Ch 1. Maxwells eq. in lossless space

$$\begin{cases} \nabla \times \bar{E} = -\dfrac{\partial \bar{B}}{\partial t} \\ \nabla \times \bar{H} = \dfrac{\partial \bar{D}}{\partial t} \\ \nabla \cdot \bar{D} = 0 \\ \nabla \cdot \bar{B} = 0 \\ \bar{B} = \mu_0 \mu \bar{H} \\ \bar{D} = \varepsilon_0 \varepsilon \bar{E} \end{cases} \Rightarrow \nabla^2 \bar{E} - \frac{1}{c^2}\frac{\partial^2 \bar{E}}{\partial t^2} = 0 \quad (1)$$

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0 \mu \varepsilon}}$$

Time harmonic

Phasors

Ex $v(t) = V_0 \cos(\omega t + \varphi) = \mathrm{Re}\{\underbrace{V_0 e^{-i\varphi}}_{V} \cdot e^{-i\omega t}\}$

$V = V_0 e^{-j\varphi}$ = complex voltage

$\bar{E}(\bar{r}, t) = \mathrm{Re}\{\bar{E}(\bar{r})\, e^{-i\omega t}\}$

$\bar{E}(\bar{r})$ = complex electric field

$\frac{\partial}{\partial t} \to -i\omega \quad (2)$

Ch 2 Helmholtz eq

(1) and (2) $\Rightarrow$

$$\nabla^2 \bar{E}(\bar{r}) + k^2 \bar{E}(\bar{r}) = \bar{0}$$

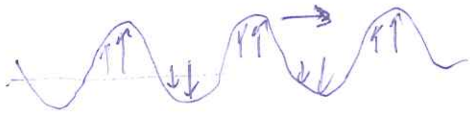
$$\nabla^2 \bar{H}(\bar{r}) + k^2 \bar{H}(\bar{r}) = \bar{0}$$

$k = \frac{\omega}{c}$ = wavenumber

Ex Plane wave

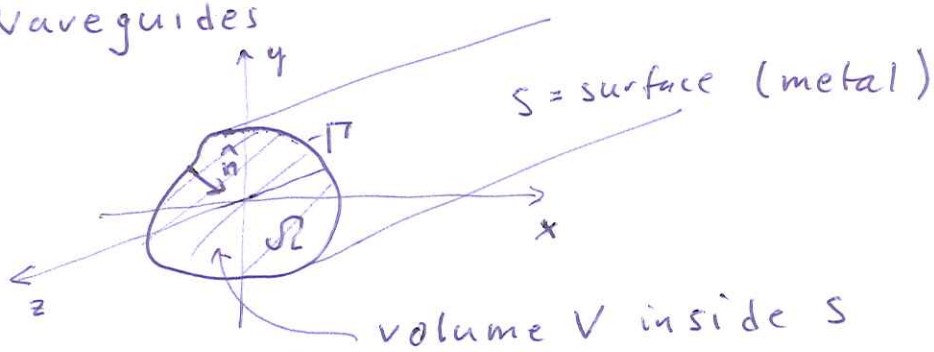
$$\bar{E}(z) = E_0 e^{ikz} \hat{x}$$

$$\Rightarrow \bar{E}(z,t) = E_0 \cos(kz - \omega t)\, \hat{x}$$



Chapters 4 & 5

Waveguides



$$\bar{r} = \bar{\rho} + z\hat{z}$$

$$\bar{\rho} = (x, y)$$

$$\bar{E} = \bar{E}_T + \hat{z} E_z$$

$$\bar{H} = \bar{H}_T + \hat{z} H_z$$

$$\nabla = \nabla_T + \hat{z}\frac{\partial}{\partial z}$$

$$\nabla_T = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y}$$

$$\nabla^2 = \nabla_T^2 + \frac{\partial^2}{\partial z^2}$$

Equations to solve

$$\nabla^2 \bar{E} + k^2 \bar{E} = \bar{0} \qquad \bar{r} \in V$$

or
$$\nabla^2 \bar{H} + k^2 \bar{H} = \bar{0}$$

$$\Rightarrow \boxed{\begin{aligned} \nabla^2 E_z + k^2 E_z &= 0 \\ \nabla^2 H_z + k^2 H_z &= 0 \end{aligned}} \qquad \bar{r} \in V$$

5.1 Boundary conditions

$$\hat{n} \times \bar{E} = \bar{0} \quad \bar{r} \in S \, , \qquad \hat{n} \cdot \bar{H} = 0 \quad \bar{r} \in S$$

$$\Rightarrow \begin{cases} E_z(\bar{r}) = 0 \\ \hat{n} \cdot \nabla H_z(\bar{r}) = 0 \end{cases} \qquad \bar{r} \in S$$

z-dependence

$$E_z(\bar{r}) = v(\bar{\rho})\, e^{ik_z z} \Rightarrow E_z(\bar{r},t) = v(\bar{\rho}) \cos(k_z z - \omega t)$$

real

$$H_z(\bar{r}) = w(\bar{\rho})\, e^{ik_z z}$$

$\boxed{\text{Chapter 4 relates } \bar{E}_T \text{ and } \bar{H}_T \text{ to } E_z \text{ and } H_z}$

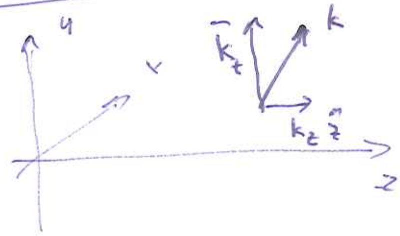
5.2 TE- and TM-waves

TM

$H_z = 0 , \quad E_z \neq 0$

$\nabla^2 E_z + k^2 E_z = 0 , \quad E_z = v(\bar{\rho})\, e^{i k_z z}$

$$\boxed{\Rightarrow \begin{cases} \nabla_T^2 v(\bar{\rho}) + k_t^2 v(\bar{\rho}) = 0 & \bar{\rho} \in \Omega \\ v(\bar{\rho}) = 0 & \bar{\rho} \in \Gamma \end{cases} \qquad (3)}$$



$k_t^2 = k^2 - k_z^2$

$k_t = \sqrt{k^2 - k_z^2}$

TE

$E_z = 0 , \quad H_z \neq 0$

$\nabla^2 H_z + k^2 H_z = 0 , \quad H_z = w(\bar{\rho})\, e^{i k_z z}$

$$\boxed{\begin{matrix} \nabla_T^2 w(\bar{\rho}) + k_t^2 w(\bar{\rho}) = 0 & \bar{\rho} \in \Omega \\ \hat{n} \cdot \nabla w(\bar{\rho}) = 0 & \bar{\rho} \in \Gamma \end{matrix}} \qquad (4)$$

Eigenvalue problems (3) and (4)

(3) and (4) are eigenvalue problems

- Infinitely many eigenvalues k_{tn}^2 and eigenfunctions $v_n(\bar{\varrho})$

$$0 < k_{t1}^2 \leq k_{t2}^2 \leq k_{t3}^2 \leq \ldots \rightarrow \infty$$

- $\{v_n(\bar{\varrho})\}_{n=1}^{\infty}$ is a complete orthogonal set on Ω

$$\Rightarrow \int_{\Omega} v_n(\bar{\varrho})\, v_m(\bar{\varrho})\, d\Omega = \begin{cases} 1 & n = m \\ 0 & n \neq m \end{cases}$$

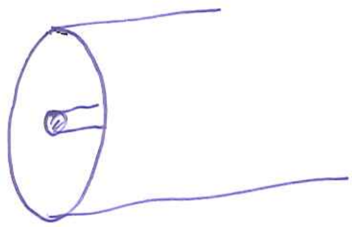
Complete $\Rightarrow$ every function $f(\bar{\varrho})$ $\bar{\varrho} \in \Omega$ and $f(\bar{\varrho}) = 0$ $\bar{\varrho} \in \Gamma$ can be written as $f(\bar{\varrho}) = \sum_{n=1}^{\infty} a_n v_n(\bar{\varrho})$

(Same for $w_n(\bar{\varrho})$ but $\hat{n} \cdot \nabla f(\bar{\varrho}) = 0$ $\bar{\varrho} \in \Gamma$)

Waveguide modes

The EM-field $\bar{E}_n, \bar{H}_n$ belonging to an eigenvalue k_{tn}^2 of (3) or (4) is a **waveguide mode**

5.3 TEM-modes



Two, or more, conductors

$\Rightarrow$ TEM can exist

$\Rightarrow$ $E_z = 0$, $H_z = 0$ $\Rightarrow$ transmission line

$k_z = k$, $k_t = 0$ see Chapter 3