



# Microwave theory, April 13, 2015

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Electrical and information technology

# Waveguide modes

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TM-waves  $\Rightarrow H_z = 0 \quad E_z = v(\boldsymbol{\rho})E^{ik_z z}$

Eigenvalue problem

$$\nabla^2 v(\boldsymbol{\rho}) + k_t^2 v(\boldsymbol{\rho}) = 0, \quad \boldsymbol{\rho} \in \Omega$$

$$v(\boldsymbol{\rho}) = 0, \quad \boldsymbol{\rho} \in \Gamma$$

Eigenvalues  $k_{tn}^2$  and eigenfunctions  $v_n(\boldsymbol{\rho})$ ,  $n = 1, 2, 3 \dots \infty$

## Entire EM-field for TM-modes

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Once all  $v_n$  are known we can construct the entire EM-field for the TE-mode (section 5.4):

$$\begin{aligned}\mathbf{E}_n^\pm(\mathbf{r}) &= (\mathbf{E}_{Tn}(\boldsymbol{\rho}) \pm v_n(\boldsymbol{\rho})\hat{\mathbf{z}}) e^{\pm ik_{zn}z} \\ \mathbf{H}_n^\pm &= \pm \mathbf{H}_{Tn}(\boldsymbol{\rho}) e^{\pm ik_{zn}z}\end{aligned}$$

where

$$\begin{aligned}\mathbf{E}_{Tn}(\boldsymbol{\rho}) &= \frac{i}{k_{tn}^2} k_{zn} \nabla_T v_n(\boldsymbol{\rho}) \\ \mathbf{H}_{Tn}(\boldsymbol{\rho}) &= Z_{nTM}^{-1} \hat{\mathbf{z}} \times \mathbf{E}_{Tn}(\boldsymbol{\rho}) \\ Z_{nTM} &= \frac{k_{zn}}{\omega \epsilon_0 \epsilon}\end{aligned}$$

# Outline for today

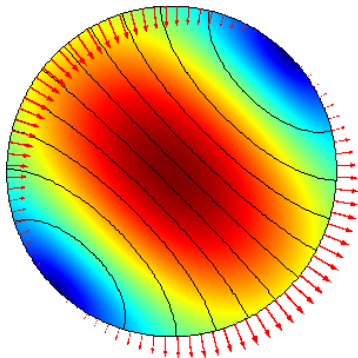
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- ▶ Circular cylindrical waveguides
- ▶ Bessel functions
- ▶ The fundamental mode  $TE_{11}$
- ▶ The mode  $TM_{01}$
- ▶ Comsol example

# Circular cylindrical waveguide

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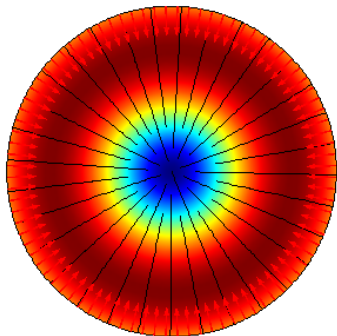
Fundamental mode  $TE_{11}$ .



# Circular cylindrical waveguide

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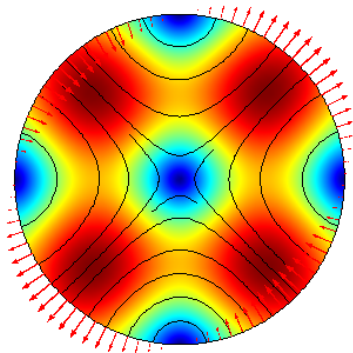
TM<sub>01</sub>.



# Circular cylindrical waveguide

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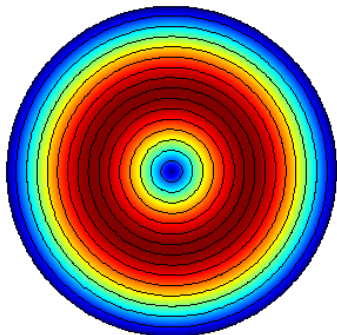
$TE_{21}$ .



# Circular cylindrical waveguide

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$TE_{01}$ .

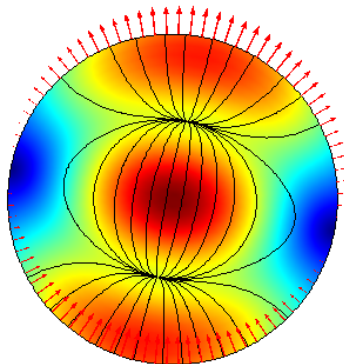




# Circular cylindrical waveguide

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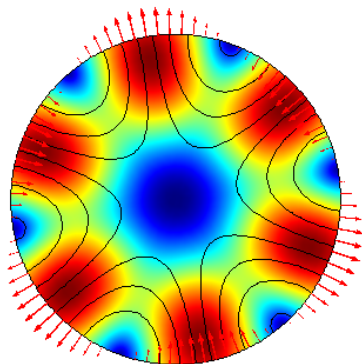
TM<sub>11</sub>.



# Circular cylindrical waveguide

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$TE_{31}$ .



# Bessel functions

