Microwave theory, March 26, 2014

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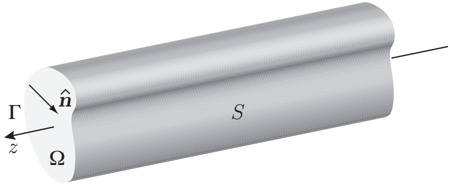
Electrical and information technology

Hollow waveguides



Three type of waves:

- TE-waves$\Rightarrow$ $E_z = 0$, $H_z = w(\boldsymbol{\rho})e^{\mathrm{i}k_z z}$
- TM-waves$\Rightarrow$ $H_z = 0$, $E_z = v(\boldsymbol{\rho})e^{\mathrm{i}k_z z}$
- TEM-waves$\Rightarrow$ $E_z = 0$ and $H_z = 0$, $\boldsymbol{E} = -\nabla\phi(\boldsymbol{\rho})e^{\mathrm{i}k_z z}$
  ONLY in waveguides with at least two conductors.

Hollow waveguides

Eigenvalue problem TE-waves

$$\nabla^2 w(\boldsymbol{\rho}) + k_t^2 w(\boldsymbol{\rho}) = 0,\ \boldsymbol{\rho} \in \Omega$$
$$\frac{\partial w(\boldsymbol{\rho})}{\partial n} = 0,\ \boldsymbol{\rho} \in \Gamma$$

Eigenvalues k_{tn}^2 and eigenfunctions $w_n(\boldsymbol{\rho})$

Hollow waveguides

Eigenvalue problem TM-waves

$$\nabla^2 v(\boldsymbol{\rho}) + k_t^2 v(\boldsymbol{\rho}) = 0,\ \boldsymbol{\rho} \in \Omega$$
$$v(\boldsymbol{\rho}) = 0,\ \boldsymbol{\rho} \in \Gamma$$

Eigenvalues k_{tn}^2 and eigenfunctions $v_n(\boldsymbol{\rho})$

Rectangular waveguide TE-modes

TE-waves ⇒
Eigenvalue problem

$$\nabla^2 w(\boldsymbol{\rho}) + k_t^2 w(\boldsymbol{\rho}) = 0,\ \boldsymbol{\rho} \in \Omega$$
$$\frac{\partial w(\boldsymbol{\rho})}{\partial n} = 0, \text{ on all four sides}$$

Rectangular waveguide TE-modes

Eigenvalues: $k_{tmn}^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$

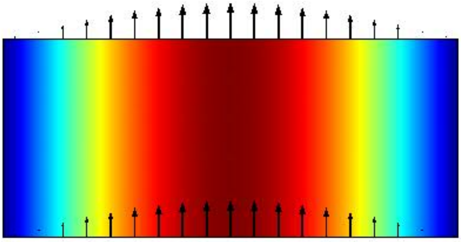
Eigenfunctions: $w_{mn}(\boldsymbol{\rho}) = A_{mn} \cos\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right)$

$m = 0, 1, 2 \ldots$, $n = 0, 1, 2 \ldots$

but $(m, n) \neq (0, 0)$

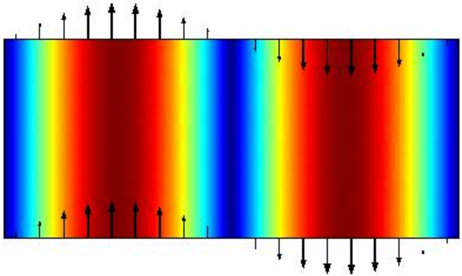
Rectangular waveguide

The TE_{10} mode. Fundamental mode!

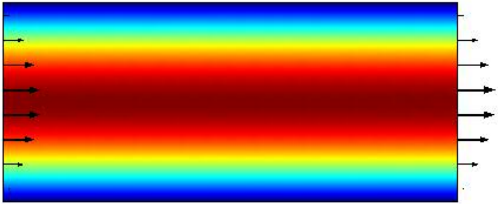


Rectangular waveguide

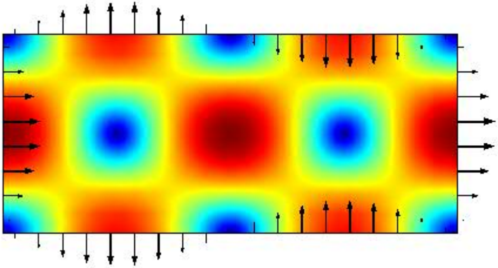
The TE_{20} mode



The TE_{01} mode



The TE_{21} mode



Cut-off frequencies

$e^{\mathrm{i}k_z z}$
$k_z = \sqrt{k^2 - k_{tmn}^2}$.

1. $k < k_{tmn} \Longrightarrow k_z$ imaginary $\Longrightarrow$ non-propagating mode
2. $k = k_{tmn} \Longrightarrow k_z = 0 \Longrightarrow$ standing wave, cut-off frequency f_c
3. $k > k_{tmn} \Longrightarrow k_z$ real $\Longrightarrow$ propagating mode

Today

- TM-wave in rectangular wavguides
- COMSOL
- Dispersion, phase- and group speeds
- The fundamental mode TE_{10}.
- Circular waveguides