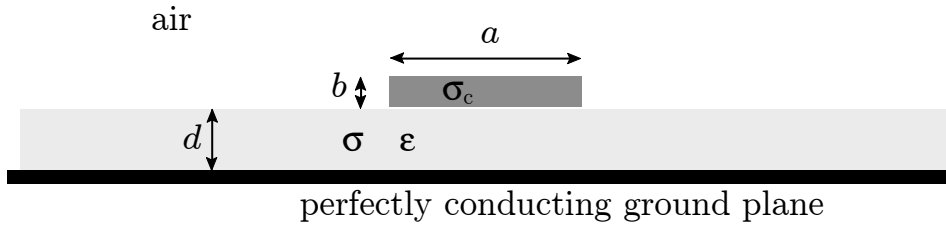


Microwave theory 2017: Problems week 4

1. Determination of R, L, G, C



Use COMSOL to determine R, L, G, C at the frequency $f = 30$ GHz for the stripline shown in the figure . The parameters are

$$\sigma_c = 4 \cdot 10^7 \text{ S/m}$$

$$\epsilon = 4$$

$$\sigma = 0.01 \text{ S/m}$$

$$a = 3 \mu\text{m}$$

$$b = 0.5 \mu\text{m}$$

$$d = 1 \mu\text{m}$$

2. Determination of S-matrix

Consider a coaxial waveguide of length 0.6 m. The inner conductor has radius 3 cm and the outer conductor has inner radius 0.1 m. Let the z -axis be the symmetry axis such that the waveguide starts at $z = 0$ and ends at $z = 0.6$ m. Between the conductors there is air, except for the region $0.2 \text{ m} < z < 0.4 \text{ m}$ where there is a non-conducting dielectric material with $\epsilon = 4$.

a) Use transmission line theory to determine the absolute value of the reflection coefficient ($|S_{11}|$) for the TEM-wave propagating in the positive z -direction. Plot this in dB scale in the frequency range 100 Mhz-2 GHz with at least 1000 frequency steps.

b) Since the medium between the conductors is lossless it must be that the incident energy is equal to the reflected energy plus the transmitted energy. It means that $|S_{11}|^2 + |S_{21}|^2 = 1$, where $S_{11} = \Gamma$ is the reflection coefficient and $S_{21} = T$ the transmission coefficient. Use this to calculate the transmission coefficient. You then have the absolute values of the S-parameters S_{11} and S_{21} . Due to the symmetry it must be that $S_{22} = S_{11}$ and $S_{12} = S_{21}$. Plot also $|S_{21}|$ in the same frequency range as $|S_{11}|$.

c) Now use COMSOL to calculate the scattering matrix. Compare with the analytic results. Use 2D axisymmetric, EM-waves and frequency domain. Plot S_{11} and S_{21} in dB scale in the frequency range 300 Mhz-400 MHz using 100 frequency steps. Compare with the Matlab graph.

d) At what frequencies is the reflection coefficient zero. Notice that a very good resolution is required in order to find these zeros from a graph.

3. Solve problem 5.4 analytically and with Comsol

a) Start by solving 5.4 analytically. Determine the lowest cut-off frequency for a waveguide with 10 cm radius and compare that with the lowest cut-off frequency $f_c = \frac{1.841}{2\pi a}c_0$ for the circular waveguide.

Hint: To obtain an approximate value of the zero of the derivative you can plot the Bessel function in Matlab.

b) Find the cut-off frequencies with Comsol and check that you get the same value as in a).

c) For the fundamental mode the electric field has a singularity at the edge of the strip. Check this with Comsol by magnifying the region around the edge.

4. Solve problem 5.6 analytically and with Comsol

Solutions

1

The extremely fine mesh gives approximately

$$n_e = 1.9146 - j0.36183$$

This gives, with $\gamma = jn_e\omega/c_0$,

$$\gamma = 2.2732 \cdot 10^2 + j1.2029 \cdot 10^3$$

and

$$Z = 39.9303 - j7.4917$$

Then

$$R = \operatorname{Re}\{\gamma Z\} = 1.8 \cdot 10^4 \Omega/\text{m}$$

(compare with the DC resistance $R = 1/(\sigma_c ab) = 1.66 \cdot 10^4 \Omega/\text{m}$)

$$L = \operatorname{Im}\{\gamma Z\}/\omega = 2.4578 \cdot 10^{-7} \text{ H/m}$$

$$G = \operatorname{Re}\{\gamma/Z\} = 0.0396 \text{ S/m}$$

and

$$C = \operatorname{Im}\{\gamma/Z\}/\omega = 1.60 \cdot 10^{-10} \text{ F/m}$$

2

Use the formula for input impedance. At $z = 0.2 \text{ m}$ you use

$$\frac{Z_{\text{in}}}{Z_0} = \frac{Z_1 \cos \beta \ell + Z_1/Z_0 j \sin \beta \ell}{Z_0 Z_1/Z_0 \cos \beta \ell + j \sin \beta \ell}$$

where $\beta = \omega\sqrt{\varepsilon}/c$, $\ell = 0.2 \text{ m}$, $Z_1/Z_0 = 1/\sqrt{\varepsilon}$. The reflection coefficient is given by

$$|\Gamma| = \left| \frac{\frac{Z_{\text{in}}}{Z_0} - 1}{\frac{Z_{\text{in}}}{Z_0} + 1} \right|$$

The reflection coefficient is zero when $\sin \beta \ell = n\pi$, $n = 1, 2, 3, \dots$. That gives frequencies $f_n = n \cdot 0.375 \text{ GHz}$.

3 problem 5.4

We only consider TE-modes. For a TE-mode $H_z(\mathbf{r}) = w(\rho, \phi)e^{ik_z z}$ where

$$\begin{aligned}\nabla_T^2 w + k_t^2 w &= 0 \\ \frac{\partial w}{\partial \rho}(a, \phi) &= \frac{\partial w}{\partial \phi}(\rho, 0) = \frac{\partial w}{\partial \phi}(a, 2\pi) = 0 \\ w(\rho, \phi) &\text{ finite}\end{aligned}$$

Separation of variables $w = f(\rho)g(\phi)$ inserted in the Helmholtz equation $\nabla_T^2 w + k_t^2 w = 0$ gives the eigenvalue problem

$$g''(\phi) + \gamma g(\phi) = 0 \quad (0.1)$$

$$g'(0) = g'(2\pi) = 0 \quad (0.2)$$

Equation (0.1) gives $g(\phi) = A \sin \sqrt{\gamma}\phi + B \cos \sqrt{\gamma}\phi$ where equation (0.2) gives $A = 0$ and $\gamma = (m/2)^2$, $m = 0, 1, \dots$. In the ρ -direction we get the Bessel differential equation

$$\begin{aligned}\frac{1}{\rho} \frac{\partial}{\partial \rho} \rho \frac{\partial f(\rho)}{\partial \rho} + \left(k_t^2 - \left(\frac{m}{2\rho} \right)^2 \right) f(\rho) &= 0 \\ f'(a) &= 0\end{aligned}$$

with the finite solutions

$$f_{mn}(\rho) = J_{m/2}(k_t \rho) = J_{m/2}(\eta_{m/2,n} \rho / a)$$

where $J'_{m/2}(\eta_{m/2,n}) = 0$. The normalized modes are given by

$$w_{mn} = B J_{m/2}(k_{tmn} \rho) \cos m\phi / 2$$

where $k_{tmn} = \eta_{m/2,n} / a$ and

$$B = \sqrt{\frac{\epsilon_m}{2\pi}} \left(\int_0^a (J_{m/2}(k_{tmn} \rho))^2 \rho d\rho \right)^{-1/2}$$

with $\epsilon_m = 2 - \delta_{m,0}$.

Cut-off frequencies

For even m we get the zeros of $J'_{m/2}$ from appendix A. For odd m we can utilize that $J'_\nu(x) = \frac{\nu}{x} J_\nu(x) - J_{\nu+1}(x)$ and determine the zeros numerically in *e.g.*, Matlab. This gives the following values of $\eta_{m/2,n}$

	n=1	n=2	n=3	n=4
m=0	3.832	7.016	10.17	13.32
m=1	1.1656	4.60	7.79	19.95
m=2	1.841	5.331	8.536	11.71
m=3	2.46	6.03	9.26	12.44

We see that $TE_{1/2,1}$ has the lowest cut-off frequency $f_{1/2,1} = c_0 1.1656 / (2\pi a)$. Without the metal plate, according to table 3.4 in the book, the TE_{11} mode has the lowest cut-off frequency $f_{1,1} = c_0 1.841 / (2\pi a)$. The cut-off frequency for the fundamental mode is then reduced by almost 40% when the metal plate is introduced.

4 Problem 5.6

a) We first determine the modes that can propagate when $a = 3$ cm and $f = 5$ GHz. The lowest cut-off frequencies are obtained from the tables of zeros for $J_m(x)$ (for TM) and $J'_m(x)$ (for TE) in appendix A

$$f_{11}^{TE} = \frac{c_0}{2\pi} \frac{1.841}{3} 10^2 = 2.93 \text{ GHz} < 5 \text{ GHz} \quad (0.3)$$

$$f_{21}^{TE} = \frac{c_0}{2\pi} \frac{3.053}{3} 10^2 = 4.86 \text{ GHz} < 5 \text{ GHz} \quad (0.4)$$

$$f_{01}^{TM} = \frac{c_0}{2\pi} \frac{2.405}{3} 10^2 = 3.83 \text{ GHz} < 5 \text{ GHz} \quad (0.5)$$

The next modes are f_{01}^{TE} and f_{11}^{TM} which both are non-propagating modes since they have cut-off frequency 6.1 GHz.

b) The waveguide is filled with a plastic material with $\sigma = 10^{-11}$ S/m and $\epsilon = 3$. The z -dependence of the fundamental mode TE_{11} is given by $e^{ik_z z}$ where $k_z = \sqrt{k^2 - k_{t11}^2}$. The wave number k is given by

$$k^2 = \left(\frac{\omega}{c_0}\right)^2 \epsilon_{ny} = \left(\frac{\omega}{c_0}\right)^2 \left(\epsilon + i \frac{\sigma}{\omega \epsilon_0}\right)$$

It is seen that $\sigma / (\epsilon \epsilon_0) \approx 10^{-11} / (3 \cdot 8.854 \cdot 10^{-12})$ In the microwave region $\sigma / (\omega \epsilon \epsilon_0) \ll 1$ and the following approximations are valid

$$\begin{aligned} k_z &= (k^2 - k_{t11}^2)^{1/2} = \left(\left(\frac{\omega}{c_0}\right)^2 \epsilon - k_{t11}^2 \right)^{1/2} \left(1 + i \frac{\sigma \omega \mu_0}{((\omega/c_0)^2 \epsilon - k_{t11}^2)} \right)^{1/2} \\ &\approx \left(\left(\frac{\omega}{c_0}\right)^2 \epsilon - k_{t11}^2 \right)^{1/2} \left(1 + i \frac{\sigma \omega \mu_0}{2((\omega/c_0)^2 \epsilon - k_{t11}^2)} \right) \end{aligned}$$

and hence $k_z = \text{Re}(k_z) + i \text{Im}(k_z)$ where

$$\text{Im}(k_z) = \frac{\sigma \omega \mu_0}{2} ((\omega/c_0)^2 \epsilon - k_{t11}^2)^{-1/2} = \frac{\sigma \eta}{2} (1 - (f_c/f)^2)^{-1/2}$$

where $f_c = c_0 \xi_{11} / (2\pi a \sqrt{\epsilon})$ och $\eta = \sqrt{\mu_0 / (\epsilon \epsilon_0)}$ = wave impedance. The numerical value is $f_c = 1.7$ GHz.