

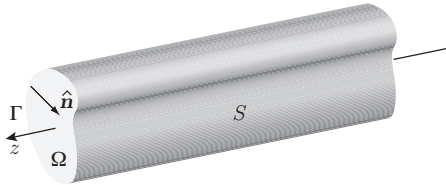


Microwave theory, April 7, 2015

Anders Karlsson, anders.karlsson@eit.lth.se

Electrical and information technology

Hollow waveguides



Three type of waves:

- ▶ TE-waves $\Rightarrow E_z = 0, H_z = w(\boldsymbol{\rho})e^{ik_z z}$
- ▶ TM-waves $\Rightarrow H_z = 0, E_z = v(\boldsymbol{\rho})e^{ik_z z}$
- ▶ TEM-waves $\Rightarrow E_z = 0$ and $H_z = 0, \mathbf{E} = -\nabla\phi(\boldsymbol{\rho})e^{ik_z z}$
(TEM only in waveguides with at least two conductors)

Hollow waveguides

Eigenvalue problem TE-waves

$$\nabla^2 w(\boldsymbol{\rho}) + k_t^2 w(\boldsymbol{\rho}) = 0, \boldsymbol{\rho} \in \Omega$$

$$\frac{\partial w(\boldsymbol{\rho})}{\partial n} = 0, \boldsymbol{\rho} \in \Gamma$$

Eigenvalues k_{tn}^2 and eigenfunctions $w_n(\boldsymbol{\rho})$

Hollow waveguides

Eigenvalue problem TM-waves

$$\nabla^2 v(\boldsymbol{\rho}) + k_t^2 v(\boldsymbol{\rho}) = 0, \boldsymbol{\rho} \in \Omega$$

$$v(\boldsymbol{\rho}) = 0, \boldsymbol{\rho} \in \Gamma$$

Eigenvalues k_{tn}^2 and eigenfunctions $v_n(\boldsymbol{\rho})$

Rectangular waveguide TE-modes

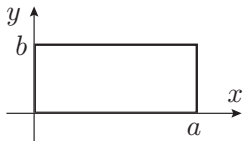


TE-waves \Rightarrow
Eigenvalue problem

$$\nabla^2 w(\boldsymbol{\rho}) + k_t^2 w(\boldsymbol{\rho}) = 0, \boldsymbol{\rho} \in \Omega$$

$$\frac{\partial w(\boldsymbol{\rho})}{\partial n} = 0, \text{ on all four sides}$$

Rectangular waveguide TE-modes



Eigenvalues: $k_{tmn}^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$

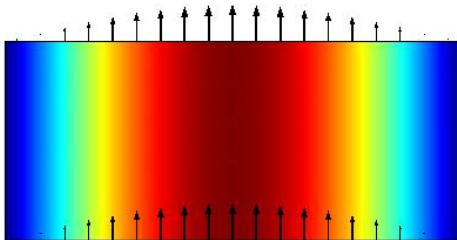
Eigenfunctions: $w_{mn}(\boldsymbol{\rho}) = A_{mn} \cos\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right)$

$m = 0, 1, 2, \dots, n = 0, 1, 2, \dots$

but $(m, n) \neq (0, 0)$

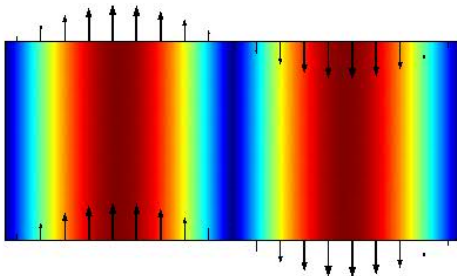
Rectangular waveguide

The TE_{10} mode. Fundamental mode!



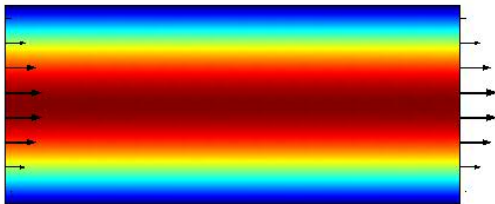
Rectangular waveguide

The TE_{20} mode



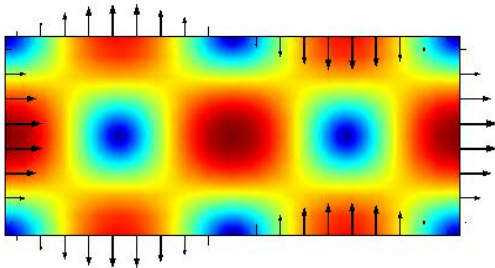
Rectangular waveguide

The TE_{01} mode



Rectangular waveguide

The TE_{21} mode



Cut-off frequencies

$$e^{ik_z z}$$

$$k_z = \sqrt{k^2 - k_{tmn}^2}.$$

1. $k < k_{tmn} \implies k_z$ imaginary \implies non-propagating mode
2. $k = k_{tmn} \implies k_z = 0 \implies$ standing wave, cut-off frequency f_c
3. $k > k_{tmn} \implies k_z$ real \implies propagating mode

Today

- ▶ TM-wave in rectangular waveguides
- ▶ COMSOL
- ▶ Dispersion, phase- and group speeds
- ▶ The fundamental mode TE_{10} .