Assignment 3

Microwave theory 2014

The solutions should be handed in no later than May 26. If you run into major problems you may ask Anders for help.

1

Determine the ratio between radius a and length d for a circular cylinder such that the lowest resonance frequency for the TE and TM modes are the same. All walls are perfectly conducting and there is vacuum inside the cavity.

2

a) What is the lowest resonance frequency of a cube with side 1 dm?
b) The lowest resonance in the cube is degenerated which means that there are more than one mode that have the same resonance frequency. How many modes are there for this resonance?

3

Consider the TM_{010}-mode in a cylindric cavity with radius a and length L .
a) At what radius r_c is the magnetic field maximal.
b) Where is the surface current density maximal?
c) Where is the surface charge density maximal?

4

A resonance cavity is a cylinder with elliptic cross section. The ellipse has major half-axis $a = 3$ cm and minor half-axis $b = 2$ cm. The length of the cylinder is 3 cm. Determine the three lowest resonance frequencies of the cavity. All walls are perfectly conducting and there is vacuum inside the cavity.

Help: One can solve this problem in COMSOL using either a 2D or a 3D calculation.

5

A dielectric resonator has radius $a = 3$ cm and height $h = 3$ cm. The resonator consists of a dielectric material with relative permittivity $\varepsilon = 30$. There is air ($\varepsilon = 1$) outside the cylinder. Determine the three lowest resonance frequencies for axial-symmetric resonances.

Help: Use 2D-axial symmetry in COMSOL. You find a similar example in the book.

6

An optical fiber has a core with index of refraction $n = 1.504$ and a cladding with index of refraction $n = 1.5$. The radius of the core is $5\,\mu$m and the radius of the cladding can be considered to be infinite. Determine the effective mode index and the phase velocity for the fundamental mode HE_{11} in the optical fiber. The wavelength of the light is 1.55 μm in vacuum. Hand in a plot of the power flow density in the cross section.

Extra problems

Problem 7 and 8 and are not compulsory but solve them if you have time.

7

a) Show that the quality factor for a cavity can be written as

$$Q = \omega_0 \mu_0 \frac{\int_V |\boldsymbol{H}|^2 dV}{R_s \int_S |\boldsymbol{H}|^2 dS}$$

where R_s is the surface resistance and ω_0 the angular resonance frequency.

b) Show that for the TM_{010} mode in a circular cylindric cavity with radius a and length L

$$R_s Q_0 = 453 \frac{L/a}{1 + L/a}$$

8

In the time domain the electric field of a resonance in a cavity is given by

$$\boldsymbol{E}(\boldsymbol{r}, t) = \boldsymbol{E}(\boldsymbol{r}) \cos(\omega t + \phi)$$

where ω is the angular resonance frequency. Now assume an axially symmetric cavity with length d, where the lowest $m = 0$ (axially symmetric) mode is used for accelerating particles. Let a particle enter the cavity at time $t = 0$ with a speed v. The particle with charge q travels along the symmetry axis and its speed is approximately constant

a) Show that the energy that is transferred from the cavity to the particle is

$$W_p = q \int_0^d E_z(0, 0, z) \cos(\omega z/v + \phi)\, dz$$

If $W_p > 0$ then the cavity gives energy to the particle and if $W_p < 0$ then the particle gives energy to the cavity.

b) Define a voltage V as $qV = W_p$ and a resistance $R = \dfrac{1}{2}\dfrac{|V|^2}{P_d}$ where P_d is the time average of the dissipated power in the cavity. Show that R/Q is only determined by the geometry of the cavity and the speed of the cavity.

c) Determine the R/Q for the TM_{010} mode in cylindrical cavity with radius a and length d. All integrals should be solved analytically.