# Microwave 2017: Assignment 1

The solutions should be handed in no later than April 26. You can either send a pdf-file or give a paper copy to Anders (anders.karlsson@eit.lth.se). If you get stuck you can send an e-mail to Anders.

### 1

A rectangular waveguide with dimension  $a \times b$ , a = 0.3 m, b = 0.15 m is filled with air. For a propagating mode the wavelength in the z-direction is given by  $\lambda_z = \frac{2\pi}{k_z}$ .

Use Matlab to plot  $\lambda_z/\lambda$  for the TE<sub>10</sub> mode as a function of frequency in the frequency interval  $[1.1f_c, 10f_c]$ , where  $f_c$  is the cut-off frequency for the TE<sub>10</sub> mode. Note: Air has  $c = 3 \cdot 10^8$  m/s.

# 2

For hollow waveguides the phase speed in the z-direction is given by  $v_{\rm p} = \frac{\omega}{k_z}$  and the group speed by the  $v_{\rm g} = \frac{k_z}{k}c^2$ . The TE<sub>11</sub>-mode propagates in a circular waveguide with radius a = 0.1 m that is filled with air.

Use Matlab to plot  $v_{\rm p}$  and  $v_{\rm g}$  as functions of frequency in the interval  $[1.1f_{\rm c}, 10f_{\rm c}]$ where  $f_{\rm c}$  is the cut-off frequency.

### 3

A TE<sub>10</sub> mode with  $\boldsymbol{E} = E_0 \sin(\pi x/a)e^{ik_z z} \boldsymbol{e}_y$  is propagating in the positive z-direction for z < 0. At z = 0 the waveguide is terminated by a perfectly conducting plate. Determine the total electric field in the waveguide.

# 4

For circular waveguides the fundamental mode  $TE_{11}$  is degenerated since there are two different polarizations of the mode.

- a) Sketch the electric field of the two different modes in the cross section of the waveguide.
- b) Assume that we need to add a filter such that only one of the modes propagates in the waveguide. One can do that by inserting a thin metallic plate in the waveguide. The width of the plate is equal to diameter of the waveguide. Explain how the plate should be inserted in order to remove the mode with the electric field in the y-direction at the symmetry axis. The mode with the electric field in the x-direction should not be affected by the plate.

# $\mathbf{5}$

A lossless transmission line has the characteristic impedance  $Z_0 = 60 \Omega$ . The line is terminated by a load impedance  $Z_b = 120 \Omega$ . Determine the reflection coefficient at the load and the standing wave ratio S.

### 6

Show that the time average of the electric energy per unit length is equal to the time average of the magnetic energy per unit length for a time harmonic wave that travels along a lossless transmission line. The easiest way to do this is to express the energies in terms of current, voltage, capacitance and inductance.

# 7

In the early days of telephone communication a major problem was that both Rand G of the cables were non-zero. The sound was then distorted and it was not possible to make telephone calls at longer distances than 10 km. In order to avoid distorsion, and by that increase the distance for telephone calls, a special type of coil, called Pupin coil, was introduced. The coil was attached so that the current in one of the wires passed through the coil. Assume that one put one coil every kilometer along the cable. Determine an expression for the inductance  $L_p$  of each coil such that the distortion was minimized. The expression for  $L_p$  should include the parameters R, L, G, and C for the transmission line.

# 8

A coaxial cable has an inner conductor with diameter 0.90 mm and an outer conductor with (inner)diameter 2.95 mm. The material between the conductors is polythene with  $\varepsilon = 2.1$  and conductivity  $\sigma = 4.0 \cdot 10^{-7}$ S/m. The conductors are silverplated. Silver has conductivity  $\sigma = 6.3 \cdot 10^{7}$ S/m.

- a) Determine R, L, G, C, the characteristic impedance  $Z_0$ , and the phase speed  $v_p$  for the cable using the analytic formulas. The frequency is f = 100 MHz.
- b) Determine the attenuation expressed in dB/100m for the cable at frequencies 100 MHz, 200 MHz, 500 MHz. Compare these results with the values given by the ELFA catalogue: 12.7 dB/100 m at 100 MHz, 18.3 dB/100 m at 200 MHz and 26.7 dB/100 m at 400 MHz. Is it the isolation between the cables or the resistivity of the cables that is the major cause to the attenuation?

<u>Comment:</u> The losses are very small and they can be neglected when you calculate  $Z_0$  and  $v_p$ . The cable is in the ELFA catalogue denoted M17/084-RG223 and is used for radio communication equipment and video/camera equipment.

#### 9

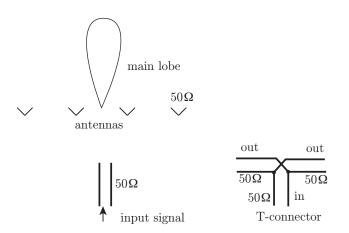


Figure 1: The four antennas in the array.

A linear array antenna consists of four identical equidistant patch antennas, each having an impedance  $Z_a = 50 \,\Omega$ . The array is a broad side antenna which means that the main lobe is directed perpendicular to the line along the array, see figure. This is obtained if the input voltages to the four antennas have the same phase. The signal is sent from a generator via a coaxial cable with characteristic impedance 50  $\Omega$ . The frequency is 1 GHz. Find a way to connect the four antennas to the generator's cable such that there is no reflected wave coming back to the generator. To your disposition you have T-connectors that connect three cables as indicated in the figure. You also have as much 50  $\Omega$  coaxial cable as you like. The insulation between the inner and outer conductor of the coaxial cable is non-magnetic ( $\mu = 1$ ) and has a relative permittivity  $\varepsilon_r = 2.25$ . Show with a figure how you connect the antennas and the cables and give the lengths in centimeters of the different cables you use.