

## Microwave theory 2015: Assignment 2

Hand in the solutions no later than May 8, 2015.

### 1

Determine the three lowest cut-off frequencies for the waveguides below by using the analytic formulas and confirm your solutions by determining the cut-off frequencies with COMSOL. Give the ten lowest cut-off frequencies obtained by COMSOL for each waveguide. Check the accuracy in the COMSOL solutions and give a rough estimate of the error.

- A rectangular waveguide with  $a = 8$  cm and  $b = 3$  cm.
- A circular waveguide with radius  $R = 5$  cm.
- A waveguide with a cross section as a half circle with radius  $R = 5$  cm.
- The bandwidth of a waveguide is the width of the frequency band where only the fundamental mode can propagate. The fundamental mode is the mode with lowest cut-off frequency. The relative bandwidth is defined by

$$B = \frac{\Delta f}{f_0}$$

where  $\Delta f$  is the band width and  $f_0$  is the cut-off frequency for the fundamental mode. Determine  $B$  for the three waveguides. There are COMSOL instructions at the end of this booklet.

### 2

The fundamental mode is the mode with the lowest cut-off frequency. It is very important in most waveguide application. By using frequencies in the bandwidth for the fundamental mode the electromagnetic fields are stable and easy to control since higher order modes cannot propagate.

- Determine the explicit expressions for the complex transverse electric and magnetic fields for the fundamental mode of a rectangular waveguide with cross-section  $a \times a/2$ . The mode is assumed to propagate in the positive  $z$ -direction and the transverse fields should be written as

$$\begin{aligned} \mathbf{E}_T(x, y, z) &= (\hat{\mathbf{x}}E_x(x, y) + \hat{\mathbf{y}}E_y(x, y)) e^{ik_z z} \\ \mathbf{H}_T(x, y, z) &= (\hat{\mathbf{x}}H_x(x, y) + \hat{\mathbf{y}}H_y(x, y)) e^{ik_z z} \end{aligned}$$

The waveguide is filled with air.

b) Determine the surface currents that run in the short wall  $x = 0$  (*i.e.*, in the  $yz$ -plane) when the fundamental mode propagates in the waveguide. Transform the current to the time domain and write it as

$$\mathbf{J}_s(y, z, t) = \text{Re}\{\mathbf{J}_s(y)e^{ik_z z}e^{-i\omega t}\} = \hat{\mathbf{y}}J_{sy}(y)\cos(k_z z - \omega t + \phi_y) + \hat{\mathbf{z}}J_{sz}(y)\cos(k_z z - \omega t + \phi_z)$$

where  $\mathbf{J}_s(y)$  is the complex surface current density,  $\phi_y$  and  $\phi_z$  are phase angles (they follow from the analysis),  $J_{sy}(y)$  and  $J_{sz}(y)$  are real amplitudes, and  $k_z$  is the longitudinal wavenumber for the fundamental mode.

Draw the vectors of the surface current density on a distance corresponding to one period, *i.e.*,  $0 < z < \lambda_g = 2\pi/k_z$ , on the wall  $x = 0$  when  $t = 0$ . The frequency should be  $f = \frac{3c_0}{4a}$ , *i.e.*, the frequency between the cut-off frequency for the fundamental mode and the second mode.

c) Determine the surface currents that run at  $x = 0$ ,  $x = a/2$  and  $x = a$  in the wall  $y = 0$ , when the fundamental mode propagates in the waveguide. Go to the time domain and write the surface current as

$$\begin{aligned} \mathbf{J}_s(a/2, z) &= \text{Re}\{\mathbf{J}_s(a/2)e^{ik_z z}e^{-i\omega t}\} \\ &= \hat{\mathbf{x}}J_{sx}(a/2)\cos(k_z z - \omega t + \phi_y) + \hat{\mathbf{z}}J_{sz}(a/2)\cos(k_z z - \omega t + \phi_z) \end{aligned}$$

Draw the vectors of the surface current density on a distance corresponding to one period in the wall  $y = 0$  along the lines  $x = a/2$ ,  $0 < z < \lambda_g = 2\pi/k_z$  and  $x = 0$ ,  $0 < z < \lambda_g = 2\pi/k_z$ , when  $t = 0$ . The frequency is the same as in b).

### 3

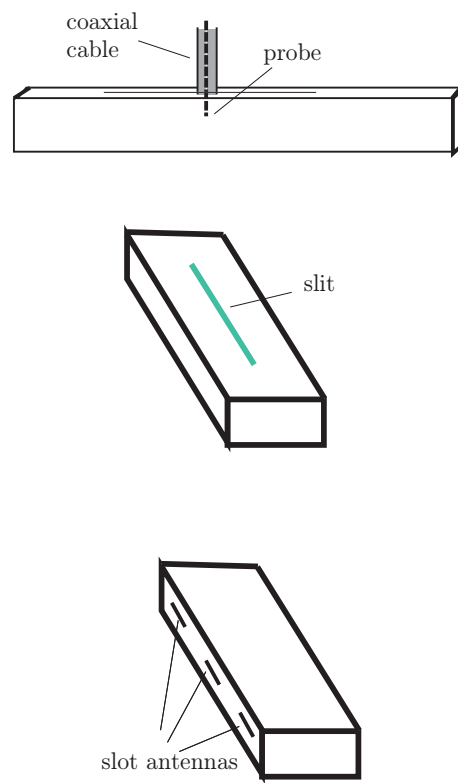
In order to measure the standing wave ratio in a rectangular waveguide one uses a standing wave meter. The probe of the meter is the inner conductor of a coaxial cable that extends into the waveguide through a long slot in the upper long wall, cf the two upper figures in figure 1. The standing wave ratio can be measured by moving the probe along the slot. Explain why the slot is placed in the middle of the upper long wall of the waveguide.

### 4

Certain array antennas used for high power radiation are made of slots in waveguides. The slots act as antenna elements since they leak electromagnetic waves from the waveguides to the outer region. The slot antennas in a rectangular waveguide are usually placed on the short wall and are directed along the waveguide, as in the lower figure in figure 1.

a) Explain why the slots are directed the way they are. Would slots that are directed perpendicular to the ones in the figure be good antennas?

b) The direction of the main beam from an array antenna depends on the phase of the fields in the slot. The array slot antenna can be used as a frequency directed antenna. This means that by changing the frequency the direction of the main beam changes. Determine the distance between the slots if the antenna is a broadside



**Figure 1:** The standing wave meter with slot and the array slot antenna.

antenna, *i.e.*, an antenna that has its main lobe in a direction perpendicular to the array. Give the distance in terms of the waveguide wavelength  $\lambda_g = 2\pi/k_z$ .

## 5

In this problem you use COMSOL.

a) A coaxial cable is normally used as a transmission line for TEM waves. At a high enough frequency the higher order modes start to propagate. The number of propagating modes increases when the frequency increases. It is possible to determine these modes and their cut-off frequencies analytically (*cf.*, problem 5.3 in the book). However, we will use COMSOL to find the cut-off frequencies and also to sketch the field distribution in the coaxial cable. In this example the cable has an outer radius  $a = 0.25$  cm of the inner conductor and an inner radius  $b = 1$  cm of the outer conductor. The volume between the conductors is filled with air ( $\epsilon_r = \mu_r = 1$ ). Find the ten lowest cut-off frequencies and make a table

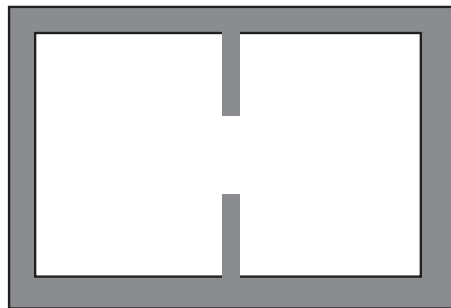
frequency	mode(TE/TM)	m-index	multiplicity
	⋮		
	⋮		

Multiple modes that have the same frequency but only differs by a rotation of 90 degrees of the fields should only be counted as one mode.

b) Out of the ten modes there are two, one TE (single) and one TM (multiplicity 2), that have the same cut-off frequency. Try to identify these two modes in your list. (The mathematical reason why they are the same is that  $J'_0(x) = J_1(x)$ .) You find COMOL instructions below.

## 6

Study first Example 5.6 in the book.



Use COMSOL to design a ridge waveguide for which the fundamental mode has a bandwidth ranging at least from 0.6 GHz to 2.4 GHz. In order to get accurate

values you need to refine the mesh around the edges of the thin walls since the electromagnetic fields vary rapidly close to edges. The waveguide in the figure does not fulfill the requirements so you need to modify it.

*Something to think about:* Try to understand why the cutoff frequency for the fundamental mode is so low. Also try to understand why the power flow densities look the way they do for the different modes.

Please hand in the geometry of the waveguide and a plot of the power flow for the fundamental mode.

## 7

At ESS the cavities are fed by klystrons. The frequency is 302 MHz or 704 MHz, depending where the cavities are located. The klystrons and cavities will be placed in two different tunnels and for this reason the connecting waveguides will be long. This leads to power losses. Use the impedance boundary condition in COMSOL to calculate the relative power loss  $\Delta P/P$  for an 18 meter long rectangular waveguide made of copper. Here  $\Delta P$  is the power loss and  $P$  is the power sent from the klystron. The frequency is 704 MHz, and the dimension of the waveguide is  $a \times b = 0.3 \times 0.15$  m<sup>2</sup>.

*Hint:* Use 2D electromagnetic waves. A similar problem is solved in the book.