

$$SER_{\text{coh}} \leq (M-1)Q\left(\frac{d/2}{\sqrt{N_0/2}}\right) = (M-1)Q\left(\sqrt{\frac{\log_2(M)E_b}{N_0}}\right) \tag{1}$$

which goes to zero for all $E_b/N_0 > 2\ln(2) = 1.4\text{ dB}$, when $M \to \infty$.

When orthogonal signaling is used, the motivation is often that it can be detected in a simple way without a coherent receiver. Therefore, let us look at the non-coherent case too, where the selection between any pair of symbols can be seen as binary modulation, with bit energy replaced by $\log_2(M)E_b$ in the expression. Hence, the pair-wise symbol-error probabilities become

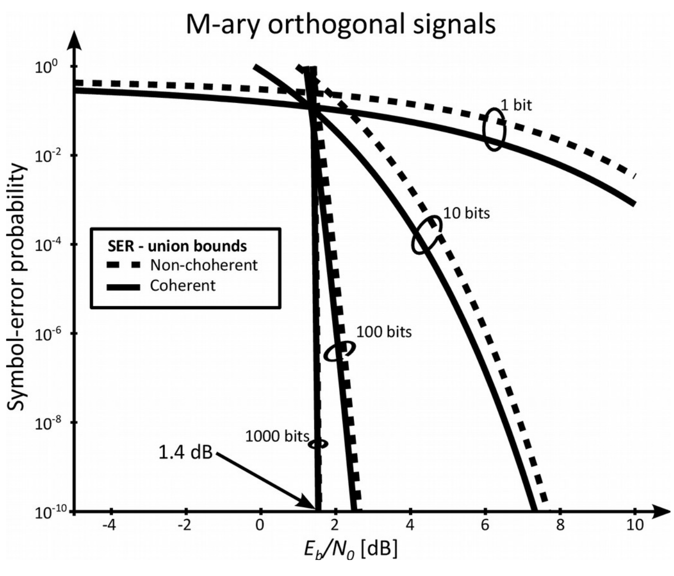
$$P_{s_i \to s_j} = \frac{1}{2}e^{-\frac{\log_2(M)E_b}{2N_0}} \tag{2}$$

and the symbol error union bound

$$SER_{\text{non-coh}} \leq \frac{M-1}{2}e^{-\frac{\log_2(M)E_b}{2N_0}}. \tag{3}$$

Even though non-coherent detection has lower performance than coherent detection, this bound also goes to zero for all $E_b/N_0 > 2\ln(2) = 1.4\text{ dB}$, when $M \to \infty$.

Below, the above union bounds are plotted for 1, 10, 100, and 1000 bits ($= \log_2(M)$):



12.8 Fading penalty

a) Using the same required BER of 10^{-5} as in problem 12.1, but now over a Rayleigh-fading channel, we need to increase the transmit powers by

   Coherent BPSK and Coherent FSK → **34.4 dB** P_{TX} increase

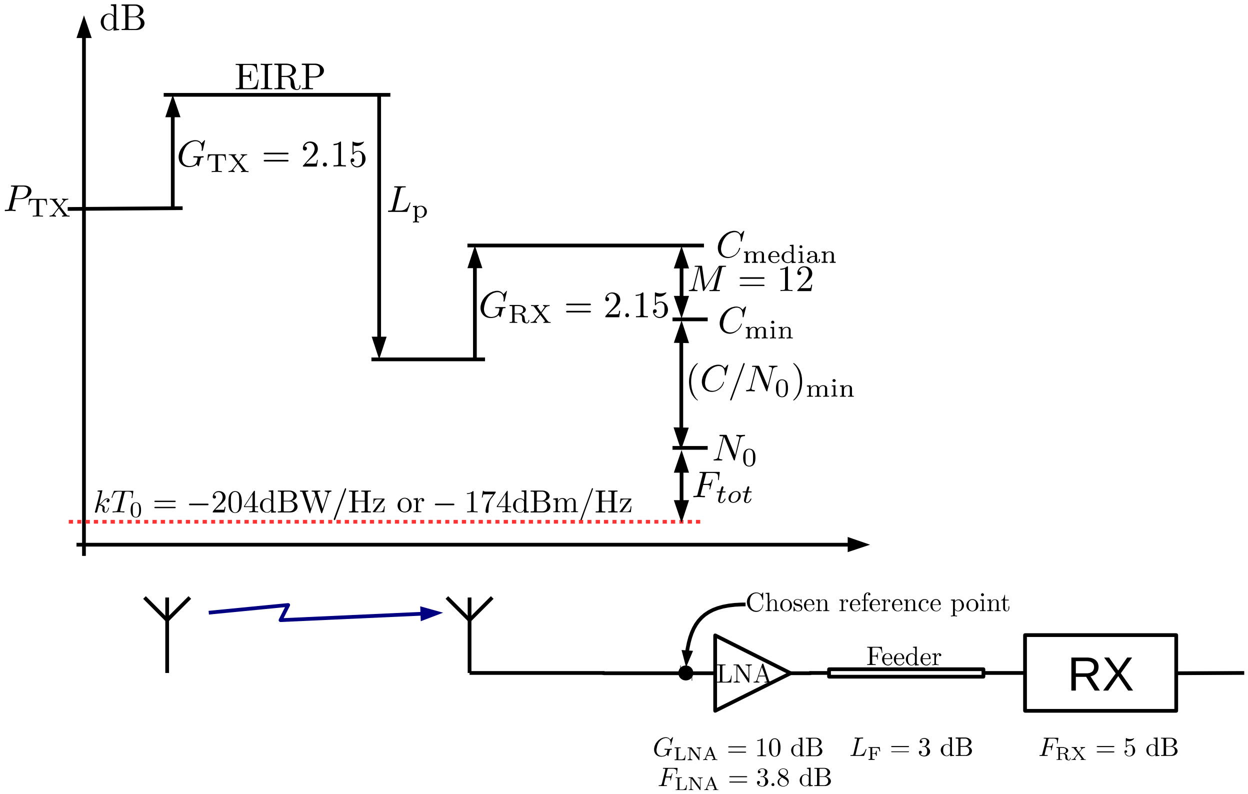
Differential BPSK and Non-coherent FSK → **36.6 dB** P_{TX} increase

b) With Ricean fading and a K_r-factor of 10, we need to increase P_{TX} by a **8.5 dB**. When $K_r \to 0$, the Rice distribution approaches the Rayleigh distribution and we get the result in a).

Extra Problems – Batch 1

1. Link budget – noise and propagation loss

Let's start with drawing a link budget for the given problem. Before we know how to draw it, we need to decide the reference point where we measure received power and place the equivalent noise source. Let us choose the MS antenna output as that reference (then we don't need the LNA or the feeder visible in the link budget – they only influence the receiver noise).
The link budget for the given system can now be drawn as (system block diagram below):



Several of the variables in the link budget are not yet calculated. Let's start with the propagation loss L_p. Since propagation takes place in a *large city* (metropolitan area) and all parameters fall within the valid ranges, we choose the Okumura propagation model.

Free space attenuation:

$$L_{\mathrm{free}} = 20\log_{10}\left(\frac{4\pi d}{\lambda}\right) = 105.5 \text{ dB}$$

Correction terms accoding to Fig. 7.12-7.14:

$$A_{\mathrm{excess}} = 28 \text{ dB}, H_{\mathrm{cb}} = -2 \text{ dB}, H_{\mathrm{cm}} = 2 \text{ dB}$$

Total pathloss becomes: $L_{\mathrm{tot}} = \mathbf{133.5 \ dB}$

Now we continue with the equivalent noise source at the reference point. The noise factor/figure of the total noise source is (where gains, attenuations and noise factors are inserted in calculations in non-dB values)

$$F_{tot} = T_a/T_0 + (F_{\mathrm{LNA}} - 1) + (F_{\mathrm{F}} - 1)/G_{\mathrm{LNA}} + (F_{\mathrm{RX}} - 1)L_{\mathrm{F}}/G_{\mathrm{LNA}} = 5.4 = \mathbf{7.3\ dB}$$

With the given minimal $E_b/N_0 = 10\ \mathrm{dB}$, we can calculate

$$(C/N_0)_{\min} = C_{\min} - N_0 = E_{b,\min} + d_b - N_0 = (E_b/N_0)_{\min} + d_b = 10 + 10\log_{10}(271 \times 10^3)$$
$$= \mathbf{64.3\ dB}$$

We now have all required values to traverse the link-budget and calculate the required power in to the BS antenna:

$$\begin{aligned} P_{TX} &= kT_0 + F_{tot} + (C/N_0)_{\min} + M - G_{RX} + L_p - G_{TX} \\ &= -204 + 7.3 + 64.3 + 12 - 2.15 + 133.5 - 2.15 = \mathbf{8.8\ dBW} = \mathbf{7.6\ W} \end{aligned}$$

2. Diversity

(a) The definition of outage probability is $p_o = \Pr\{(C/N_0) < (C/N_0)_{\min}\}$. Given the exponential probability distribution function for C and a constant N_0, an equivalent expression is

$$p_o = \Pr\{C < C_{min}\} = cdf_C(C_{min}) = 1 - \exp(-C_{\min}/\bar{C})$$

from which we can calculate the required minimum power

$$\bar{C} = \frac{C_{\min}}{-\ln(1-p_o)}$$

and dividing by N_0 gives

$$\bar{C}/N_0 = \frac{C_{\min}/N_0}{-\ln(1-p_o)} = [\text{in dB}] = C_{\min}/N_0 - 10\log_{10}(-\ln(1-p_o))$$

(b) If we require outage less than p_o when we have K independently fading antennas and select the strongest one, the corresponding outage on any one of the K antennas only has to be $(p_o)^{1/K}$ (see hint). This gives (using the result from a):

$$\bar{C}/N_0 = \frac{C_{\min}/N_0}{-\ln(1-p_o^{1/K})} = [\text{in dB}] = C_{\min}/N_0 - 10\log_{10}(-\ln(1-p_o^{1/K}))$$

(c) Subtracting the result from (b) from the result from (a) gives the diversity gain in dB, when using K antennas and RSSI selection, as

$$G_K = -10\log_{10}(-\ln(1-p_o)) + 10\log_{10}(-\ln(1-p_o^{1/K}))\ .$$

3. 8PSK receiver input quality

To achieve a BER of 10^{-4} when using 8PSK over a non-fading channel with additive white Gaussian noise, we need (Slide 33, Lecture 06)

$$10^{-4} \approx \frac{2}{3}Q\left(\sqrt{0.87\frac{E_b}{N_0}}\right)$$

which gives $E_b/N_0 \approx 11.7$ dB. Using the relation $C = [dB] = E_b + d_b$, we get

$$\frac{C}{N_0} = [dB] = \frac{E_b}{N_0} + d_b$$

which, with a data rate of $d_b = 10 \times 10^3 = 40$ dB, gives

$$\frac{C}{N_0} = 11.7 + 40 = 51.7 \text{ dB}.$$

Note that the bandwidth is not required to solve the problem.

4. 2ASK and RSSI diversity

This problem can be solved entirely by a simple noise calculation combined with Figure 13.11 in the textbook.

The noise power spectral density is $N_0 = -204 + 10\log_{10}(1000/290) \approx -199$ dB at the receiver input (our reference point).

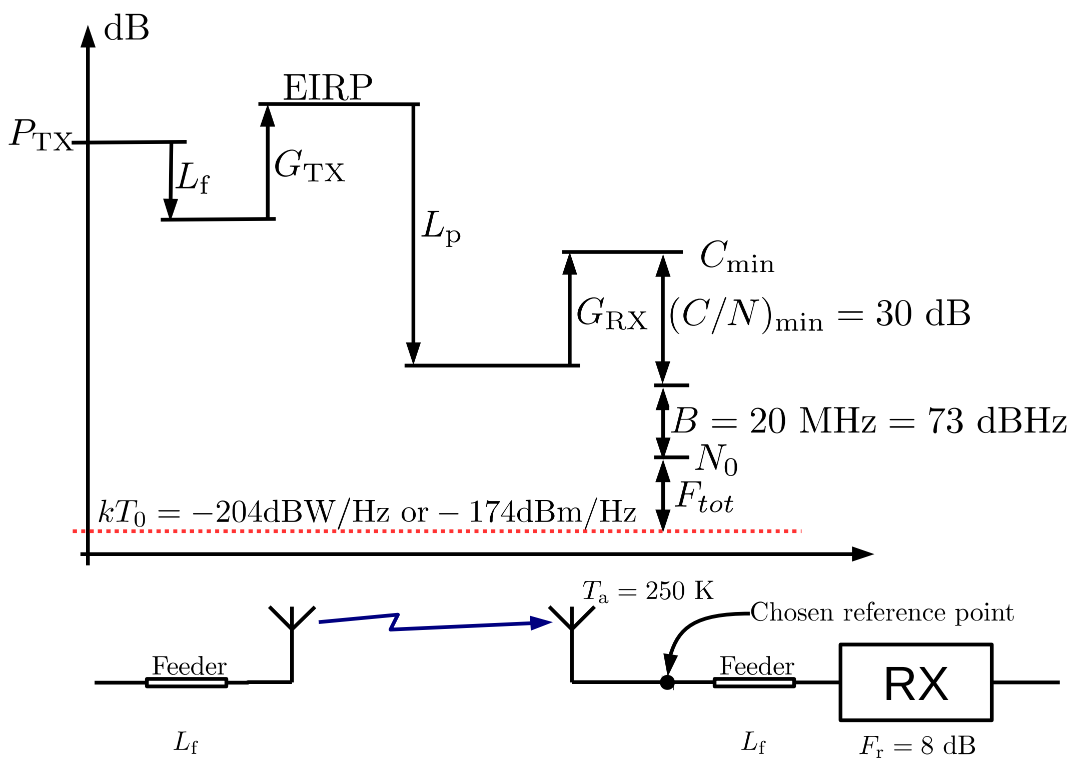
(a) With a single antenna, Fig. 13.11, gives an average $\text{SNR} = E_b/N_0 \approx 25$ dB and corresponding average received power, at a data rate $d_b = 10$ kbit/sek = 40 dB, becomes

$$C = 25 + 40 + (-199) = -134 \text{ dBW} = -104 \text{dBm}.$$

(b) Again referring to Fig. 13.11, we get an average $\text{SNR} = E_b/N_0 \approx 15$ dB when two antennas and RSSI diversity is employed, which implies a diversity gain of around $25 - 15 = 10$dB and a corresponding reduction of the required average received power, i.e., $C \approx -134 - 10 = -124$ dBW or -94dBm.

5. Radio link in Utopia

Let's start by drawing a link budget for the studied system (have chosen the receiver antenna output as the reference point).



There are a number of attenuations and gains which we need to calculate:

$$L_f = \frac{33}{100} \times 12 \approx 4 \text{ dB}$$

$$G_{TX} = G_{RX} = 10\log_{10}\left(0.55\frac{\pi^2 D^2}{\lambda^2}\right) \approx 23.8 \text{ dB}$$

$$L_p = 20\log_{10}\left(\frac{4\pi d}{\lambda}\right) \approx 118.5 \text{ dB}$$

What remains to complete the link budget is the total noise figure, at the reference point,

$$F_{tot} = \frac{T_a}{T_0} + (L_f - 1) + (F_r - 1)L_f$$

where we do all calculations in non-dB and convert to dB at the end, i.e.,

$$F_{tot} = \frac{250}{290} + (2.5 - 1) + (6.3 - 1) \times 2.5 \approx 15.6 = 12 \text{ dB}$$

Traversing the link-budget gives us the (not too realistic)

$$P_{TX} = -204 + 12 + 73 + 30 - 23.8 + 118.5 - 23.8 + 4 \approx -14 \text{ dBW} = 16 \text{ dBm}$$

A real radio link will probably have much smaller parabolic antennas, resulting in larger transmit power.