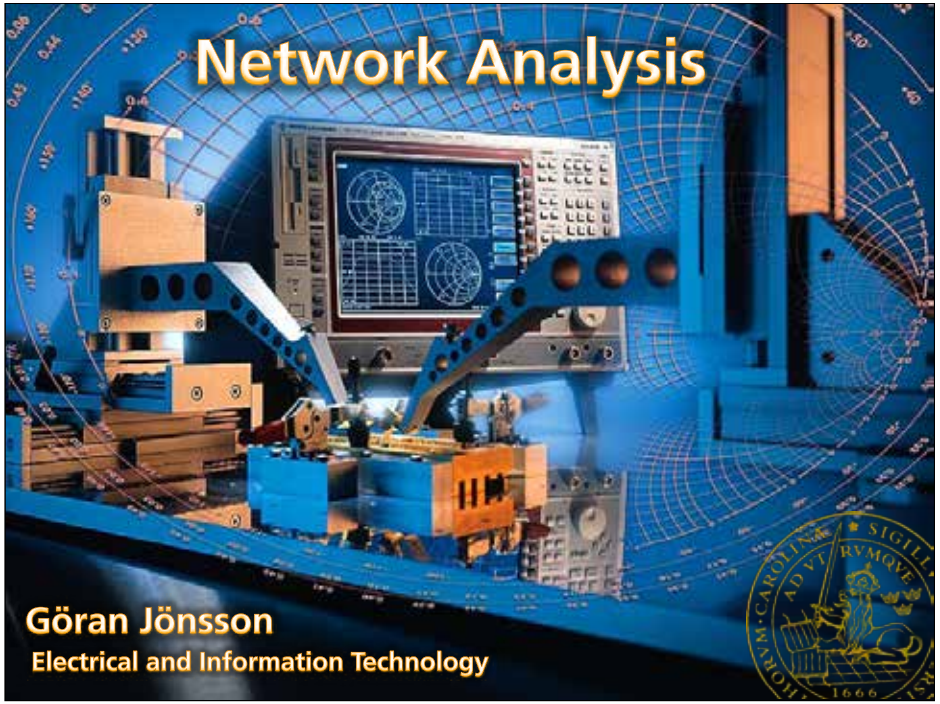


Network Analysis

Göran Jönsson

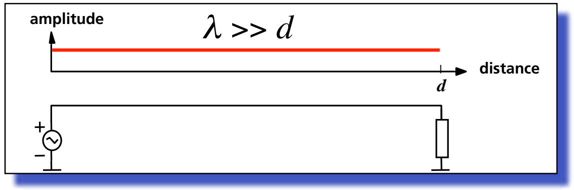
Electrical and Information Technology

Contents

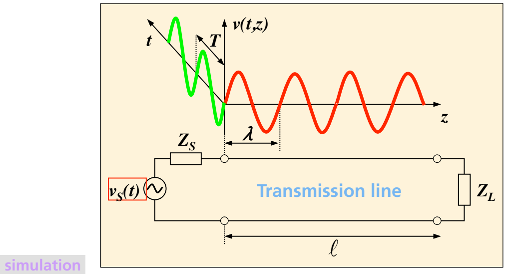
- **Transmission Lines**
- **The Smith Chart**
- **Vector Network Analyser (VNA)**
  - ✓ structure
  - ✓ calibration
  - ✓ operation
- **Measurements**

Waves on Lines

- **If the wavelength to be considered is significantly greater compared to the size of the circuit the voltage will be independent of the location.**



but this is not true at short wavelengths = high frequencies…



$$v = \frac{\lambda}{T} = \lambda \cdot f$$

$$\Rightarrow \lambda[\mathrm{m}] = \frac{v}{f} = \frac{300}{f[\mathrm{MHz}]}$$

The voltage or the current is a function of both time and distance

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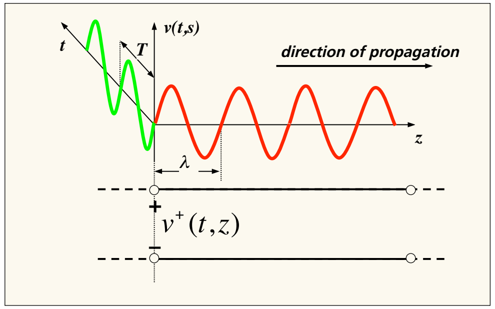
Travelling Voltage Wave on a Lossless Line



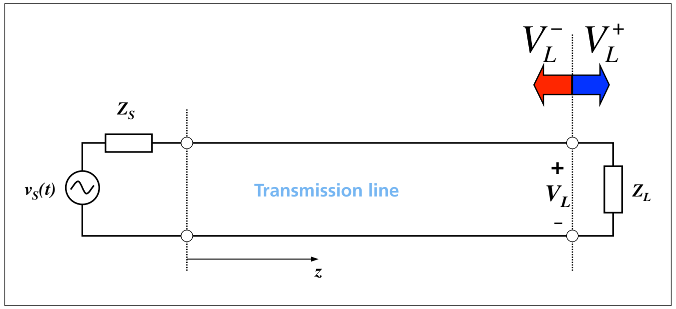
$$v^+(t,z) = \left|V_0^+\right| \cos(\omega t - \beta z + \phi_0^+) = \mathrm{Re}\left[V_0^+ e^{j(\omega t - \beta z)}\right]$$

– **where** $V_0^+ = \left|V_0^+\right| e^{j\phi_0^+}$ **= the complex amplitude of** $v^+(t,z)$ **at** $z = 0$

Reflection Coefficient

- **Definition:**

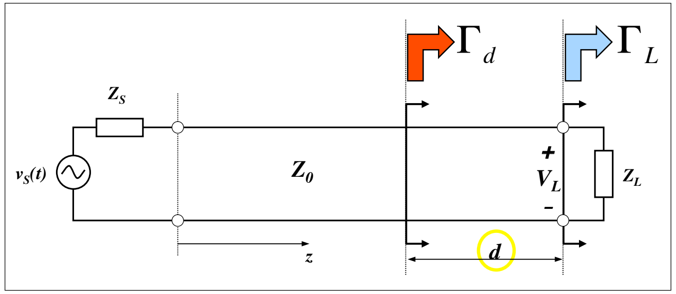
$$\Gamma = \frac{\text{reflected voltage wave}}{\text{incident voltage wave}} = \frac{V^- e^{\gamma z}}{V^+ e^{-\gamma z}}$$



Reflection Coefficient

- **At an arbitrary location *d* at the line the reflection coefficient is**

$$\Gamma_d = \Gamma_L e^{-2\gamma d} = \Gamma_L e^{-2\alpha d} e^{-2j\beta d}$$

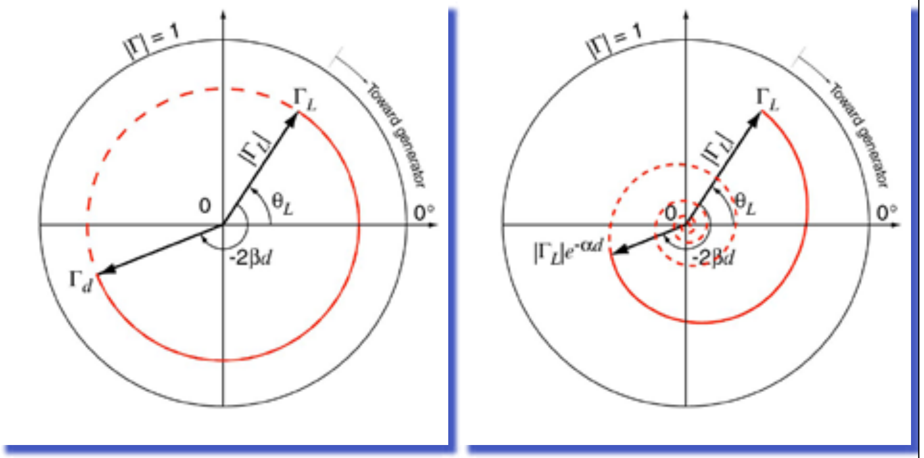


Reflection Coefficient

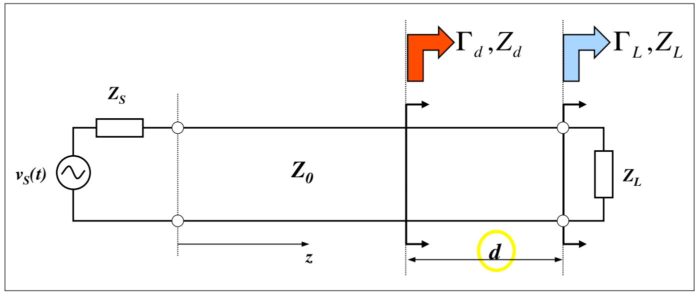
Implies a rotation in the polar Γ-plane

- **Polar diagram** $\Gamma_d = \Gamma_L e^{-2\gamma d} = \Gamma_L e^{-2\alpha d} e^{\boxed{-2j\beta d}}$

Lossless transmission line **Lossy transmission line**



Conversion of Reflection Coefficient to Impedance

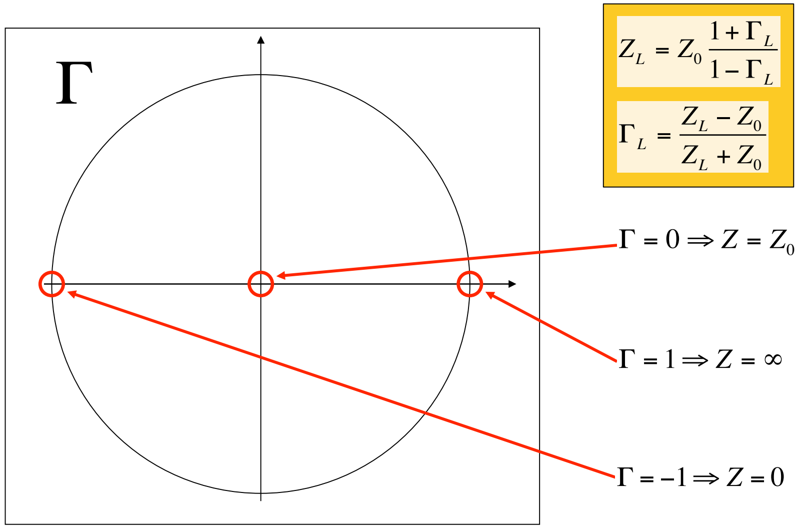


$$\Gamma_d = \frac{Z_d - Z_0}{Z_d + Z_0} \Rightarrow Z_d = Z_0 \frac{1+\Gamma_d}{1-\Gamma_d}$$

Reflection Coefficient – Load Impedance

$$Z_L = Z_0 \frac{1+\Gamma_L}{1-\Gamma_L}$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

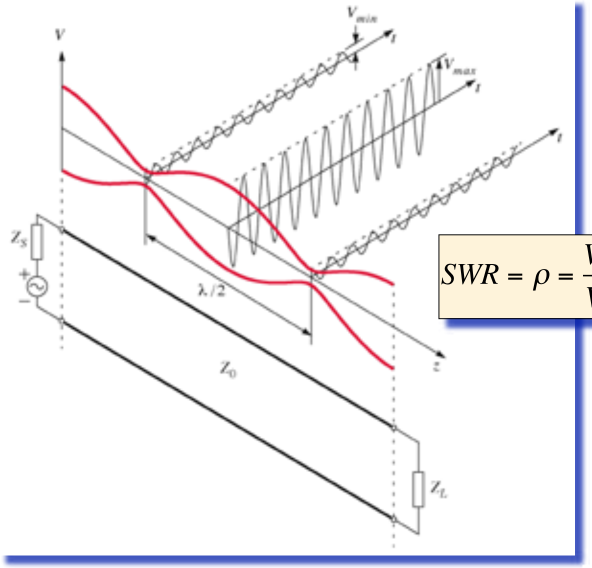


$\Gamma = 0 \Rightarrow Z = Z_0$

$\Gamma = 1 \Rightarrow Z = \infty$

$\Gamma = -1 \Rightarrow Z = 0$

Standing-Wave Ratio

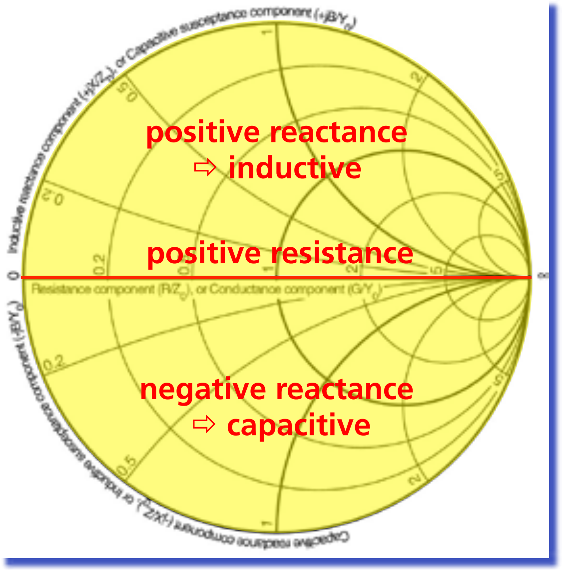


$$SWR = \rho = \frac{V_{max}}{V_{min}} = \frac{I_{max}}{I_{min}} = \frac{1+|\Gamma|}{1-|\Gamma|}$$

The Smith Chart

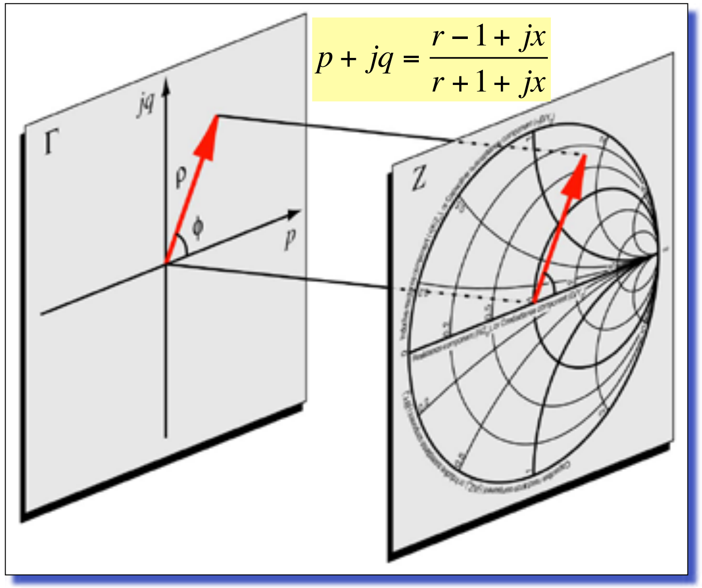
The chart was
invented by
Phillip Smith
in the early
1930-ties

Transform
between
Γ- and Z-plane



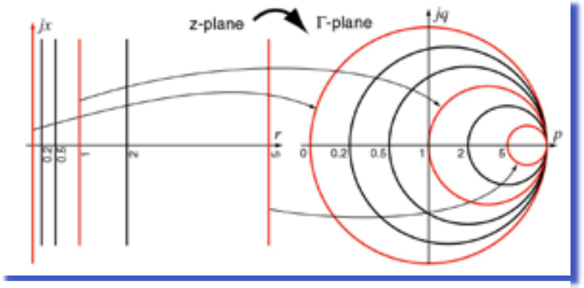
The Smith Chart

$$p + jq = \frac{r-1+jx}{r+1+jx}$$

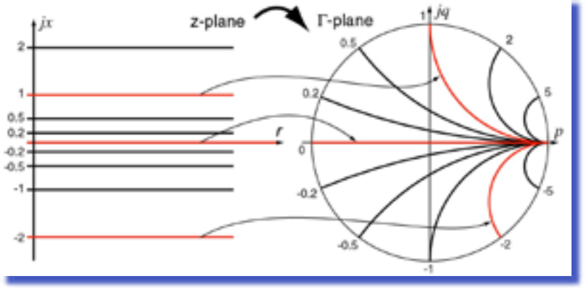


The Smith Chart Circles

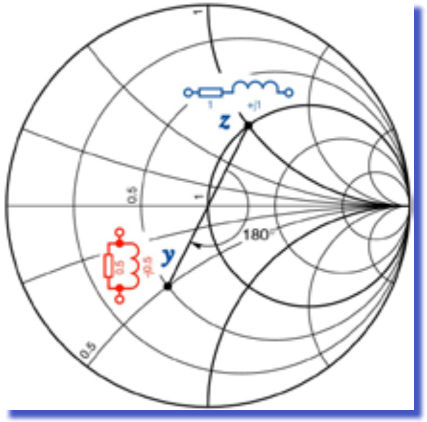
- Constant
  resistance lines ⇨
  resistance circles

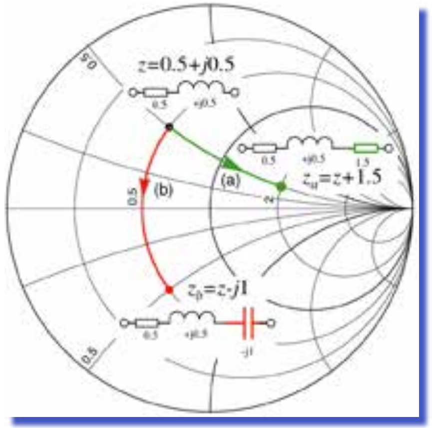


- Constant
  reactance lines ⇨
  reactance circles



Example of Smith Chart Usage





Conversion
impedance ⇨ admittance

$$z = \frac{1}{y}$$

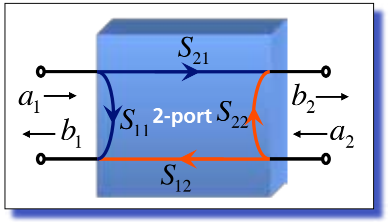
$$\Gamma(y) = \frac{1/y-1}{1/y+1} = \\ = -\frac{y-1}{y+1} = \\ = -\Gamma(z) = e^{j\pi}\Gamma(z)$$

- Series connection
  - Addition of resistance:
    - motion at constant reactance circle
  - Addition of reactance:
    - motion at constant resistance circle

Definition of S-parameters

- **Model:**

a_x = incident wave
b_x = reflected wave



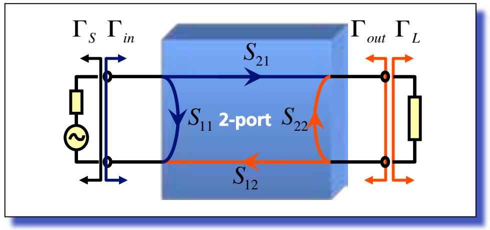
$$\begin{cases} b_1 = s_{11}\cdot a_1 + s_{12}\cdot a_2 \\ b_2 = s_{21}\cdot a_1 + s_{22}\cdot a_2 \end{cases} \qquad \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

$$S_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0} \qquad S_{12} = \left.\frac{b_1}{a_2}\right|_{a_1=0}$$

$$S_{21} = \left.\frac{b_2}{a_1}\right|_{a_2=0} \qquad S_{22} = \left.\frac{b_2}{a_2}\right|_{a_1=0}$$

IMPORTANT!
The definition utilizes
50Ω as
reference impedance

Measurement of S-parameters



$$\Gamma_{in} = S_{11} + S_{12}S_{21}\frac{\Gamma_L}{1-S_{22}\Gamma_L} = S_{11}\Big|_{\Gamma_L=0} \qquad \Gamma_{out} = S_{22} + S_{12}S_{21}\frac{\Gamma_S}{1-S_{11}\Gamma_S} = S_{22}\Big|_{\Gamma_S=0}$$

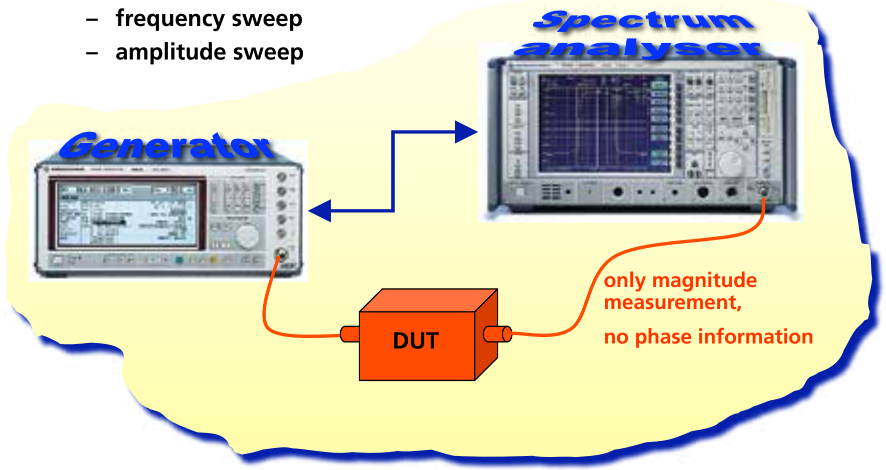
The S-parameters are easily measured if the ports are terminated by the reference impedance $Z_0 = 50\Omega$ (Γ_L respectively $\Gamma_S = 0$)

$$S_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0} \qquad S_{12} = \left.\frac{b_1}{a_2}\right|_{a_1=0}$$

$$S_{21} = \left.\frac{b_2}{a_1}\right|_{a_2=0} \qquad S_{22} = \left.\frac{b_2}{a_2}\right|_{a_1=0}$$

Scalar Network Analysis

- Characterising the **D**evice **U**nder **T**est properties
- Spectrum analyser + sweep generator
  - frequency sweep
  - amplitude sweep

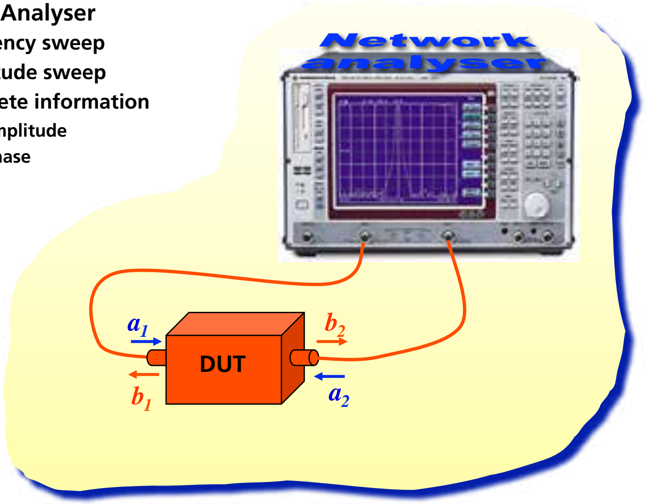


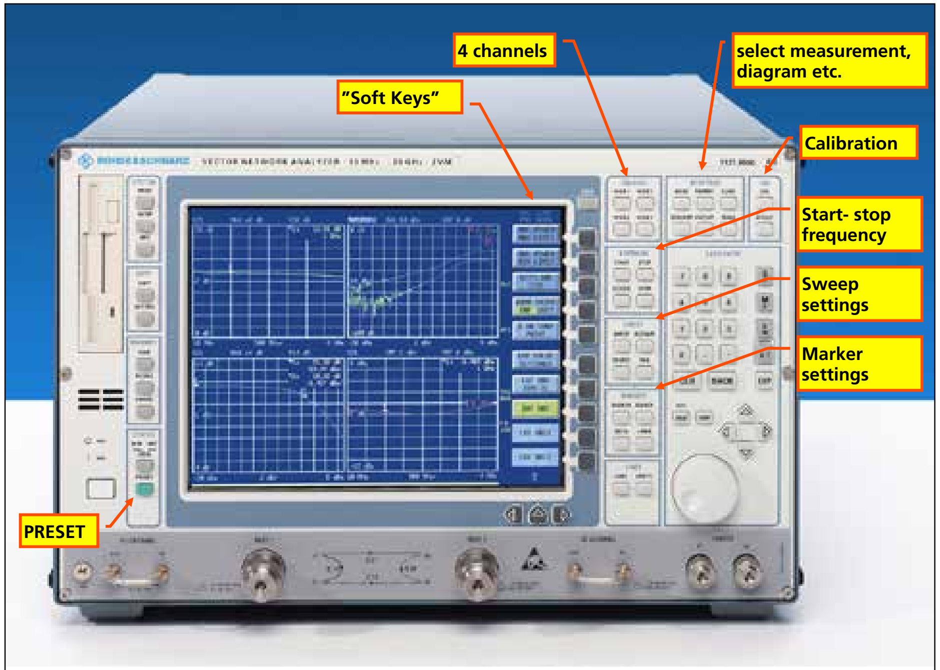
only magnitude measurement,

no phase information

Vector Network Analysis

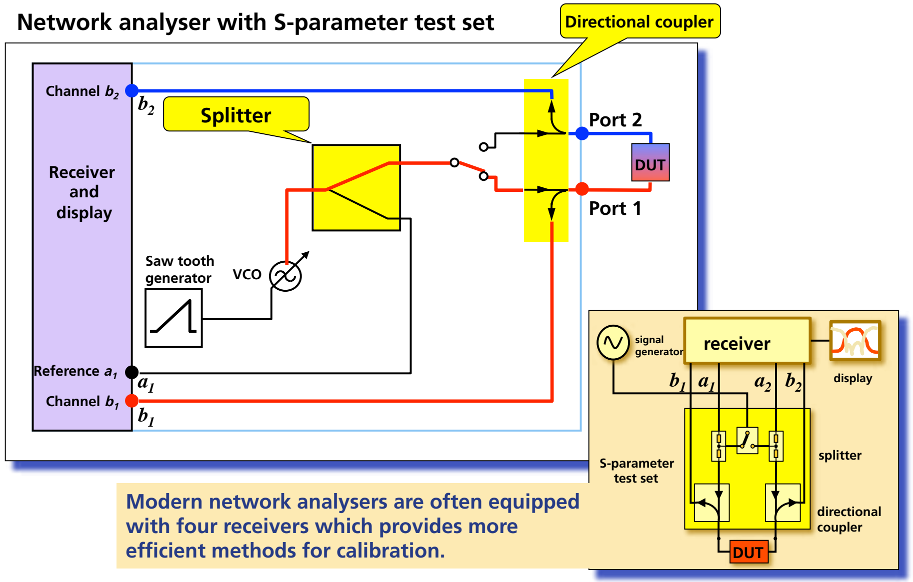
- Characterising the **D**evice **U**nder **T**est properties
- Network Analyser
  - frequency sweep
  - amplitude sweep
  - complete information
    - amplitude
    - phase





The Vector Network Analyser Structure

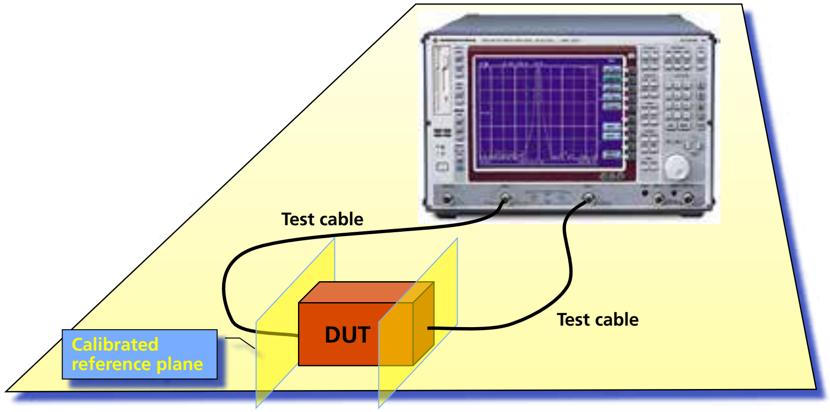
Network analyser with S-parameter test set



Modern network analysers are often equipped with four receivers which provides more efficient methods for calibration.

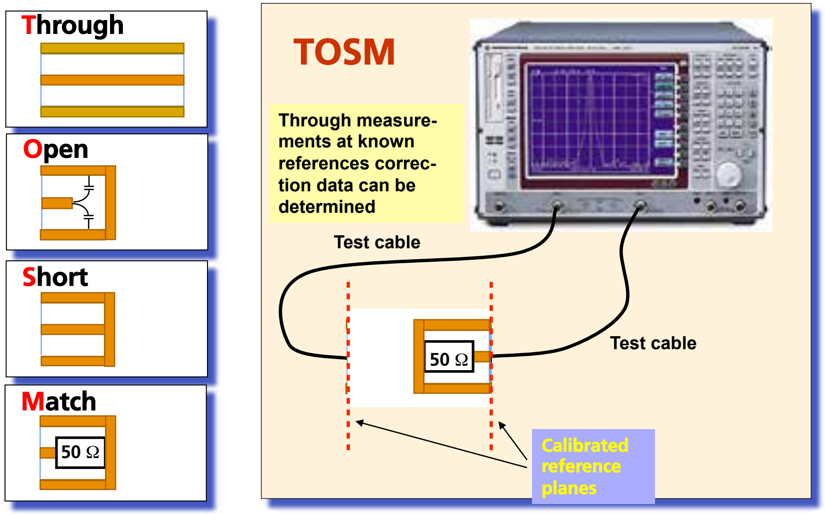
Calibration

- Attenuation and phase shift in the test cables must be compensated
- Calibrated reference planes are therefore created where the device under test is connected



Calibration

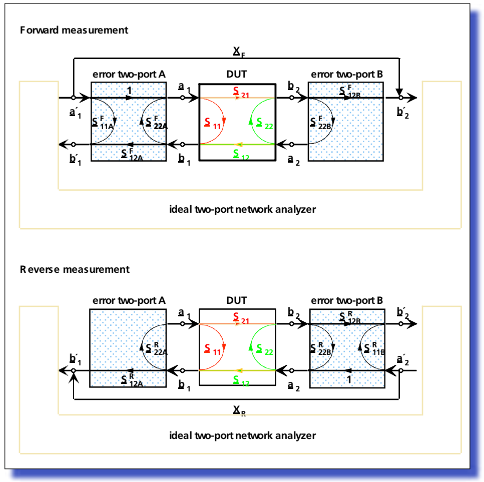
- Calibrated reference planes will be created where the DUT is to be connected



Through measurements at known references correction data can be determined

Calibration

TOSM - Classical Full Two-Port Calibration



Extension of the one port error model by 3 additional error terms for forward direction yields 6 error terms. Adding a similar model for reverse direction yields the classical

⇨12-term error model (TOSM)

- Load matches
- Transmission losses of receiver
- Device independent crosstalks

Be careful about torn connectors!

- The wear and tear when connectors are connected and disconnected may result in measurement errors.
  - always check that the connectors are clean
  - only turn the socket or the nut
    - the contact pin may never spin round
  - always use a torque wrench
    - the connector may never be fastened by other tools if you tighten up to hard the thread is harmed
- Test cables and connectors for professional use are only used for a limited period until they will be exchanged or reconditioned.