Microwave theory, April 10, 2014

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Electrical and information technology

Last week

- The vector basis functions
- Circular cylindric waveguides

Outline

- Power transport in waveguides
- Losses
- Determination of R, L, G and C

The vector basis functions

TM-waves: $H_z = 0$, $E_z(\boldsymbol{r}) = v(\boldsymbol{\rho})e^{\mathrm{i}k_z z}$
TE-waves: $E_z = 0$, $H_z(\boldsymbol{r}) = w(\boldsymbol{\rho})e^{\mathrm{i}k_z z}$
Eigenvalue problems give eigenfunctions $v_n(\boldsymbol{\rho})$ and k_{zn}

The vector basis functions

The total electromagnetic fields for the TM-waves can be written as

$$\begin{cases} \boldsymbol{E}_{n\nu}^{\pm}(\boldsymbol{r}) = \{\boldsymbol{E}_{Tn\nu}(\boldsymbol{\rho}) \pm v_n(\boldsymbol{\rho})\hat{\boldsymbol{z}}\}\, e^{\pm \mathrm{i} k_{zn} z} \\ \boldsymbol{H}_{n\nu}^{\pm}(\boldsymbol{r}) = \pm \boldsymbol{H}_{Tn\nu}(\boldsymbol{\rho}) e^{\pm \mathrm{i} k_{zn} z} \end{cases} \qquad \nu = \mathrm{TM}$$

The vector basis functions

The total electromagnetic fields for the TE-waves can be written as

$$\begin{cases} \boldsymbol{E}^{\pm}_{n\nu}(\boldsymbol{r}) = \boldsymbol{E}_{Tn\nu}(\boldsymbol{\rho})e^{\pm \mathrm{i}k_{zn}z} \\ \boldsymbol{H}^{\pm}_{n\nu}(\boldsymbol{r}) = \left\{\pm \boldsymbol{H}_{Tn\nu}(\boldsymbol{\rho}) + w_n(\boldsymbol{\rho})\hat{\boldsymbol{z}}\right\} e^{\pm \mathrm{i}k_{zn}z} \end{cases} \qquad \nu = \mathrm{TE}$$

The vector basis functions

The transverse components $\boldsymbol{E}_T$ and $\boldsymbol{H}_T$

$$\begin{cases} \boldsymbol{E}_{Tn\nu}(\boldsymbol{\rho}) = \begin{cases} \frac{\mathsf{i}}{k_{tn}^2} k_{zn} \nabla_T v_n(\boldsymbol{\rho}), & \nu = \mathsf{TM} \\ -\frac{\mathsf{i}\omega}{k_{tn}^2} \mu_0 \mu \hat{z} \times \nabla_T w_n(\boldsymbol{\rho}), & \nu = \mathsf{TE} \end{cases} \\ \boldsymbol{H}_{Tn\nu}(\boldsymbol{\rho}) = Z_{n\nu}^{-1} \hat{z} \times \boldsymbol{E}_{Tn\nu}(\boldsymbol{\rho}) \end{cases}$$

The vector basis functions

The mode impedances:

$$Z_{n\nu} = \begin{cases} \dfrac{k_{zn}}{\omega \epsilon_0 \epsilon}, & \nu = \text{TM} \\ \\ \dfrac{\omega \mu_0 \mu}{k_{zn}}, & \nu = \text{TE} \end{cases} \tag{1}$$

The vector basis functions

Notice that at frequencies much higher than the cut-off frequency

$$k_{zn} = \sqrt{k^2 - k_{tn}^2} \approx k \Rightarrow$$

$$Z_{n\nu} \approx \eta_0 \eta = \sqrt{\frac{\mu_0}{\epsilon_0 \epsilon}} = \text{wave impedance} \Rightarrow$$

The waveguide mode is almost a plane wave propagating in the positive $z-$direction

Circular waveguide

TE-modes

$$E_z = 0, \qquad H_z = A_{mn} J_m(k_{tmn}\rho) \begin{pmatrix} \cos m\phi \\ \sin m\phi \end{pmatrix}$$

$$k_{tmn} = \frac{\eta_{mn}}{a},\ J'_m(\eta_{mn}) = 0,\ n = 1,\ 2\ldots$$

cut-off frequency, $k_z = 0 \Rightarrow k = k_{tmn} \Rightarrow f_c = c\dfrac{\eta_{mn}}{2\pi a}$

Fundamental mode TE_{11} $f_c = c\dfrac{1.841}{2\pi a}$

$a = 15$ mm gives $f_c = 5.9$ GHz

Circular waveguide

TM-modes

$$H_z = 0, \qquad E_z = B_{mn} J_m(k_{tmn}\rho) \begin{pmatrix} \cos m\phi \\ \sin m\phi \end{pmatrix}$$

$k_{tmn} = \dfrac{\xi_{mn}}{a}$, $J_m(\xi_{mn}) = 0$, $n = 1,\, 2 \ldots$

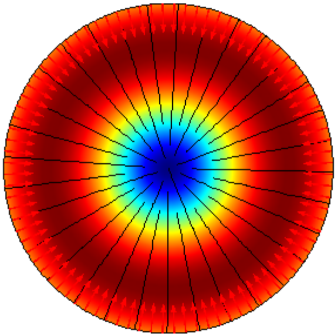
cut-off frequency: $f_c = c\dfrac{\xi_{mn}}{2\pi a}$

Lowest cut-off frequency TM_{01} $f_c = c\dfrac{2.405}{2\pi a}$

$a = 15$ mm gives $f_c = 7.65$ GHz

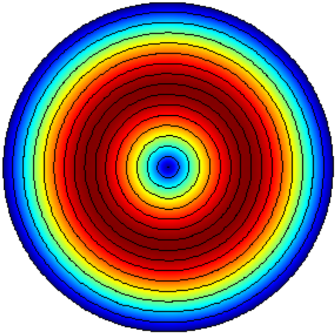
Transverse electric field $\boldsymbol{E}_T$.

TM_{01}.



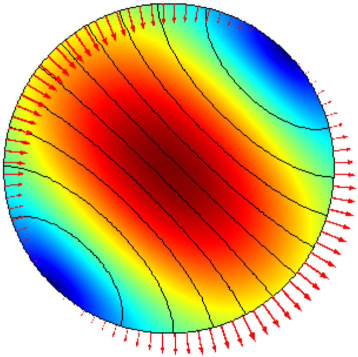
Transverse electric field $\boldsymbol{E}_T$.

TE_{01}.



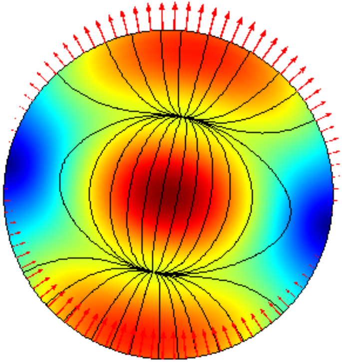
Transverse electric field $\boldsymbol{E}_T$.

Fundamental mode TE_{11}.



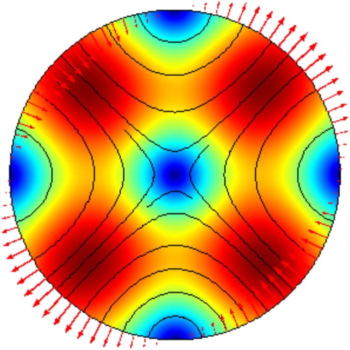
Transverse electric field $\boldsymbol{E}_T$.

TM_{11}.



Transverse electric field $\boldsymbol{E}_T$.

TE_{21}.



Transverse electric field $\boldsymbol{E}_T$.

TE_{31}.