Microwave 2016: *Assignment 1*

The solutions should be handed in no later than April 11. You can either send a pdf-file or give a paper copy to Anders (anders.karlsson@eit.lth.se). On the problem sessions on Tuesdays you can get some guidance from Anders if you are stuck.

Problem 1

Solve problem 3.8 in the book

Problem 2

A lossless transmission line has the characteristic impedance $Z_0 = 60\,\Omega$. The line is terminated by a load impedance $Z_b = 120\,\Omega$. Determine the reflection coefficient at the load and the standing wave ratio S.

Problem 3

Show that the time average of the electric energy per unit length is equal to the time average of the magnetic energy per unit length for a time harmonic wave that travels along a lossless transmission line. The easiest way to do this is to express the energies in terms of current, voltage, capacitance and inductance.

Problem 4

Long ago the telephone wires were hanging in the air attached to poles. A problem with these wires was that they were not lossless since both R and G were non-zero. That implied that the signals were distorted and already after 10 km the distorsion had severely affected the sound of a human voice. In order to avoid distorsion, and by that increase the distans for telephone calls, a special type of coil, called Pupin coil, was used. Suppose that one likes to avoid distortion along a transmission line by using Pupin coils. The coils are placed every kilometer along the line. Determine an expression for the inductance L_p of each coil such that there is no distortion of the waves. The expression for L_p should include the parameters R, L, G, and C for the transmission line.

Problem 5

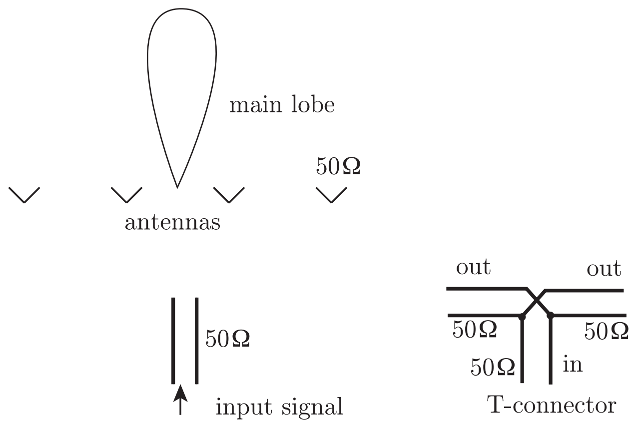


Figure 1: The four antennas in the array.

A linear array antenna consists of four identical equidistant patch antennas, each having an impedance $Z_a = 50\,\Omega$. The array is a broad side antenna which means that the main lobe is directed perpendicular to the line along the array, see figure. This is obtained if the input voltages to the four antennas have the same phase. The signal is sent from a generator via a coaxial cable with characteristic impedance 50 Ω. The frequency is 1 GHz. Find a way to connect the four antennas to the generator's cable such that there is no reflected wave coming back to the generator. To your disposition you have T-connectors that connect three cables as indicated in the figure. You also have as much 50 Ω coaxial cable as you like. The insulation between the inner and outer conductor of the coaxial cable is non-magnetic ($\mu = 1$) and has a relative permittivity $\varepsilon_r = 2.25$. Show with a figure how you connect the antennas and the cables and give the lengths in centimeters of the different cables you use.

Problem 6

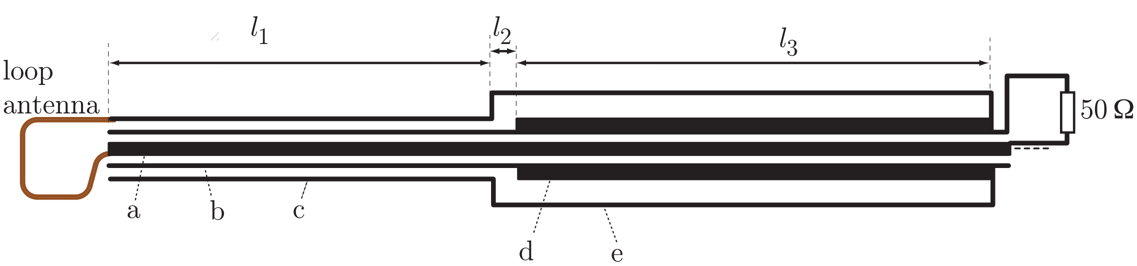
A coaxial cable has an inner conductor with diameter 0.90 mm and an outer conductor with (inner)diameter 2.95 mm. The material between the conductors is polythene with $\varepsilon = 2.1$ and conductivity $\sigma = 4.0 \cdot 10^{-7}$S/m. The conductors are silverplated. Silver has conductivity $\sigma = 6.3 \cdot 10^{7}$S/m.
a) Determine R, L, G, C, the characteristic impedance Z_0, and the phase speed v_p for the cable using the analytic formulas. The frequency is $f = 100$ MHz.
b) Determine the attenuation expressed in dB/100m for the cable at frequencies 100 MHz, 200 MHz, 500 MHz. Compare these results with the values given by the ELFA catalogue: 12.7 dB/100 m at 100 MHz, 18.3 dB/100 m at 200 MHz and 26.7 dB/100 m at 400 MHz. Is it the isolation between the cables or the resistivity of the cables that is the major cause to the attenuation?
c) Determine R, L, G, C using COMSOL when the frequency is $f = 100$ MHz. How many correct digits can you get with COMSOL compared to the analytic results?

Comment: The losses are very small and they can be neglected when you calculate Z_0 and v_p. The cable is in the ELFA catalogue denoted M17/084-RG223 and is used for radio communication equipment and video/camera equipment.

Problem 7

A two-wire transmission line consists of two identical parallel circular conductors. The conductors are made of copper and the material between the cables is polythene with $\varepsilon = 2.1$ and conductivity $\sigma = 4.0 \cdot 10^{-7}$ S/m. The radius of the conductors is 2 mm and the distance between the centers of the wires is $c = 5$ mm.
a) Determine R, L, G and C for the cable using the analytic formulas. The frequency is $f = 100$ MHz.
b) Determine R, L, G and C using COMSOL when the frequency is $f = 100$ MHz. How many correct digits can you get with COMSOL compared to the analytic results?
Note: Make sure that the computational domain is large enough by comparing results from two different sizes of the domain.

Problem 8



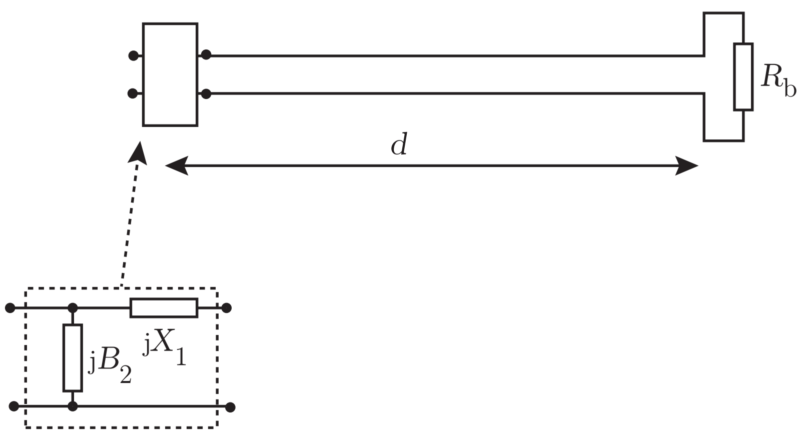
In synchrotron accelerators the electron travel in a circular ring and radiate light. The radiated energy equals the loss of kinetic energy of the electrons. There are a number of cavities along the ring that compensates for the loss of energy by accelerating the electrons. At the Max IV accelerator the cavities are designed such that the lowest resonance frequency (the fundamental frequency) is 100 MHz and it is the electric field of this resonance that accelerates the electrons. The beam acts as a source and excites resonances with higher frequencies than 100 MHz in the cavity (so called higher order modes). These modes might damage the beam if they grow strong. For this reason one plans to have a damper for the higher order modes. The damper consists of a loop antenna, a coaxial cable with a 50 Ω load and a notch filter (bandstop filter). The design is given in the figure where a is a solid circular cylindric conductors and b, c, d and e are hollow circular cylindric conductors. There is vacuum in all of the structure, and the radius and lengths are as follows:

$$\text{radius of}\, a = 4.3\,\text{mm}$$
$$\text{inner radius of}\, b = 10\,\text{mm}$$
$$\text{outer radius of}\, b = 11\,\text{mm}$$
$$\text{inner radius of}\, c = 17\,\text{mm}$$
$$\text{inner radius of}\, d = 10\,\text{mm}$$
$$\text{outer radius of}\, d = 17.5\,\text{mm}$$
$$\text{inner radius of}\, e = 38.1\,\text{mm}$$
$$\ell_1 = 279.5\,\text{mm},\ \ell_2 = 20\,\text{mm},\ \ell_3 = 354\,\text{mm}$$

a) Determine the characteristic impedance of the inner coaxial cable (the one between a and b).
b) Why is the inner coaxial cable terminated by a 50 Ω load?
c) The notch filter is the outer coaxial structure that consists of three parts with lengths ℓ_1, ℓ_2 and ℓ_3. Describe how you can obtain the input impedance of this filter by viewing the filter as three coaxial cables in series. Write a Matlab program that calculates the input impedance and plot it in the frequency range 10 MHz-700 MHz. Notice that the input impedance of the notch filter must be purely reactive.
d) Draw an equivalent discrete circuit where the loop antenna is a voltage source and where the notch filter and the inner coaxial cable are replaced by impedances.
e) Explain why the design will fullfil its purpose, *i.e.*, to prevent the electromagnetic field of the fundamental resonance to escape from the cavity and to let the other resonances be absorbed by the load.
f) The notch filter will also prevent some other frequencies to escape from the cavity. One should avoid to have resonances at these frequencies. Determine the two lowest frequencies above the fundamental frequency that are stopped.

Problem 9

A resistive load $R_b = 150\,\Omega$ is connected to a line with $Z_0 = 50\,\Omega$.
a) Mark the point for the input impedance at the load in the Smith chart.
b) Determine the input impedance at a distance $\ell = 0.3\lambda$ from the load by using the Smith chart.
c) Start at the load and move towards the generator until you reach the position where the input impedance is purely resistive. What is the input impedance at that position and what is the distance, d to the load, measured in wavelengths?



d) Use the Smith chart to determine the series reactance X_1 and shunt susceptance B_2 you need to add at the distance d from the load in order for the input impedance to be 50 Ω at this position, see figure. Compare your result with the expressions in section 3.7.3. Notice that there are two solutions. It is enough that you present one.