# Microwave theory, lecture 3, 2016

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Electrical and information technology

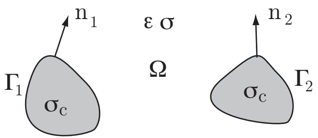
# Last lecture

- Standing wave ratio (SWR)
- Lossy line

# Today

- Determination of $R$, $L$, $G$, $C$
- COMSOL example
- $S-$parameters
- Smith chart
- Introduction to hollow waveguides

# $R,\ L,\ G,\ C$




Solve Laplace equation $\nabla^2\Phi(x,y)$ in $\Omega$.

$$\nabla^2\Phi(x,y) = 0$$

$$\Phi(x,y) = \begin{cases} V/2 & \text{on } \Gamma_1 \\ -V/2 & \text{on } \Gamma_2 \end{cases}$$

$$C = \frac{Q}{V} = \frac{\oint_{\Gamma_1} \rho_S \, d\ell}{V}$$

or

$$C = 2\frac{W_e}{V^2}$$

# $L$, $G$, $R$

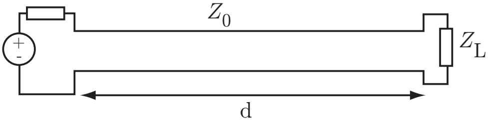
$L = \frac{\mu_0\epsilon_0\epsilon}{C}$ (if losses are small)

$$G = \frac{C\sigma}{\epsilon_0\epsilon}$$

$$R = R_s\frac{\int_{\Gamma_1+\Gamma_2}(\rho_S)^2\,d\ell}{\left(\int_{\Gamma_1}\rho_S\,d\ell\right)^2}$$

$R_S = \sqrt{\frac{\omega\mu_0}{2\sigma_c}}$=surface resistance.

# Smith chart



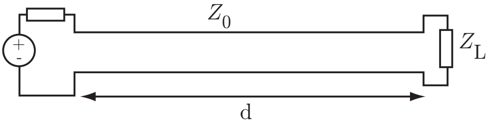

$$\Gamma(0) = \frac{Z(0) - Z_0}{Z(0) + Z_0} = \frac{Z_L - Z_0}{Z_L + Z_0} e^{-2j\beta d} = \Gamma_d e^{-2j\beta d}$$

Normalize with $Z_0$

$$z(0) = \frac{Z(0)}{Z_0} = \frac{R(0) + jX(0)}{Z_0} = r(0) + jx(0)$$

$$z_L = \frac{Z_L}{Z_0}$$

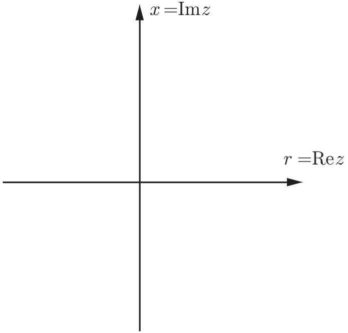
# Smith chart

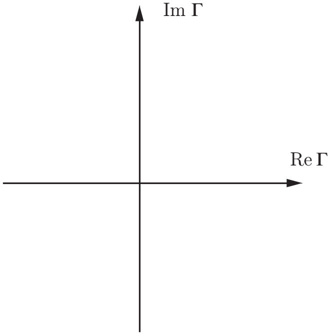



$$\Gamma(0) = \frac{z(0) - 1}{z(0) + 1} = \frac{z_L - 1}{z_L + 1} e^{-2j\beta d}$$

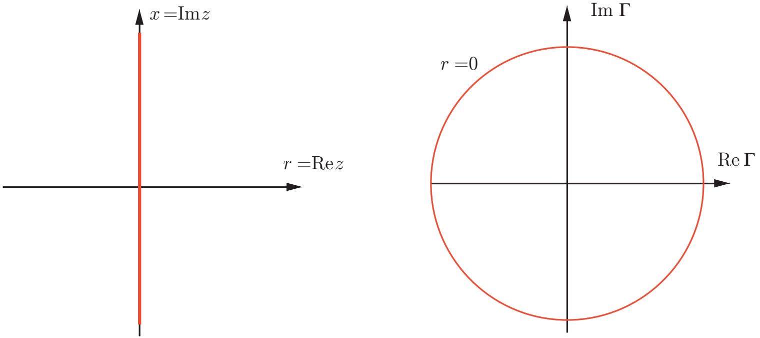
Plot $\Gamma(0)$, $r(0)$, $x(0)$ in the same diagram $\Rightarrow$ Smith chart!

# Smith chart

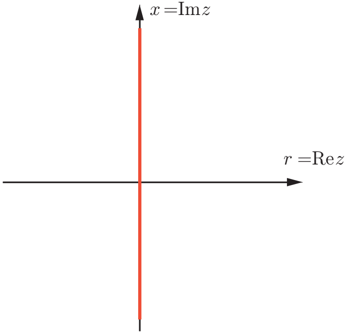


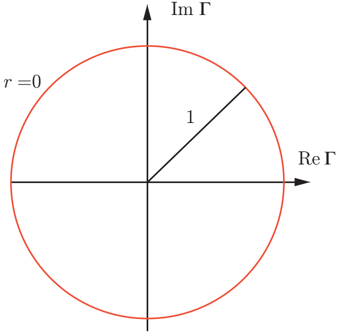
# Smith chart


$x =\text{Im}\, z$
$r =\text{Re}\, z$
$\text{Im}\,\boldsymbol{\Gamma}$
$r =0$
$\text{Re}\,\boldsymbol{\Gamma}$

# Smith chart

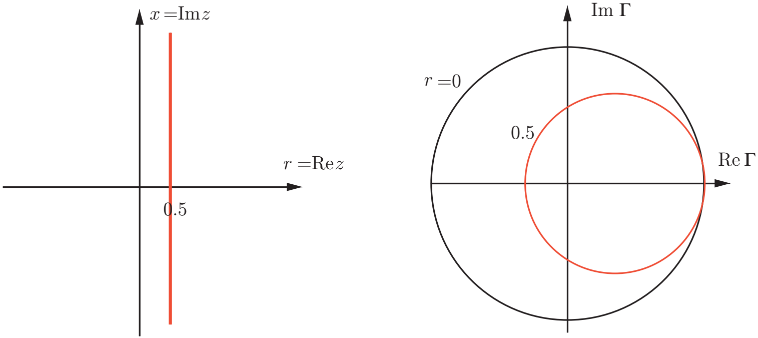


$x = \mathrm{Im}\, z$
$r = \mathrm{Re}\, z$

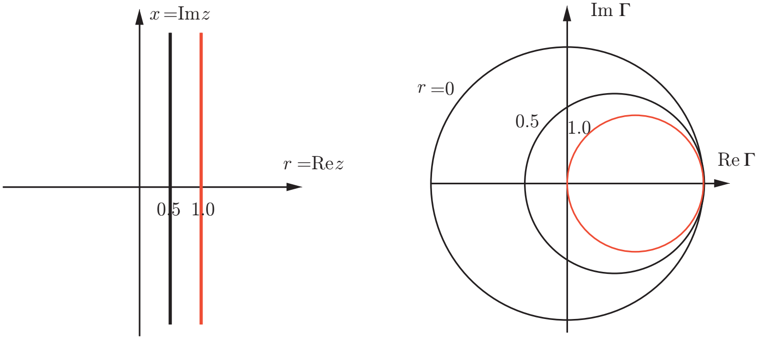



$\mathrm{Im}\, \mathbf{\Gamma}$
$r = 0$
1
$\mathrm{Re}\, \mathbf{\Gamma}$

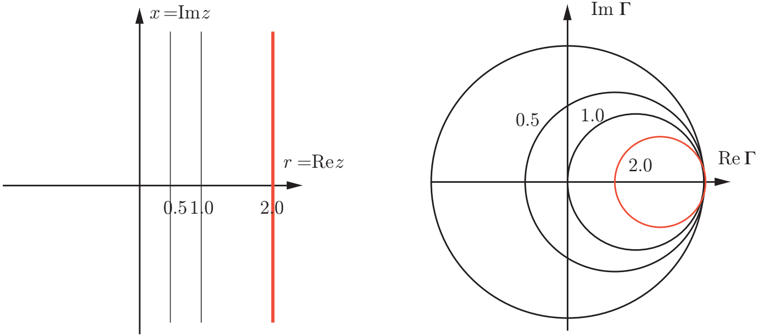
# Smith chart


$x = \mathrm{Im}\, z$
$r = \mathrm{Re}\, z$
0.5
$\mathrm{Im}\, \Gamma$
$r = 0$
0.5
$\mathrm{Re}\, \Gamma$

# Smith chart

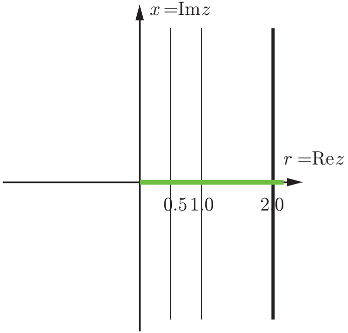


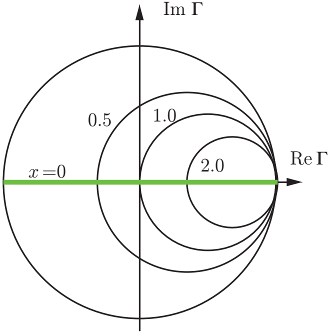
# Smith chart



$x = \mathrm{Im}\, z$
$r = \mathrm{Re}\, z$
0.5
1.0
2.0
$\mathrm{Im}\, \Gamma$
$\mathrm{Re}\, \Gamma$
0.5
1.0
2.0

# Smith chart



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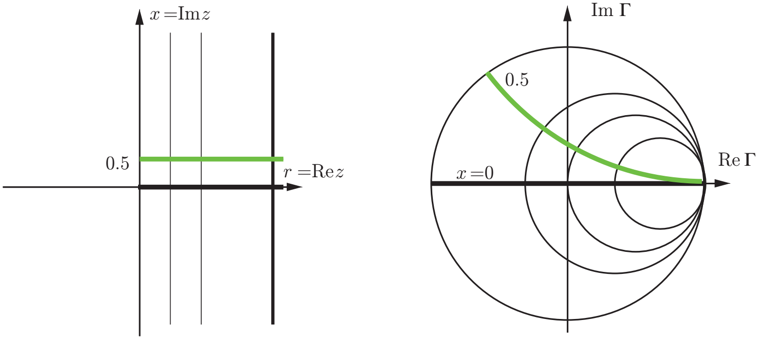

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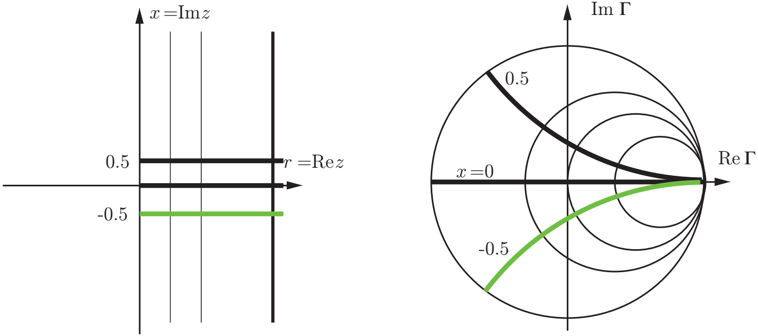
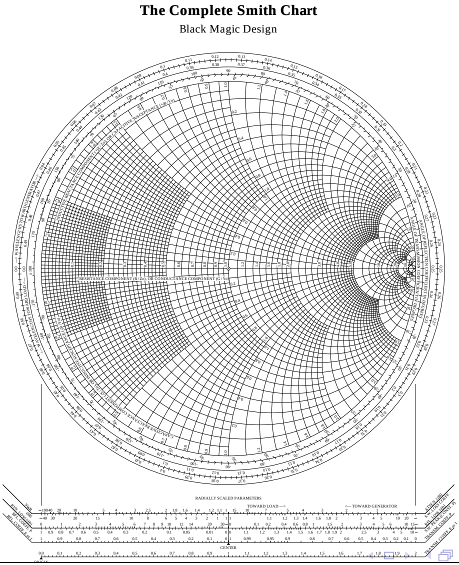
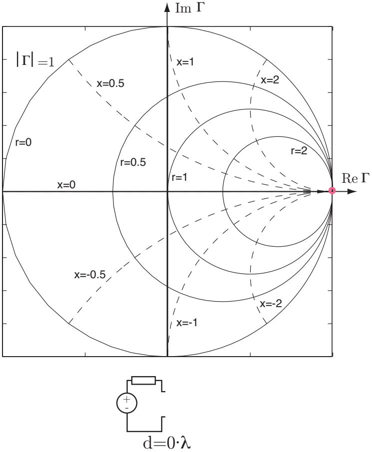

$x = \mathrm{Im}\, z$
0.5
$r = \mathrm{Re}\, z$
-0.5
$\mathrm{Im}\, \Gamma$
0.5
$\mathrm{Re}\, \Gamma$
$x = 0$
-0.5

# Smith chart

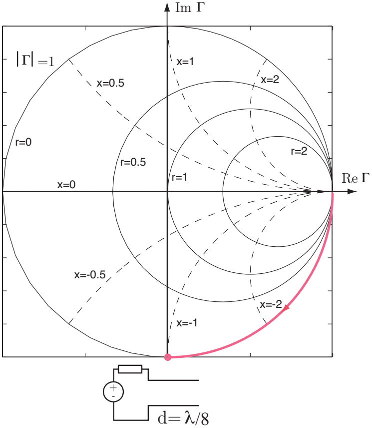

The Complete Smith Chart
Black Magic Design
RESISTANCE COMPONENT (R/Zo), OR CONDUCTANCE COMPONENT (G/Yo)
RADIALLY SCALED PARAMETERS
TOWARD LOAD —>
<— TOWARD GENERATOR
CENTER

# Smith chart: example 1



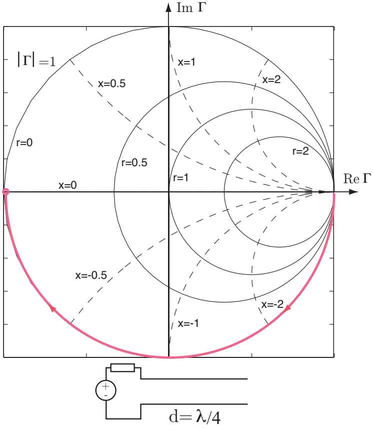

$$\beta d = 0 \Rightarrow \Gamma(0) = \Gamma_d = 1$$

# Smith chart: example 1



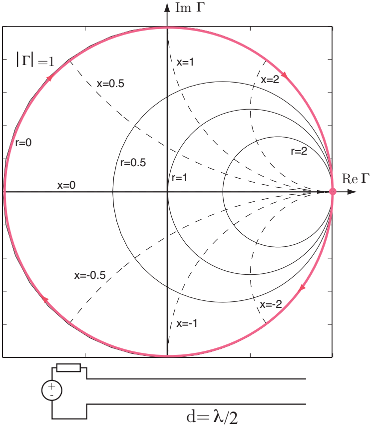

$$\beta d = \pi/4 \Rightarrow \Gamma(0) = -i$$

# Smith chart: example 1



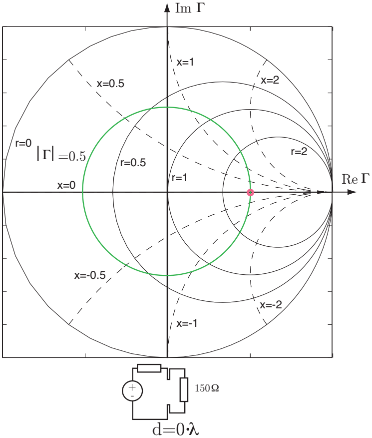

$$\beta d = \pi/2 \Rightarrow \Gamma(0) = -1$$

# Smith chart: example 1



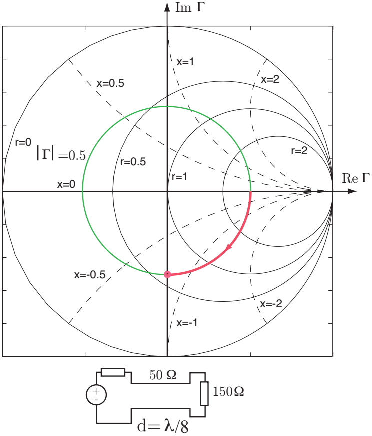

$$\beta d = \pi \Rightarrow \Gamma(0) = \Gamma_d = 1$$

# Smith chart: example 2



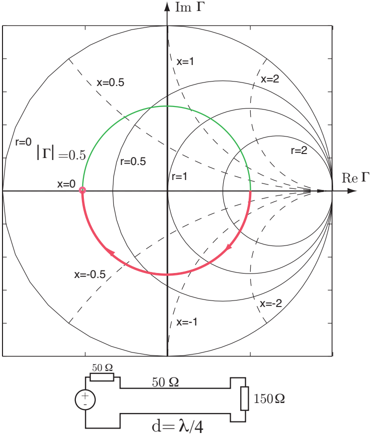

$$\beta d = 0 \Rightarrow \Gamma(0) = \Gamma_d = \frac{150-50}{150+50} = 0.5$$

# Smith chart: example 2



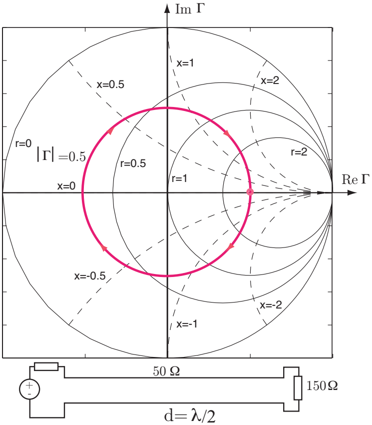

$$\beta d = \pi/4 \Rightarrow \Gamma(0) = -0.5\text{i}$$

# Smith chart: example 2




$$\beta d = \pi/2 \Rightarrow \Gamma(0) = -0.5$$

# Smith chart: example 2




$$\beta d = \pi \Rightarrow \Gamma(0) = \Gamma_d = 0.5$$

The admittance diagram

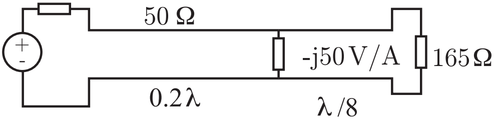
$Y = \dfrac{1}{Z} = G + \mathrm{j}B =$admittance

$G =$conductance, $B =$susceptance.

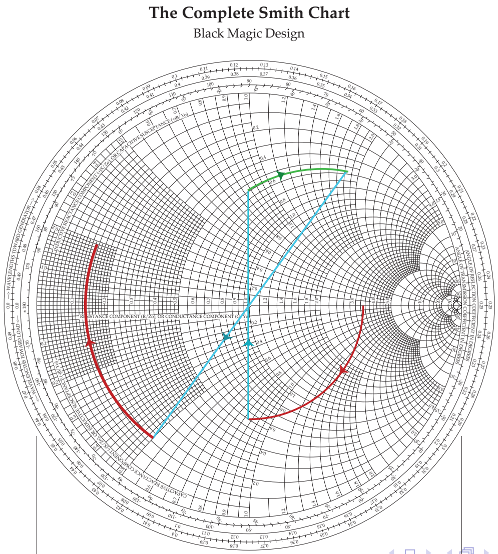
$$\Gamma(0) = \frac{z(0)-1}{z(0)+1} = \frac{1-y(0)}{1+y(0)} = -\frac{y(0)-1}{y(0)+1}$$

Go to $-\Gamma$ in Smith chart and exchange $g$ for $r$ and $b$ for $x$.

# Smith chart



# Smith chart


The Complete Smith Chart
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RESISTANCE COMPONENT (R/Zo), OR CONDUCTANCE COMPONENT (G/Yo)
WAVELENGTHS TOWARD GENERATOR
WAVELENGTHS TOWARD LOAD
ANGLE OF REFLECTION COEFFICIENT IN DEGREES
ANGLE OF TRANSMISSION COEFFICIENT IN DEGREES

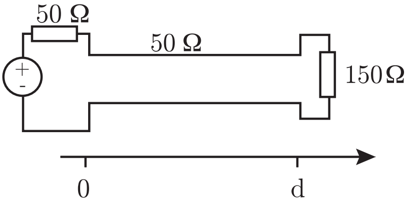
# Impedance match by Smith chart

Goal: Try to reach $z = 1 \Rightarrow \Gamma = 0$.
Rules:

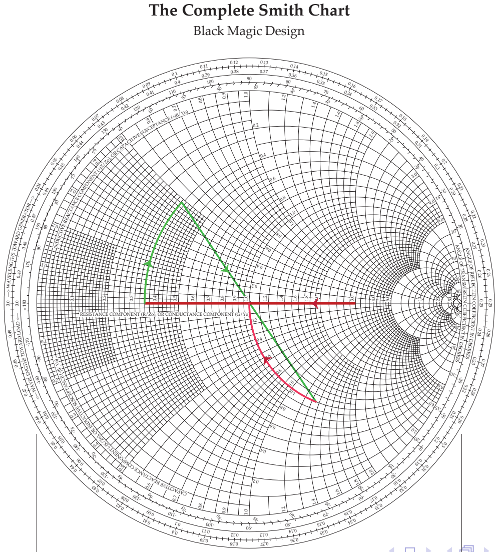
- You cannot add resistance, only reactive components.
- You are allowed to move along the transmission line.
- You are allowed to go to admittance diagram and add susceptance.

# Impedance match by Smith chart example

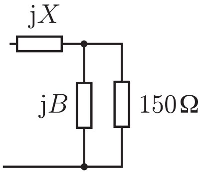
Goal: Match with two reactive components at $z = d$.



# Impedance match by Smith chart example



# Impedance match by Smith chart example




$$B = \frac{0.475}{50} = 9.5 \text{ mA/V and } X = 1.38 \cdot 50 = 69 \text{ V/A.}$$

$$Z = \mathrm{j}69 + \frac{1}{\frac{1}{150} + \mathrm{j}\frac{0.475}{50}} \approx 49.5\,\Omega - \mathrm{j}1.5 \text{ V/A}$$