Temporal Characteristics of Fading
Small-scale fading
Doppler shifts

Frequency of received signal:
\[ f = f_0 + \nu \]

where the Doppler shift is
\[ \nu = -f_0 \frac{v_r}{c} \cos(\theta) \]

The maximum Doppler shift
\[ \nu_{\text{max}} = f_0 \frac{v_r}{c} \]

Receiving antenna moves with speed \( v_r \) at an angle \( \theta \) relative to the propagation direction of the incoming wave, which has frequency \( f_0 \).

The relationship \( \nu_{\text{max}} = f_0 \frac{v_r}{c} \) is based on several assumptions – e.g., static IOs, no double reflections on moving objects, etc.
Small-scale fading
Doppler shifts

How large is the maximum Doppler frequency in the following cases:

• at pedestrian speeds for 5.2 GHz WLAN,
• and at highway speeds using GSM 900?

\[ \nu_{max} = f_0 \frac{\nu_r}{c} \]

- \( f_0 = 5.2 \times 10^9 \text{ Hz} \), \( \nu_r = 5 \text{ km/h, (1.4 m/s)} \) \( \rightarrow \) 24 Hz
- \( f_0 = 900 \times 10^6 \text{ Hz} \), \( \nu_r = 110 \text{ km/h, (30.6 m/s)} \) \( \rightarrow \) 92 Hz
The beating effect

Consider a GSM 900, and an Rx moving at 40 km/h.

\[ v_{\text{max}} = f_0 \frac{v}{c} \]

- \( f_0 = 900 \times 10^6 \) Hz, \( v = 40 \text{ km/h}, (11.1 \text{ m/s}) \rightarrow 33.33 \text{ Hz} \)

Each 30 ms the envelop of the received signal will go to zero.

Superposition of two carriers with different frequencies (beating)
Small-scale fading
Example: the time-variant two-path model

\[ E_1 = \hat{E}_1(j2\pi ft_c \left\{ 1 - \frac{v}{c} \cos(\theta_1) \right\}) \]

\[ E_2 = \hat{E}_2(j2\pi ft_c \left\{ 1 - \frac{v}{c} \cos(\theta_2) \right\}) \]

\[ d_{01}, d_{02}: \text{distances at } t=0 \]
\[ k_1, k_2: \text{wavenumbers} \]
\[ |k_1| = |k_2| = k_0 = 2\pi/\lambda \]

The two components have different Doppler shifts!
The Doppler shifts will cause a random frequency modulation
Small-scale fading
Example: the time-variant two-path model

Equivalent cases:
• Two superimposed incident waves + moving Rx
• Superimposing two signals with different Doppler shifts at the Rx antenna

Besides the distribution, the temporal behavior is needed to characterize small-scale fading.
Radio Channels (some properties)

Path loss
- Attenuation due to distance

Large-scale fading
- Variation around the mean due to IOs

Small-scale fading
- Frequency dispersion of the signal due to the time-varying nature of the channel
- Time dispersion of the signal due to the multipaths

- Slow fading
- Fast fading
- Flat fading
- Frequency-selective fading

The final effect is a function of the characteristics of BOTH: the channel, and the transmitted signal
Doppler spectra

Incoming waves from several directions (relative to movement or RX)
All waves of equal strength in this example, for simplicity.

The received signal may occupy the $f_0 - v_{max}$ to $f_0 + v_{max}$ frequency band.
Jakes spectrum

Jakes spectrum
Isotropic uncorrelated scattering

RX movement

Uncorrelated amplitudes and phases
Uniform incoming power distribution (isotropic)

Doppler spectrum at center frequency \( f_0 \).

\[ S_D(v - f_0) \]

\[ f_0 - v_{\text{max}} \quad f_0 \quad f_0 + v_{\text{max}} \]
Doppler spectrum vs. the time-autocorrelation function

\[ S_v \]

\[ f_0 - v_{\max} \quad f_0 \quad f_0 + v_{\max} \]

Condensed parameters:
- Coherence time
- Doppler spectrum vs. the time-autocorrelation function
- Time-autocorrelation function
- Doppler power spectrum

\[ \rho_t(\Delta t) \]

\[ \rho_t(0) \]

\[ T_c \]
The channel time correlation

- Time correlation – how static is the channel?
- For a uniform scattering environment the time correlation of the Re- and Im-components is described by a “zeroth-order Bessel function of the first kind”

\[ \rho(\Delta t) = E\left\{ a(t) a^*(t + \Delta t) \right\} \propto J_0\left(2\pi v_{\text{max}} \Delta t\right) \]

- The time correlation for the amplitude is

\[ \rho(\Delta t) \propto J_0^2\left(2\pi v_{\text{max}} \Delta t\right) \]
The channel time correlation (example)

Example: Assume that an MS is located in a fading dip. On average, what distance should the MS move so that it is no longer influenced by this fading dip?

\[
\rho = 0 \text{ (complete decorrelation)}
\]

\[
\rho = 0.5
\]

\[
\rho = 0.18\lambda
\]

\[
\rho = 0.38\lambda
\]
Temporal Dependence of small-scale fading

What about the length and the frequency of fading dips?

Important for system design
- block length
- code design
- choice of modulation
Temporal Dependence of small-scale fading (cont.)

These curves are for Rayleigh fading & isotropic uncorrelated scattering (Jakes' doppler spectrum).

Example: assume a multipath environment where the received signal has a Rayleigh distribution and the Doppler spectrum has Jakes shape. Compute the LCR and the ADF for a maximum Doppler frequency=50Hz, and $r/r_{\text{min}}=0.1$.

a) $0.25 = \text{LCR} / 50 \implies \text{LCR}=12.5$

b) $0.04 = \text{ADF} \times 50 \implies \text{ADF}=0.8$ msec
Temporal Nature of the Channel

- Caused by the motion of the Tx, Rx, or IOs.
- Characterizes the time-varying nature of the channel in a small-scale region (How fast does the channel change?) ➔ The Coherence Time $T_c$ (defined as the time during which the channel is time invariant) is used as a measure.
- Characterizes the frequency-dispersion of the channel (How does the Doppler shifts spread the frequency components of the transmitted signal?) ➔ The Doppler spread $D_s$ (defined as the Maximum Doppler Shift) is used as a measure.
- As a general approximation:

$$T_c = \frac{1}{D_s}$$
The coherence time

Given the time correlation of a channel, we can define the Coherence Time $T_C$.

$\rho_t(\Delta t)$

$\rho_t(0)$

$\frac{\rho_t(0)}{2}$

$T_C$

$\Delta t$

$T_C$ shows us over how long time we can assume that the channel is fairly constant.

If the Symbol Time $T_s$ is much shorter than $T_C$, it will experience the same channel (i.e., it will get attenuated by the channel, however, it will not get distorted due to fading).
If the bandwidth of the base band signal $W$ is much greater than $D_s$, the effect of the Doppler spread on the received signal is negligible (no distortion).
Slow fading vs. Fast fading

\[ T_c \gg T_s \quad \text{OR} \quad D_s \ll W \]

⇒ Slow Fading (i.e., the channel fading rate is less than the symbol rate which results only in SNR degradation)

\[ T_c < T_s \quad \text{OR} \quad D_s > W \quad \text{or comparable} \]

⇒ Fast Fading (i.e., the channel fading rate is greater than the symbol rate which results in distortion degradation)

Mitigation Techniques:
- Robust modulation (schemes that don’t require phase tracking, and reduces the detector’s integration time)
- Increasing the symbol rate
- Coding and Interleaving
Radio Channels (some properties)

Path loss
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Small-scale fading
- Frequency dispersion $T_c$, $D_s$
- Time dispersion of the signal due to the multipaths

- Slow fading $T_c \gg T_s$ or $D_s \ll W$
- Fast fading $T_c < T_s$ or $D_s > W$
  or comparable values

Flat fading
Frequency-selective fading

The final effect is a function of the characteristics of BOTH: the channel, and the transmitted signal
Measurement example

- Measurement in the lab
- Center frequency 3.2 GHz
- Measurement bandwidth 200 MHz, 201 frequency points
- 60 measurement positions, spaced 1 cm apart
- Measured with a vector network analyzer
Coherence time, measured

Assume 1 m/s, $v_{\text{max}} = f_0 \frac{v}{c}$ = 10.7 Hz

Compare $1/(2\pi v_{\text{max}}) = 0.014$ s
Probability density function
This channel has highly dynamic scatterers, both Tx and Rx are moving.
Vehicle-to-Vehicle channel characteristics

Time-delay characteristics:
- Rapidly varying channel
- Discrete components carry significant energy and change delay bin with time
- Diffuse components following following LOS

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Time variant impulse response

Let's take a closer look at the Doppler shifts here.
Scattering function, $t=8.5-8.65$ s
Doppler-delay characteristics

Doppler-delay characteristics:

- Discrete components: small Doppler spread, but can change delay bin rapidly
- Diffuse components: large delay and Doppler spread
- Time-variant Doppler spectrum → WSSUS conditions are violated

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Impulse response, two cars, same direction 90 km/h

Fading of a single scatterer