Microwave 2017: *Assignment 2*

Hand in the solutions no later than May 4, 2017.

1

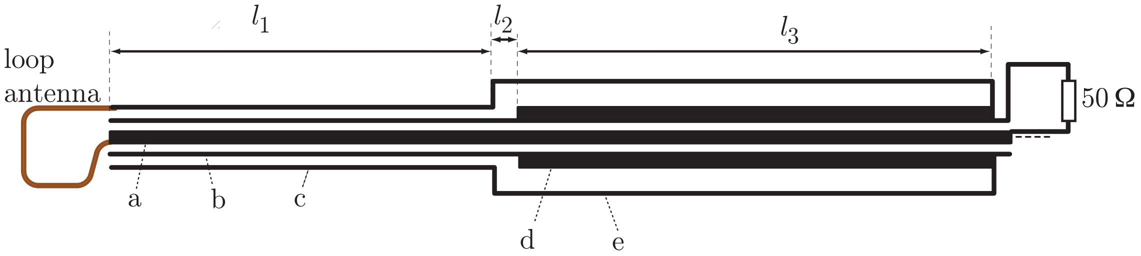
A two-wire transmission line consists of two identical parallel circular conductors. The conductors are made of copper and the material between the cables is polythene with $\varepsilon = 2.1$ and conductivity $\sigma = 4.0 \cdot 10^{-7}$ S/m. The radius of the conductors is 2 mm and the distance between the centers of the wires is $c = 5$ mm.

a) Determine R, L, G and C for the cable using the analytic formulas. The frequency is $f = 100$ MHz.

b) Determine R, L, G and C using COMSOL when the frequency is $f = 100$ MHz. How many correct digits can you get with COMSOL compared to the analytic results?

   Note: Make sure that the computational domain is large enough by comparing results from two different sizes of the domain.

2



In synchrotron accelerators the electron travel in a circular ring and radiate light. The radiated energy equals the loss of kinetic energy of the electrons. There are a number of cavities along the ring that compensates for the loss of energy by accelerating the electrons. At the storage rings of Max IV the cavities are designed such that the lowest resonance frequency (the fundamental frequency) is 100 MHz and it is the electric field of this resonance that accelerates the electrons. The beam acts as a source and excites resonances with higher frequencies than 100 MHz in the cavity (so called higher order modes). These modes might damage the beam if they grow strong. For this reason one might introduce a damper for the higher order modes. The damper consists of a loop antenna, a coaxial cable with a 50 Ω load and a notch filter (bandstop filter). The design is given in the figure where a is a solid circular cylindric conductors and b, c, d and e are hollow circular cylindric conductors. There is vacuum in all of the structure, and the radius and lengths are as follows:

$$\begin{aligned}
&\text{radius of } a = 4.3\,\text{mm}\\
&\text{inner radius of } b = 10\,\text{mm}\\
&\text{outer radius of } b = 11\,\text{mm}\\
&\text{inner radius of } c = 17\,\text{mm}\\
&\text{inner radius of } d = 10\,\text{mm}\\
&\text{outer radius of } d = 17.5\,\text{mm}\\
&\text{inner radius of } e = 38.1\,\text{mm}\\
&\ell_1 = 279.5\,\text{mm},\ \ell_2 = 20\,\text{mm},\ \ell_3 = 354\,\text{mm}
\end{aligned}$$

a) Determine the characteristic impedance of the inner coaxial cable (the one between a and b).

b) Why is the inner coaxial cable terminated by a 50 Ω load?

c) The notch filter is the outer coaxial structure that consists of three parts with lengths ℓ_1, ℓ_2 and ℓ_3. Describe how you can obtain the input impedance of this filter by viewing the filter as three coaxial cables in series. Write a Matlab program that calculates the input impedance and plot it in the frequency range 10 MHz-700 MHz. Notice that the input impedance of the notch filter must be purely reactive.

d) Draw an equivalent discrete circuit where the loop antenna is a voltage source and where the notch filter and the inner coaxial cable are replaced by impedances.

e) Explain why the design will fullfil its purpose, *i.e.*, to prevent the electromagnetic field of the fundamental resonance to escape from the cavity and to let the other resonances be absorbed by the load.

f) The notch filter will also prevent some other frequencies to escape from the cavity. One should avoid to have resonances at these frequencies. Determine the two lowest frequencies above the fundamental frequency that are stopped.

3

Determine the three lowest cut-off frequencies for the waveguides described below by using the analytic formulas and confirm your solutions by determining the cut-off frequencies with COMSOL. Give the ten lowest cut-off frequencies obtained by COMSOL for each waveguide. Check the accuracy in the COMSOL solutions and give a rough estimate of the error.

a) A rectangular waveguide with $a = 8$ cm and $b = 3$ cm.

b) A circular waveguide with radius $R = 5$ cm.

c) A waveguide with a cross section as a half circle with radius $R = 5$ cm.

d) The bandwidth of a waveguide is the width of the frequency band where only the fundamental mode can propagate. The fundamental mode is the mode with lowest cut-off frequency. The relative bandwidth is defined by

$$B = \frac{\Delta f}{f_0}$$

where Δf is the band width and f_0 is the cut-off frequency for the fundamental mode. Determine B for the three waveguides.

4

The fundamental mode is the mode with the lowest cut-off frequency. It is very important in most waveguide application. By using frequencies in the bandwidth for the fundamental mode the electromagnetic fields are stable and easy to control since higher order modes cannot propagate.

a) Determine the explicit expressions for the complex transverse electric and magnetic fields for the fundamental mode of a rectangular waveguide with cross-section $a \times a/2$. The mode is assumed to propagate in the positive $z-$direction and the transverse fields should be written as

$$\begin{aligned} \boldsymbol{E}_T(x,y,z) &= (\hat{\boldsymbol{x}} E_x(x,y) + \hat{\boldsymbol{y}} E_y(x,y))\, e^{ik_z z} \\ \boldsymbol{H}_T(x,y,z) &= (\hat{\boldsymbol{x}} H_x(x,y) + \hat{\boldsymbol{y}} H_y(x,y))\, e^{ik_z z} \end{aligned}$$

The waveguide is filled with air.

b) Determine the surface currents that run in the short wall $x = 0$ (*i.e.*, in the yz-plane) when the fundamental mode propagates in the waveguide. Transform the current to the time domain and write it as

$$\boldsymbol{J}_s(y,z,t) = \mathrm{Re}\{\boldsymbol{J}_s(y) e^{ik_z z} e^{-i\omega t}\} = \hat{\boldsymbol{y}} J_{sy}(y)\cos(k_z z - \omega t + \phi_y) + \hat{\boldsymbol{z}} J_{sz}(y)\cos(k_z z - \omega t + \phi_z)$$

where $\boldsymbol{J}_s(y)$ is the complex surface current density, ϕ_y and ϕ_z are phase angles (they follow from the analysis), $J_{sy}(y)$ and $J_{sz}(y)$ are real amplitudes, and k_z is the longitudinal wavenumber for the fundamental mode.

Draw the vectors of the surface current density on a distance corresponding to one period, *i.e.*, $0 < z < \lambda_g = 2\pi/k_z$, on the wall $x = 0$ when $t = 0$. The frequency should be $f = \dfrac{3c_0}{4a}$, *i.e.*, the frequency between the cut-off frequency for the fundamental mode and the second mode.

5

Certain array antennas used for high power radiation are made of slots in waveguides. The slots act as antenna elements since they leak electromagnetic waves from the waveguides to the outer region. The slot antennas in a rectangular waveguide are usually placed on the short wall and are directed along the waveguide, as in the lower figure in figure 1.

a) Explain why the slots are directed the way they are. Would slots that are directed perpendicular to the ones in the figure be good antennas?

b) The direction of the main beam from an array antenna depends on the phase of the fields in the slot. The array slot antenna can be used as a frequency directed antenna. This means that by changing the frequency the direction of the main beam changes. Determine the distance between the slots if the antenna is a broadside antenna, *i.e.*, an antenna that has its main lobe in a direction perpendicular to the array. Give the distance in terms of the waveguide wavelength $\lambda_g = 2\pi/k_z$.

6

In this problem you use COMSOL.

a) A coaxial cable is normally used as a transmission line for TEM waves. At a high enough frequency the higher order modes start to propagate. The number of propagating modes increases when the frequency increases. It is possible to determine these modes and their cut-off frequencies analytically (*cf.*, problem 5.3 in the book). However, we will use COMSOL to find the cut-off frequencies and also to sketch the field distribution in the coaxial cable. In this example the cable has an outer radius $a = 0.25\,\text{cm}$ of the inner conductor and an inner radius $b = 1\,\text{cm}$ of the outer conductor. The volume between the conductors is filled with air ($\varepsilon_r = \mu_r = 1$). Find the ten lowest cut-off frequencies and make a table

   Multiple modes that have the same frequency but only differs by a rotation of 90 degrees of the fields should only be counted as one mode.

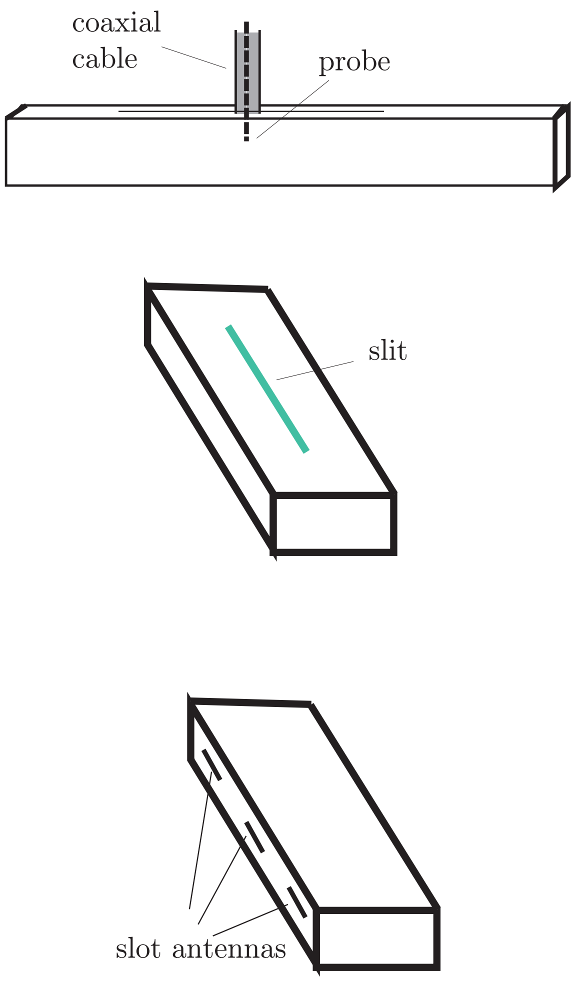


Figure 1: The standing wave meter with slot and the array slot antenna.

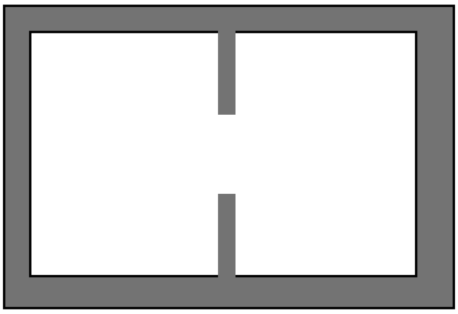
b) Out of the ten modes there are two, one TE (single) and one TM (multiplicity 2), that have the same cut-off frequency. Try to identify these two modes in your list. (The mathematical reason why they are the same is that $J_0'(x) = -J_1(x)$.)

You find COMOL instructions below.

7

Study first Example 5.6 in the book.
Use COMSOL to design a ridge waveguide for which the fundamental mode has a bandwidth ranging at least from 0.6 GHz to 2.4 GHz. In order to get accurate values you need to refine the mesh around the edges of the thin walls since the electromagnetic fields vary rapidly close to edges. The waveguide in the figure does not fulfill the requirements so you need to modify it.

frequency	mode(TE/TM)	m-index	multiplicity
	⋮ ⋮		



Something to think about: Try to understand why the cutoff frequency for the fundamental mode is so low. Also try to understand why the power flow densities look the way they do for the different modes.

Please hand in the geometry of the waveguide and a plot of the power flow for the fundamental mode.

8

At ESS they will use klystrons to generate the waves for the cavities. The frequency is 302 MHz in the early part of the LINAC and 704 MHzin the late part. The klystrons and cavities will be placed in two different tunnels and for this reason the connecting waveguides will be long. This leads to power losses. Use the impedance boundary condition in COMSOL to calculate the relative power loss $\Delta P/P$ for an 18 meter long rectangular waveguide made of copper. Here ΔP is the power loss and P is the power sent from the klystron. The frequency is 704 MHz, and the dimension of the waveguide is $a \times b = 0.3 \times 0.15$ m^2.
Hint: Use 2D electromagnetic waves. A similar problem is solved in the book.