## Assignment 3

## Microwave theory 2016

The solutions should be handed in no later than May 16. If you run into major problems you may ask Anders for help.

## 1

Determine the ratio between radius $a$ and length $d$ for a circular cylinder such that the lowest resonance frequency for the TE and TM modes are the same. All walls are perfectly conducting and there is vacuum inside the cavity. Confirm your result with Comsol.

## 2

Consider the $\mathrm{TM}_{010}$-mode in a cylindric cavity with radius $a$ and length $L$.
a) At what radius $r_{c}$ is the magnetic field maximal.
b) Where is the surface current density maximal?
c) Where is the surface charge density maximal?
d) Confirm you results from a Comsol 2D axisymmetric eigenfrequency calculation. You can plot the quantities along lines by using line graph. You find line graph under Results $\rightarrow 1 \mathrm{D}$ Plot Group. If you dont find surface charge density then you can plot the absolute value of the normal component of the $D$-field, or equivalently, the norm of the the $D$-field along the surface.

## 3

A resonance cavity is a cylinder with elliptic cross section. The ellipse has major half-axis $a=3 \mathrm{~cm}$ and minor half-axis $b=2 \mathrm{~cm}$. The length of the cylinder is 3 cm .
a) Determine the three lowest resonance frequencies of the cavity by using Comsol. All walls are perfectly conducting and there is vacuum inside the cavity.
b) Let the walls be made of iron (this is a material you find in Comsol). Use impedance boundary condition in Comsol and find the three frequencies again, and the Q -factors for the three resonances.

Help: One can solve problem a) in COMSOL using either a 2 D or a 3D calculation. For b) you need a 3D calculation.
TWO MORE PROBLEMS ON NEXT PAGE!

A dielectric resonator has radius $a=3 \mathrm{~cm}$ and height $h=3 \mathrm{~cm}$. The resonator consists of a dielectric material with relative permittivity $\varepsilon=30$. There is air $(\varepsilon=1)$ outside the cylinder. Determine the three lowest resonance frequencies for axial-symmetric resonances.

Help: Use 2D-axial symmetry in COMSOL. You find a similar example in the book. It is sometimes tricky to get the correct values.

## 5

An optical fiber has a core with index of refraction $n=1.504$ and a cladding with index of refraction $n=1.5$. The radius of the core is $5 \mu \mathrm{~m}$ and the radius of the cladding can be considered to be infinite. Use COMSOL to determine the effective mode index and the phase velocity for the fundamental mode $\mathrm{HE}_{11}$ in the optical fiber. The wavelength of the light is $1.55 \mu \mathrm{~m}$ in vacuum. Hand in a plot of the power flow density in the cross section.

Help: Use an outer circle with radius much large ( $\sim 5-10$ times) than the radius of the core in order to get a finite computational domain. Use scattering boundary condition on the outer circle.

