



Microwave theory, April 2, 2014

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Electrical and information technology

Waveguide modes

TM-waves $\Rightarrow H_z = 0$ $E_z = v(\boldsymbol{\rho})E^{ik_z z}$

Eigenvalue problem

$$\nabla^2 v(\boldsymbol{\rho}) + k_t^2 v(\boldsymbol{\rho}) = 0, \boldsymbol{\rho} \in \Omega$$

$$v(\boldsymbol{\rho}) = 0, \boldsymbol{\rho} \in \Gamma$$

Eigenvalues k_{tn}^2 and eigenfunctions $v_n(\boldsymbol{\rho})$, $n = 1, 2, 3 \dots \infty$

Entire EM-field for TM-modes

Once all v_n are known we can construct the entire EM-field for the TE-mode (section 5.4):

$$\begin{aligned}\mathbf{E}_n^\pm(\mathbf{r}) &= (\mathbf{E}_{Tn}(\boldsymbol{\rho}) \pm v_n(\boldsymbol{\rho})\hat{\mathbf{z}}) e^{\pm ik_{zn}z} \\ \mathbf{H}_n^\pm &= \pm \mathbf{H}_{Tn}(\boldsymbol{\rho}) e^{\pm ik_{zn}z}\end{aligned}$$

where

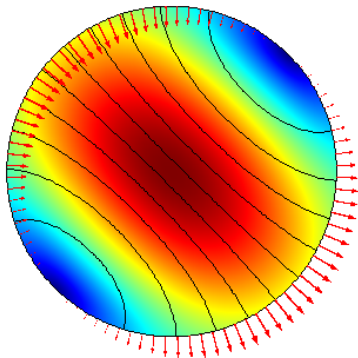
$$\begin{aligned}\mathbf{E}_{Tn}(\boldsymbol{\rho}) &= \frac{i}{k_{tn}^2} k_{zn} \nabla_T v_n(\boldsymbol{\rho}) \\ \mathbf{H}_{Tn}(\boldsymbol{\rho}) &= Z_{nTM}^{-1} \hat{\mathbf{z}} \times \mathbf{E}_{Tn}(\boldsymbol{\rho}) \\ Z_{nTM} &= \frac{k_{zn}}{\omega \epsilon_0 \epsilon}\end{aligned}$$

Outline for today

- ▶ Circular cylindrical waveguides
- ▶ Bessel functions
- ▶ The fundamental mode
- ▶ Comsol example

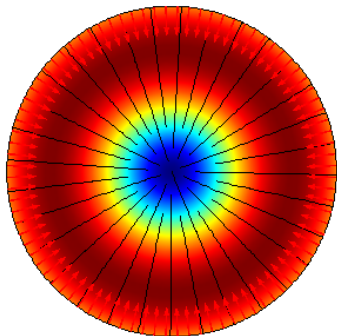
Circular cylindrical waveguide

Fundamental mode TE_{11} .



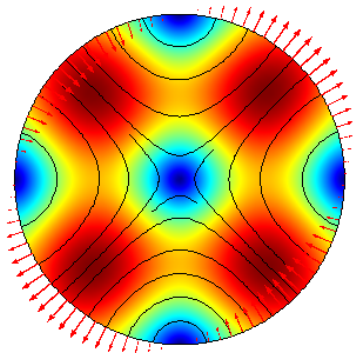
Circular cylindrical waveguide

TM₀₁.



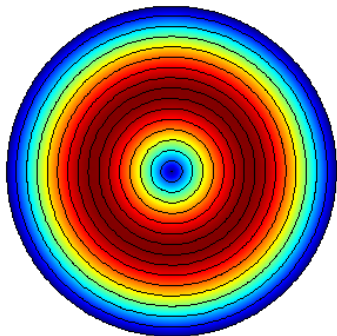
Circular cylindrical waveguide

TE_{21} .



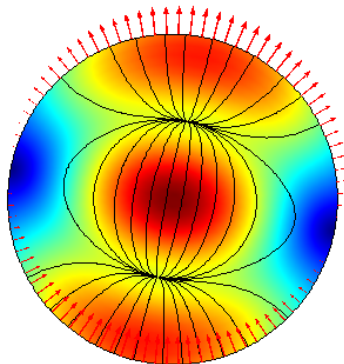
Circular cylindrical waveguide

TE_{01} .



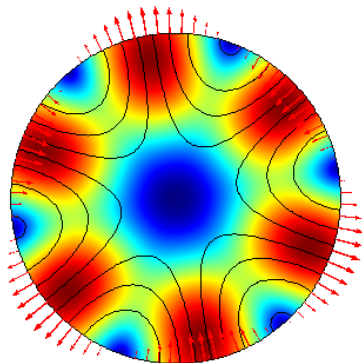
Circular cylindrical waveguide

TM₁₁.



Circular cylindrical waveguide

TE_{31} .



Bessel functions

