Microwave theory, March 17, 2014

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Electrical and information technology

Homepage

All information is on the course home page http://www.eit.lth.se/

Examination

- Hand in problems. Three assignments.
- Project.
- Oral exam for grades 4 and 5.

Assignments

The assignments should be solved individually. You are allowed to

- discuss the problem with others
- ask the teacher (Anders) if you get stuck

You are not allowed to copy solutions!

Project

- Groups of two students (or, if needed three or one).
- All students have to contribute to the project.
- All students have to take part in the oral presentation of the project.
- Discussions with the teacher (Anders) are recommended.
- You are not allowed to copy results from other groups.

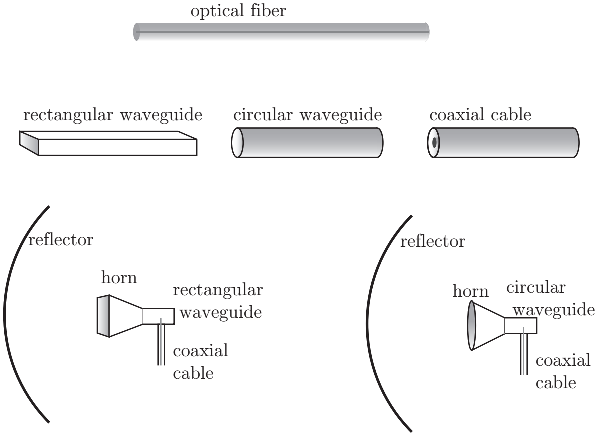
COMSOL Multiphysics 4.4

- Finite element method program
- Installed on most of the computers in the basement of the E-building
- Install it from LDC. See information on homepage
- Introduction to COMSOL Tuesday 25/3 in E:Uranus

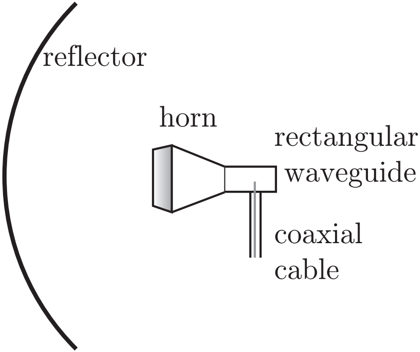
Microwave theory

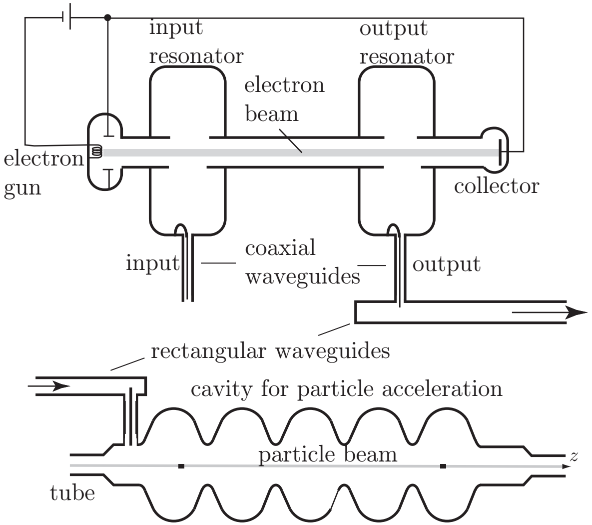
- Maxwell equations
- Transmission lines
- Waveguides
- Resonance cavities
- Dielectric waveguides
  - Optical fibers
  - Dielectric resonators

Example


optical fiber
rectangular waveguide
circular waveguide
coaxial cable
reflector
horn
rectangular
waveguide
coaxial
cable
reflector
horn
circular
waveguide
coaxial
cable

Example




input
resonator
output
resonator
electron
beam
electron
gun
collector
coaxial
waveguides
input
output
rectangular waveguides
cavity for particle acceleration
particle beam
z
tube

The Maxwell equations

Time domain!

$$\nabla \times \boldsymbol{E}(\boldsymbol{r},t) = -\frac{\partial \boldsymbol{B}(\boldsymbol{r},t)}{\partial t}$$
$$\nabla \times \boldsymbol{H}(\boldsymbol{r},t) = \boldsymbol{J}(\boldsymbol{r},t) + \frac{\partial \boldsymbol{D}(\boldsymbol{r},t)}{\partial t}$$

$\boldsymbol{E}$=electric field
$\boldsymbol{D}$=electric flux density
$\boldsymbol{H}$=magnetic field
$\boldsymbol{B}$=magnetic flux density
$\boldsymbol{J}$=current density

The Maxwell equations

What else is needed in order to find $\boldsymbol{E}$ and $\boldsymbol{H}$ in a typical problem?

- Sources
- Constitutive relations: $\boldsymbol{D} = \epsilon_0 \epsilon \boldsymbol{E}$, $\boldsymbol{B} = \mu_0 \mu \boldsymbol{H}$, $\boldsymbol{J} = \sigma \boldsymbol{E}$
- Boundary conditions

The Maxwell equations

Frequency domain!

$$\nabla \times \boldsymbol{E}(\boldsymbol{r}) = -\mathrm{j}\omega \boldsymbol{B}(\boldsymbol{r})$$
$$\nabla \times \boldsymbol{H}(\boldsymbol{r}) = \boldsymbol{J}(\boldsymbol{r}) + \mathrm{j}\omega \boldsymbol{D}(\boldsymbol{r})$$

Phasors: examples

Transformation between time and frequency domain:

$$
\begin{aligned}
\boldsymbol{E}(\boldsymbol{r},t) = \boldsymbol{E}_0(\boldsymbol{r})\cos(\omega t+\phi) &\longleftrightarrow \boldsymbol{E}(\boldsymbol{r}) = \boldsymbol{E}_0(\boldsymbol{r})e^{\mathrm{j}\phi}\\
v(t) = V_0\cos(\omega t+\pi/4) &\longleftrightarrow V = V_0 e^{\mathrm{j}\pi/4}\\
\frac{\partial}{\partial t} &\longleftrightarrow \mathrm{j}\omega
\end{aligned}
$$

$\boldsymbol{E}(\boldsymbol{r}) =$complex electric field
$V =$complex voltage

Helmholz equation

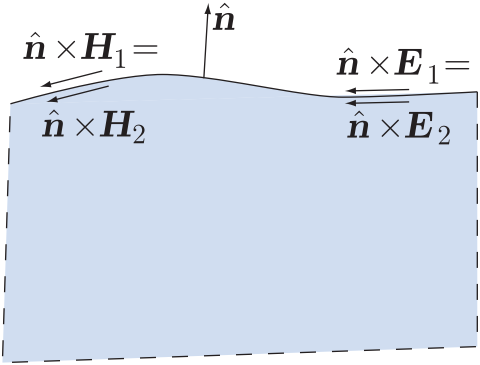
Maxwell equations in the frequency domain lead to

$$\nabla^2 \boldsymbol{E}(\boldsymbol{r}) + k^2 \boldsymbol{E}(\boldsymbol{r}) = \boldsymbol{0}$$
$$\nabla^2 \boldsymbol{H}(\boldsymbol{r}) + k^2 \boldsymbol{H}(\boldsymbol{r}) = \boldsymbol{0}$$

$k = \omega\sqrt{\mu_0 \mu \epsilon_0 \epsilon} = \dfrac{\omega}{c}$ =wave number in the material.
Helmholtz equations for $\boldsymbol{E}$ and $\boldsymbol{H}$ are vital for waveguides and cavities!

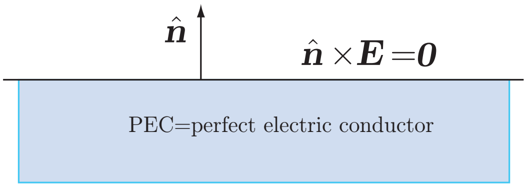
Boundary conditions

Boundary between two materials
Tangential component of $\boldsymbol{E}$ is the same on both sides!
Tangential component of $\boldsymbol{H}$ is the same on both sides!



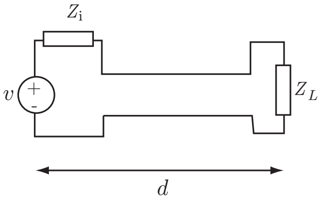
Boundary conditions

Boundary to a perfect conductor
Tangential component of $\boldsymbol{E}$ is zero!



Circuits

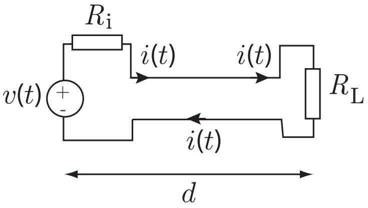
Discrete circuits!



$d \ll \lambda$ $\Rightarrow$discrete circuit $\Rightarrow$ **Circuit theory**.

Circuits

Discrete circuits!

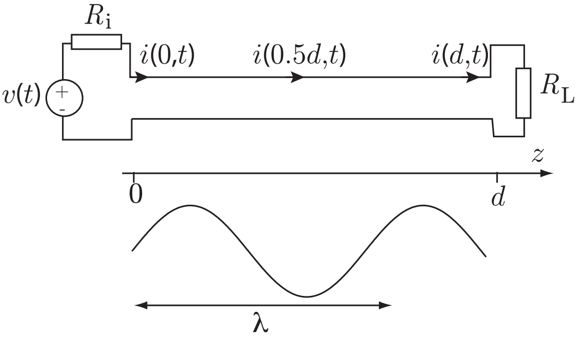


$d \ll \lambda$ $\Rightarrow$discrete circuit $\Rightarrow$ **Circuit theory**.

Discrete circuits?

- Analogue telephone line? $d \approx$1 km, $f_{\max} \sim 6$ kHz, $\lambda = 50$ km $\Rightarrow d \ll \lambda$ Yes!
- Telephone line with ADSL? $d \approx$1 km, $f_{\max} \sim 1$ MHz, $\lambda = 300$ m $\Rightarrow d > \lambda$ No!

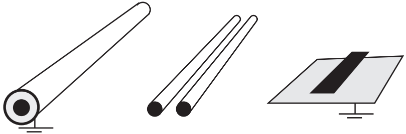
Circuits



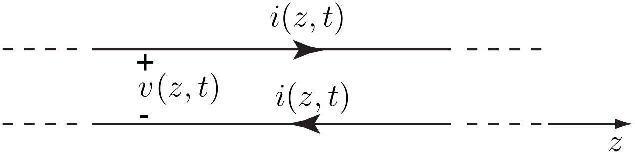
$d > 0.1\lambda \Rightarrow$ **Transmission line**.

Transmission lines

- Coaxial cables
- Parallel wires (e.g. twisted pairs for LAN)
- Micro strips in integrated circuits and on PCB.
- ...

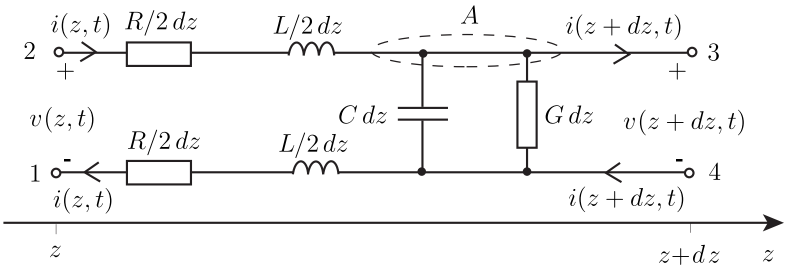


Transmission lines



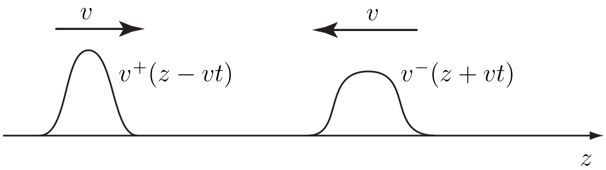
- Voltage $v(z,t)$ and current $i(z,t)$. Waves!
- Distributed line parameters (R, L, G, C)

Transmission line parameters



R =resistance per unit length
L =inductance per unit length
G =conductance per unit length
C =capacitance per unit length

Waves along transmission line



The voltage and current are superpositions of waves traveling to the left and to the right! They travel with the speed of light.