LUNDS UNIVERSITET
Lunds Tekniska Högskola

Electrical and Information Technology, LTH
Professor Bjorn Landfeldt
Chair of Network Architecture and Services

EITN095 Take Home exam

The exam consists of questions on optimisation, heuristics and discrete event simulation. In order to obtain a 4, at least one question on each topic should be correctly solved. In order to obtain a 5, all questions should be correctly solved.

Write a report with your findings and include all source code in an archive. The deadline for the exam is August 10, 2018. The report and archive should be uploaded to the EIT moodle server.

1. Optimisation

UniCAM is a company producing a certain type of DSLR camera. UniCAM has devised 5 different packages to sell its cameras. The packages are labeled as PI, PII, PIII, PIV, and PV. They differ in the type and number of professional lenses offered with the camera. UniCAM has a limited number of each lens in stock. Each package has a certain price, and projected to be sold to a maximum number of customers (i.e. demands). See the following two tables for more complete information:

	PI	PII	PIII	PIV	PV	# in stock in 1000s
Macro Lens	2	0	0	0	0	36
Prime Lens	0	2	2	2	1	216
Wide Lens	0.2	1	0	0.5	0	18
Price in 10 k SEK	4	3	2	2	1	

The interpretation of the third row is that Wide lens is provided in 20% of all PI packages, in all PII packages, in 50% of the PIV packages and none of the PIII and PV packages

Table 1: Capacity and price information

Address: Box 118, 221 00 Lund *Visiting Address:* Ole Römers Väg 3 *Phone:* +46 46-222 44 07
Email: bjorn.landfeldt@eit.lth.se

	PI	PIII	PI+PII+PIII	PIV+PV
Maximum Demand in 1000s	16	2	34	28

Table 2: Demand information

UniCAM is interested in determining the number of each package to be sold, such that its total profit is maximized.

a) Formulate a linear program (LP) for the problem described above.

b) Implement your LP in MATLAB, solve it and obtain the optimal number of each package, and the total profit.

c) Implement sensitivity analysis and find the shadow prices for each constraint. This must be done for both capacity constraints (Table 1) and demand constraints (Table 2).

d) Using the shadow price corresponding to the maximum demand for package PIII, perform an extended experiment by incrementing the maximum demand for PIII with a step size of 0.6, run the LP with the new demand, and record the achieved profit. Repeat this until the profit becomes constant (i.e. flattened). Then, using your recorded data, draw a diagram showing the relationship between the change of maximum demand for PIII and the total profit. Explain your observations from this diagram about the number of times the shadow price changes before the profit becomes constant.

e) In this part, our goal is to find a range in which the variation of price for package PI does not affect the optimal solution found in part (b) (note: by optimal solution, we mean the optimal number of each package, not the amount of optimal profit). More specifically, using the original price of PI (that is 4, according to Table 1), you should find two values α and β such that $\alpha \le 4 \le \beta$ and the optimal solution does not change within this range.

Hint: implement an iterative simulation procedure, which is initiated with the original price 4. Then, to find β, the procedure increases the price (by a very small value in each iteration), finds the new solution and checks if this solution remains unchanged. Similar procedure is applied to find α, but using iterative decrease of the price.

2. Optimisation

An Integer program is given as follows:

$$Maximise: \ z = x_1 + 5x_2$$
$$subject \ to: \quad 2x_1 - x_2 \le 4$$
$$-x_1 + x_2 \le 1$$
$$x_1 + 4x_2 \le 12$$
$$x_1 \in Z^+, x_2 \in Z^+ (positive \ integers)$$

a) Implement this integer program in MATLAB, solve it, and report the optimal solution (i.e. x1, x2, and z).
b) Relax the problem to a linear program (i.e. x_1 and x_2 are real numbers). Solve this problem using MATLAB, and obtain the solution. From this optimal solution, identify all possible roundings of x_1 and x_2 to integer values. Which pair of feasible rounding for x_1 and x_2 gives a maximum value for the objective function? How is this solution compared to the one obtained in part (a)?
c) Again, relax the problem to real-valued variables. Our goal is to obtain the optimal integer solution in a semi-automatic way. You are allowed to use the MATLAB solver to solve linear programs only (not any integer program). Therefore, you need to apply the branch-and-bound method to determine a new sub-problem in each step, formulate this sub-problem as a linear program in MATLAB and solve it. If the solution is fractional, then make new branches, and so on so forth (see Lecture-O2 to find an example). In your answer to this task, you should draw a branch-and-bound tree, which depicts all steps of the branch-and-bound method you applied, and the solution obtained in each step. In case, you terminate (i.e. cut) a branch in the tree, you should explain why it is terminated.

3. Problem 1 in discrete event simulation

In this problem you shall use antithetic variables to simulate a simple system. Simulate the following queuing system:
- There are two servers
- The service times are uniformly distributed between 0 and 2
- The time between the arrivals are exponentially distributed with mean 1.2 seconds

Find the mean number of customers in the queuing system. Supplement your code to your results.

4. Problem 2 in discrete event simulation

Assume that you have a random number generator that gives uniformly distributed numbers between 0 and 1. Describe how you can form random numbers with the following distributions with the use of the uniformly distributed numbers.

1. Geometrical distribution, that is you shall get an integer N where
   $P(N = k) = \alpha^k (1 - \alpha)$ for $k = 0, 1, 2, \ldots$

We assume that $0 < \alpha < 1$.

2. A real number with the following density function:

$$f(t) = \begin{cases} 0.5t & \text{if } 0 \leq t \leq 1 \\ 0.5 & \text{if } 1 \leq t \leq 2 \\ 1.5 - 0.5t & \text{if } 2 \leq t \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

5. Problem Meta Heuristics 1

Implement a Simulated Annealing program for a production management problem described as follows:

A manufacturing company decides on when to produce a good, how much to produce to satisfy a demand forecast while minimizing its total costs. Below, you find detail information as well as a formulation of the optimization problem.

Constraints:
• Maximum number of goods, G, that can be produced in any period
• Must satisfy a fixed demand vector D over a time length of L periods •
Initially, at time t = 0, there is no good in stock
Costs:
• CF : fixed cost of production in periods $t \in \{1, 2, ..., L\}$
• CP : production cost proportional to the amount of goods produced in
  periods $t \in \{1, 2, ..., L\}$
• CS: cost of storage proportional to the amount of goods stored at the end
  of periods $t \in \{1, 2, ..., L\}$

Problem formulation

$$Minimise : \sum_{t=1}^{L} C_p(t)X(t) + C_F(t)Y(t) + C_s(t)Z(t)$$

$$subject\ to : Z(t-1) + X(t) = D(t) + Z(t)\ \forall t \in \{1, ..., L\}$$

$$X(t) \leq GY(t) \qquad \forall t \in \{1, ..., L\}$$

$$Z(t) = 0 \qquad for\ t = 0$$

$$Y(t)\ is\ binary,\ X(t)\ and\ Z(t)\ are\ positive\ integers$$

where, X is the vector representing the amount of goods produced in each period, Y is the decision vector representing whether or not we should produce at any time period, Z is the storage vector representing the amount of goods stored at the end of each time period. D is the demand forecast vector.

Input data: your implemented procedure must work for any given set of parameters as far as the demand vector is feasible and as far as $L \leq 12$. A sample input data for validation purpose is as follows:

$G = 10$, $L = 6$, and

	t = 1	t = 2	t = 3	t = 4	t = 5	t = 6
C_P	3	4	3	4	4	5
C_F	12	15	30	23	19	45
C_S	1	1	1	1	1	1
D	6	7	4	6	3	8

Note that initially (i.e. at time t = 0) the stock is empty (i.e. $Z(t = 0) = 0$).

Output data: optimal solution in terms of number of goods produced in each period and the resulting total cost. Also indicate the total number of iterations performed by your simulated annealing program.

A general procedure for Simulated Annealing is specified in Lecture-H2. You will need to adapt it to the above problem, by designing suitable operators for neighbor generation, evaluation function, neighbor acceptance criteria, temperature schedule, number of iterations per temperature, and the termination criteria.