



Microwave theory, April 13, 2016

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Electrical and information technology

Yesterday

- ▶ The vector basis functions
- ▶ Circular cylindric waveguides

Today

- ▶ Power transport in waveguides
- ▶ Losses
- ▶ Determination of R , L , G and C

The vector basis functions

TM-waves: $H_z = 0$, $E_z(\mathbf{r}) = v(\boldsymbol{\rho})e^{ik_z z}$

TE-waves: $E_z = 0$, $H_z(\mathbf{r}) = w(\boldsymbol{\rho})e^{ik_z z}$

Eigenvalue problems give eigenfunctions $v_n(\boldsymbol{\rho})$ and k_{zn}

The vector basis functions

The total electromagnetic fields for the TM-waves can be written as

$$\begin{cases} \mathbf{E}_{n\nu}^{\pm}(\mathbf{r}) = \{\mathbf{E}_{Tn\nu}(\boldsymbol{\rho}) \pm v_n(\boldsymbol{\rho})\hat{\mathbf{z}}\} e^{\pm ik_{zn}z} \\ \mathbf{H}_{n\nu}^{\pm}(\mathbf{r}) = \pm \mathbf{H}_{Tn\nu}(\boldsymbol{\rho}) e^{\pm ik_{zn}z} \end{cases} \quad \nu = \text{TM}$$

The vector basis functions

The total electromagnetic fields for the TE-waves can be written as

$$\begin{cases} \mathbf{E}_{n\nu}^{\pm}(\mathbf{r}) = \mathbf{E}_{Tn\nu}(\boldsymbol{\rho})e^{\pm ik_{zn}z} \\ \mathbf{H}_{n\nu}^{\pm}(\mathbf{r}) = \{\pm \mathbf{H}_{Tn\nu}(\boldsymbol{\rho}) + w_n(\boldsymbol{\rho})\hat{\mathbf{z}}\} e^{\pm ik_{zn}z} \end{cases} \quad \nu = \text{TE}$$

The vector basis functions

The transverse components \mathbf{E}_T and \mathbf{H}_T

$$\begin{cases} \mathbf{E}_{Tn\nu}(\boldsymbol{\rho}) = \begin{cases} \frac{i}{k_{tn}^2} k_{zn} \nabla_T v_n(\boldsymbol{\rho}), & \nu = \text{TM} \\ -\frac{i\omega}{k_{tn}^2} \mu_0 \mu \hat{z} \times \nabla_T w_n(\boldsymbol{\rho}), & \nu = \text{TE} \end{cases} \\ \mathbf{H}_{Tn\nu}(\boldsymbol{\rho}) = Z_{n\nu}^{-1} \hat{z} \times \mathbf{E}_{Tn\nu}(\boldsymbol{\rho}) \end{cases}$$

The vector basis functions

The mode impedances:

$$Z_{n\nu} = \begin{cases} \frac{k_{zn}}{\omega\epsilon_0\epsilon}, & \nu = \text{TM} \\ \frac{\omega\mu_0\mu}{k_{zn}}, & \nu = \text{TE} \end{cases} \quad (1)$$

The vector basis functions

Notice that at frequencies much higher than the cut-off frequency

$$k_{zn} = \sqrt{k^2 - k_{tn}^2} \approx k \Rightarrow$$

$$Z_{nv} \approx \eta_0 \eta = \sqrt{\frac{\mu_0}{\epsilon_0 \epsilon}} = \text{wave impedance} \Rightarrow$$

The waveguide mode is almost a plane wave propagating in the positive z -direction

Circular waveguide

TE-modes

$$E_z = 0, \quad H_z = A_{mn} J_m(k_{tmn} \rho) \begin{pmatrix} \cos m\phi \\ \sin m\phi \end{pmatrix}$$

$$k_{tmn} = \frac{\eta_{mn}}{a}, \quad J'_m(\eta_{mn}) = 0, \quad n = 1, 2, \dots$$

$$\text{cut-off frequency, } k_z = 0 \Rightarrow k = k_{tmn} \Rightarrow f_c = c \frac{\eta_{mn}}{2\pi a}$$

$$\text{Fundamental mode TE}_{11} \quad f_c = c \frac{1.841}{2\pi a}$$

$$a = 15 \text{ mm gives } f_c = 5.9 \text{ GHz}$$

Circular waveguide

TM-modes

$$H_z = 0, \quad E_z = B_{mn} J_m(k_{tmn} \rho) \begin{pmatrix} \cos m\phi \\ \sin m\phi \end{pmatrix}$$

$$k_{tmn} = \frac{\xi_{mn}}{a}, \quad J_m(\xi_{mn}) = 0, \quad n = 1, 2, \dots$$

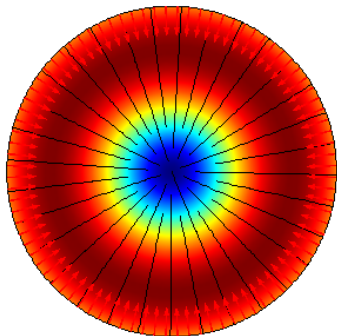
$$\text{cut-off frequency: } f_c = c \frac{\xi_{mn}}{2\pi a}$$

$$\text{Lowest cut-off frequency TM}_{01} \quad f_c = c \frac{2.405}{2\pi a}$$

$$a = 15 \text{ mm gives } f_c = 7.65 \text{ GHz}$$

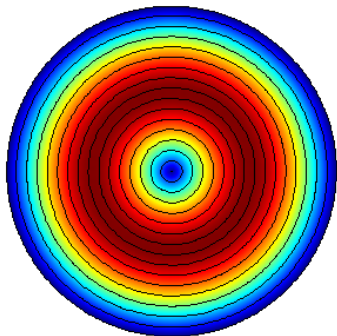
Transverse electric field E_T .

TM₀₁.



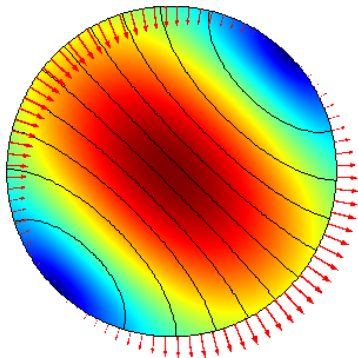
Transverse electric field E_T .

TE₀₁.



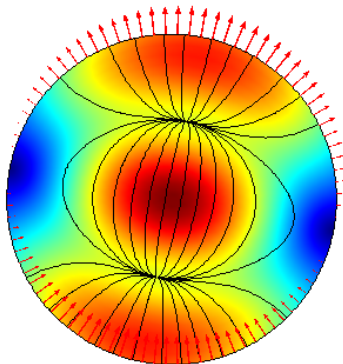
Transverse electric field E_T .

Fundamental mode TE_{11} .



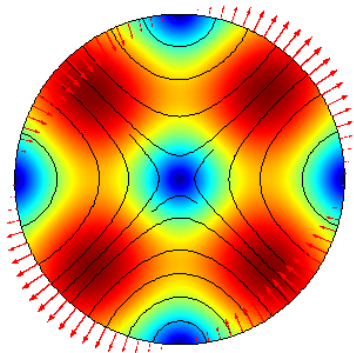
Transverse electric field E_T .

TM₁₁.



Transverse electric field E_T .

TE₂₁.



Transverse electric field E_T .

TE₃₁.

