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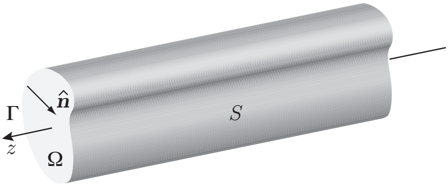
Anders Karlsson, anders.karlsson@eit.lth.se

Electrical and information technology

Outline

- Waveguides
- Eigenvalue problems for TE and TM
- TEM modes
- Rectangular waveguide

Hollow waveguide

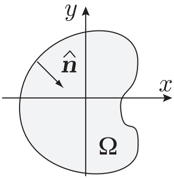


Two different type of waves can propagate in this waveguide!

TM-waves ⇒ transverse magnetic field ⇒ $H_z = 0$

TE-waves ⇒ transverse electric field ⇒ $E_z = 0$

TM-waves

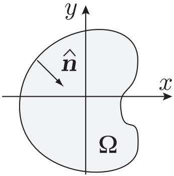


TM$\Rightarrow$ $H_z = 0$ (transverse magnetic field), $E_z(\boldsymbol{r}) = v(\boldsymbol{\rho})e^{\mathrm{i}k_z z}$, $\boldsymbol{\rho} = (x, y)$

$$\nabla_T^2 v(\boldsymbol{\rho}) + k_t^2 v(\boldsymbol{\rho}) = 0,\ \boldsymbol{\rho} \in \Omega$$
$$v(\boldsymbol{\rho}) = 0,\ \boldsymbol{\rho}\,\text{on}\,\Gamma$$

Eigenvalue problem!
Eigenvalues: k_{tn}^2
Eigenfunctions: v_n, $n = 1, 2 \ldots \infty$

TE-waves

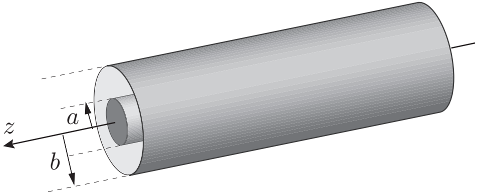


TE$\Rightarrow$ $E_z = 0$ (transverse electric field), $H_z(\boldsymbol{r}) = w(\boldsymbol{\rho})e^{\mathrm{i}k_z z}$

$$\nabla_T^2 w(\boldsymbol{\rho}) + k_t^2 w(\boldsymbol{\rho}) = 0,\ \boldsymbol{\rho} \in \Omega$$
$$\hat{\boldsymbol{n}} \cdot \nabla w(\boldsymbol{\rho}) = 0,\ \boldsymbol{\rho}\,\text{on}\,\Gamma$$

Eigenvalue problem for $w(\boldsymbol{\rho})$!
Eigenvalues: k_{tn}^2
Eigenfunctions: w_n, $n = 1, 2 \ldots \infty$

Hollow waveguide with two conductors



Three different type of waves can propagate in this waveguide!
TM-waves ⇒ transverse magnetic field ⇒ $H_z = 0$, $E_z = v(\boldsymbol{\rho})e^{\mathrm{i}k_z z}$
TE-waves ⇒ transverse electric field ⇒ $E_z = 0$, $H_z = w(\boldsymbol{\rho})e^{\mathrm{i}k_z z}$
TEM-wave ⇒ transverse electric and magnetic field $H_z = 0$ and $E_z = 0$. $\boldsymbol{E} = -\nabla\phi(\boldsymbol{\rho})e^{\mathrm{i}kz}$

Scheme TM

1. Solve eigenvalue problem for $v_n(\rho)$ and k_{tn}^2.
2. Propagation constant $k_{zn} = \sqrt{k^2 - k_{tn}^2}$
3. $E_{zn}(\boldsymbol{r}) = v_n(\boldsymbol{\rho})e^{\mathrm{i}k_{zn}z}$
4. E_x, E_y, H_x, H_y are obtained from Chapter 4

Scheme TE

1. Solve eigenvalue problem for $w_n(\rho)$ and k_{tn}^2.
2. Propagation constant $k_{zn} = \sqrt{k^2 - k_{tn}^2}$
3. $H_{zn}(\boldsymbol{r}) = w_n(\boldsymbol{\rho})e^{\mathrm{i}k_{zn}z}$
4. E_x, E_y, H_x, H_y are obtained from Chapter 4

$z-$dependence

All components must have the same $z-$dependence $e^{\mathrm{i}k_z z}$ (positive z-direction), or $e^{-\mathrm{i}k_z z}$ (negative z-direction). We solve for E_z or H_z

$$E_z(\boldsymbol{r}) = v(\boldsymbol{\rho})e^{\mathrm{i}k_z z}$$

Cut-off frequency

$f_{cn} = \frac{c}{2\pi} k_{tn}$ =cut-off frequency
Three cases

1. $f < f_{cn} \Rightarrow k_{zn}$ imaginary $\Rightarrow$ non-propagating mode
2. $f = f_{cn} \Rightarrow k_{zn} = 0 \Rightarrow$ Cut-off
3. $f > f_{cn} \Rightarrow k_{zn} > 0$ and real $\Rightarrow$ propagating mode