



EITN90 Radar and Remote Sensing

Lecture 11: An overview of radar imaging

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Learning outcomes of this lecture

In this lecture we will

- ▶ See how images can be formed from radar signals collected at different locations.
- ▶ Learn what determines the achievable resolution of the image.
- ▶ See the basics of a few imaging algorithms.
- ▶ Observe typical phenomena in radar images.

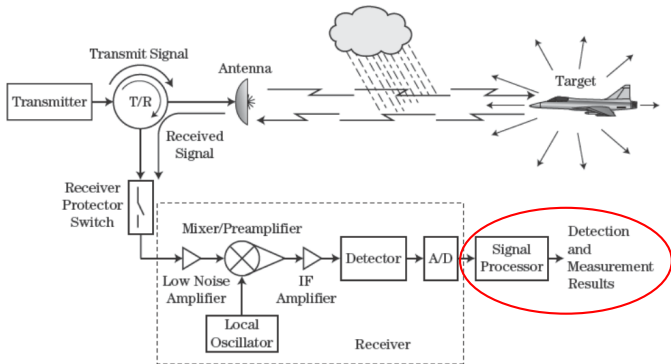


FIGURE 1-1 ■ Major elements of the radar transmission/reception process.

Outline

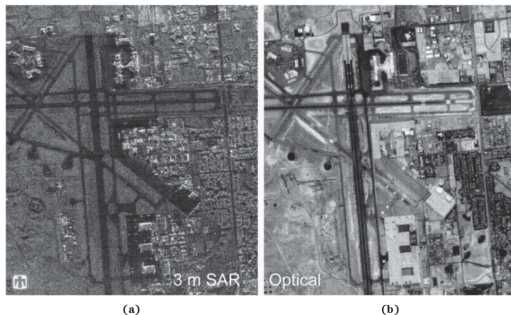
- 1 General imaging considerations**
- 2 Resolution relationships and sampling requirements**
- 3 Data collection**
- 4 Image formation**
- 5 Image phenomenology**
- 6 Conclusions**

Outline

- 1 General imaging considerations**
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Radar vs optical photo

FIGURE 21-1 ■
Two renderings of
Albuquerque Airport.
(a) Synthetic
Aperture Radar
image, and
(b) optical overhead
photograph.
(Images courtesy of
Sandia National
Laboratories.)



Some benefits of radar (cost of providing its own illumination, long measurement time, computationally demanding):

- ▶ Weather tolerant.
- ▶ Emphasizes man-made objects.
- ▶ Provides fine resolution at large range.
- ▶ Can penetrate foliage and other obstacles.
- ▶ Requires coherent operation.
- ▶ Polarization may provide additional information.

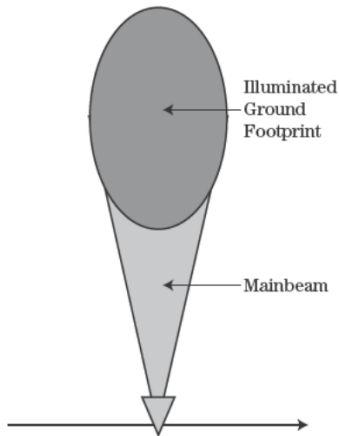
Synthetic Aperture Radar (SAR)

SAR can be thought of in at least three different aspects:

- ▶ SAR as a large synthetic antenna aperture.
- ▶ SAR as range-Doppler imaging.
- ▶ SAR as a signal processing exercise.

SAR as a large synthetic antenna aperture

FIGURE 21-2 ■
Overhead view
illustrates radar
platform moving
from left to right with
side-looking
antenna having main
beam and ground
footprint pointed
towards the top.

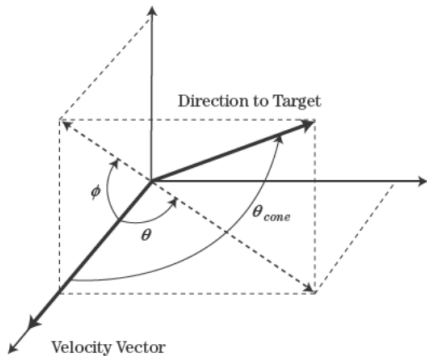


The cross-range resolution can be seen as the effective beam width of an antenna with

$$\Delta CR \approx R\theta_3 \approx R \frac{\lambda}{2D_{\text{SAR}}}$$

SAR as range-Doppler imaging

FIGURE 21-3 ■
Coordinate system showing azimuth, elevation, and cone angles, and vectors for platform velocity and direction to target.



The platform is moving in a certain direction relative to the scene, causing a Doppler shift $f_d = 2v \cos \theta_{\text{cone}} / \lambda$ in the received signal.

$$\Delta CR = R \Delta \theta = R \frac{\lambda \Delta f_d}{2v \sin \theta} = R \frac{\lambda}{2v T_d \sin \theta} = R \frac{\lambda}{2D_{\text{SAR}} \sin \theta}$$

In the book, the broadside angle $\theta_b = \theta + 90^\circ$ is used, resulting in $\sin \theta \rightarrow \cos \theta_b$.

SAR as a signal processing exercise

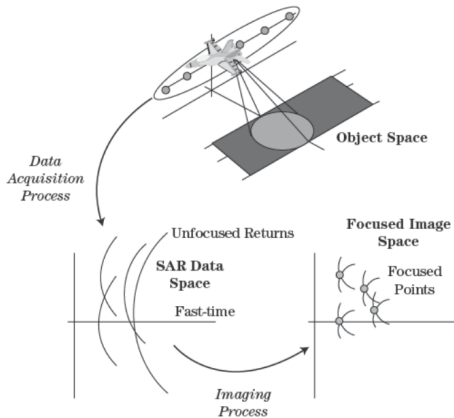


FIGURE 21-5 ■
The SAR process involves three spaces and two transformations. Data acquisition provides the transformation from scene reflectivity to raw data, while image formation operates on the raw data to yield the final SAR image.

The image is the result of the relation between three data spaces:

- ▶ Object space, defined by the scatterers.
- ▶ Data space, the raw radar measurements.
- ▶ Image space, the final synthetic image.

Down-range and cross-range coordinates

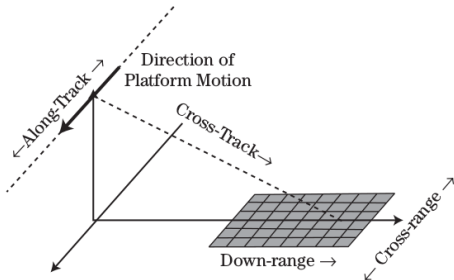


FIGURE 21-4 ■ Location in a SAR image is usually defined as a function of along-track and cross-track coordinates, or down-range and cross-range coordinates. For side-looking collections these coordinate systems are nearly equivalent.

- ▶ The down-range coordinate corresponds to fast time (delay for individual pulses).
- ▶ The cross-range coordinate corresponds to slow time (phase difference between pulses).

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Down-range and cross-range resolutions

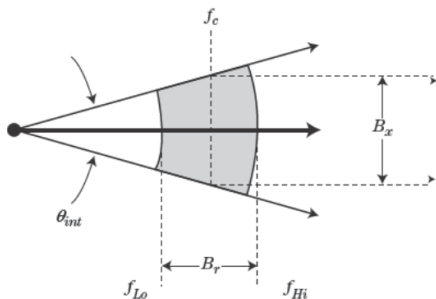


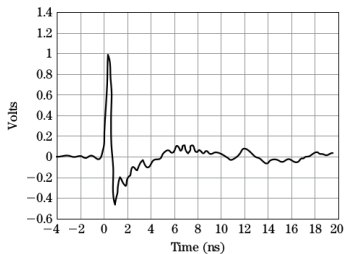
FIGURE 21-6 ■ Data support for a SAR collection is two dimensional and is a function of the waveform frequencies and integration angle. These quantities may be depicted in a polar coordinate system.

The resolutions in down-range and cross-range can be written

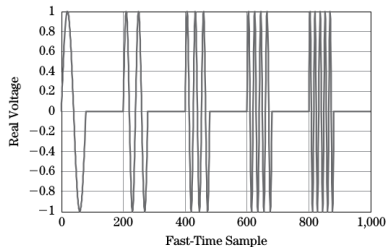
$$\Delta R = \frac{c}{2B_r}$$

$$\Delta CR = \frac{c}{2B_x} = \frac{c}{2f_c \theta_{int}} = \frac{R\lambda_c}{2D_{SAR}}$$

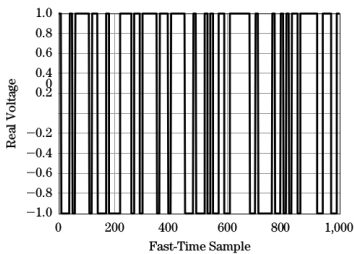
Different waveforms



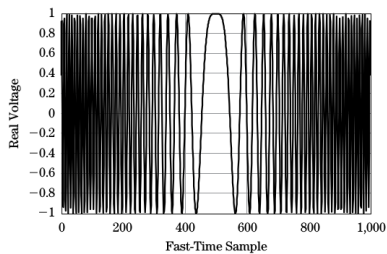
(a)



(b)



(c)



(d)

FIGURE 21-7 ■ Four options for wideband waveforms are (a) a short-time pulse, (b) a stepped-frequency series of pulses, (c) a bi-phase modulated pulse, or (d) a linear FM pulse.

Illumination options for data collection

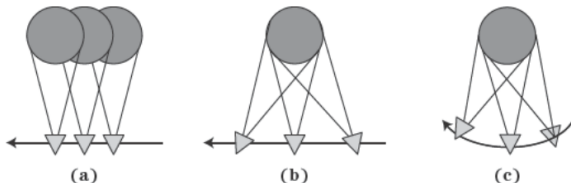


FIGURE 21-8 ■ Three collection options for achieving integration angle are (a) stripmap, (b) spotlight with mechanical or electronic steering of the real antenna beam onto a scene of interest, or (c) spotlight realized by flying a circle about the scene of interest.

Both the flightpath and the direction of the antenna can affect the total illumination of the scene.

Integration angle

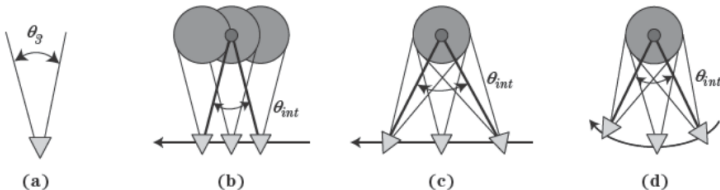


FIGURE 21-9 ■ Given the beamwidth of a real antenna (a), the stripmap integration angle (b) is limited by the real beamwidth, while spotlighting with linear (c) or circular (d) flight paths can realize integration angles greatly exceeding the real antenna beamwidth.

In the stripmap approach, θ_{int} is limited by the real antenna beam. Spotlight scenarios can greatly improve the integration angle.

The linear path and the circular path give very different signal histories, giving rise to different algorithms.

Cross-range sampling requirement

When using sampling distance d , the phase change between two samples is

$$\Delta\psi = 2\pi \left(\frac{2\delta R}{\lambda} \right) = 4\pi \frac{d \sin(\theta_3/2)}{\lambda}$$

To keep the successive phase changes $|\Delta\psi| < \pi$ requires

$$d < \frac{\lambda}{4 \sin(\theta_3/2)} \approx \frac{\lambda}{2\theta_3} \approx \frac{\Delta CR}{2}$$

For wideband pulses, use the minimum wavelength λ_{\min} .

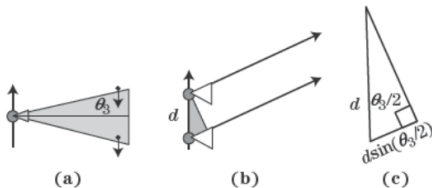


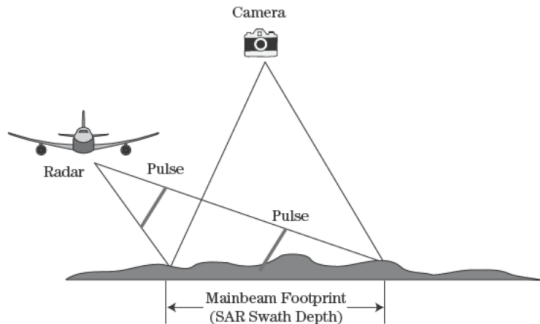
FIGURE 21-10 ■ Geometry for deriving the along-track sampling requirements for stripmap mode, (a) platform moving from bottom to top with antenna pointed to the right, (b) distance between along-track sample points d , and (c) differential range change to a scatterer on the leading edge of the mainbeam.

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Radar vs camera

FIGURE 21-11 ■
Geometry of
overhead
photography of a
scene of interest as
compared to a
typical SAR
collection.



A radar sensor is side-looking, so that the received pulses end up in different range bins for good down-range resolution.

Radar vs camera

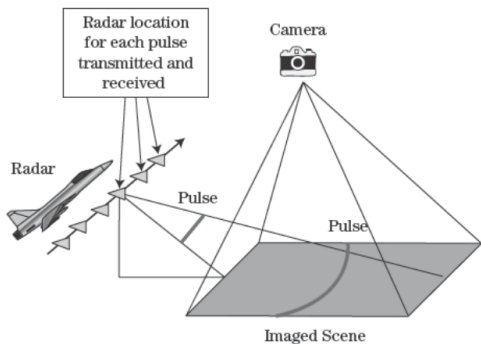
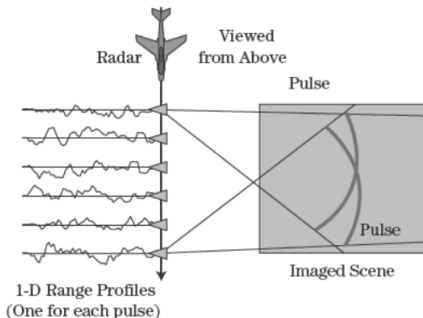


FIGURE 21-12 ■
The camera generates a 2-D image in one take, whereas the radar must collect data pulse-by-pulse over a long dwell.

A radar sensor collects data pulse-by-pulse instead of 2D image-by-2D image.

Stop-and-hop model

FIGURE 21-13 ■
Every pulse illuminates the entire scene of interest and produces a 1-D, complex range profile.



A stream of I/Q data is recorded (phase is important). The radar is moving continuously, but each signal is considered to be recorded in a stationary frame.

Cross-range phase changes

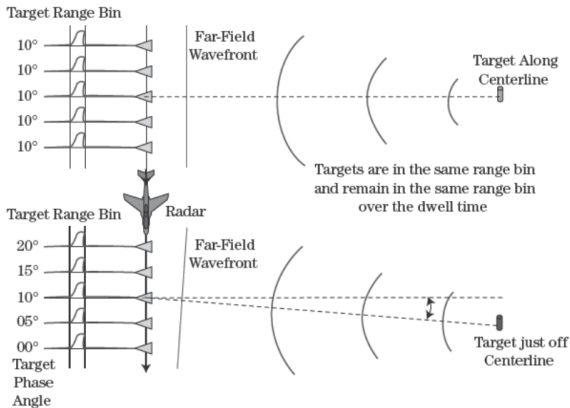


FIGURE 21-14 ■ Small changes in the cross-range location of a scatterer cause a negligible change in the apparent range bin for that return but a significant change in the induced pulse-to-pulse phase progression for that return.

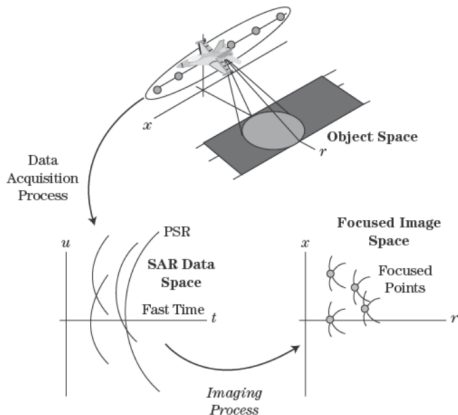
A target off the centerline causes phase changes in cross-range, corresponding to the Doppler shift. The target may still be in one single range bin.

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Stripmap coordinates

FIGURE 21-15 ■
The SAR process
as depicted in
Figure 21-5 with
axes defined.



- ▶ Platform moves linearly, parameterized by position u .
- ▶ Complex reflectivity $g(x, r)$.
- ▶ Complex data $d(u, r)$.
- ▶ Complex image $f(x, r)$.

Slant range

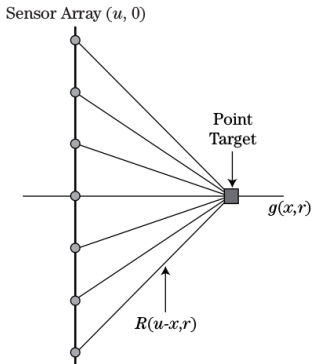


FIGURE 21-16 ■
Variation in slant range to a scatterer at (x, r) due to platform motion.

The slant range $R(u; x, r)$ is given by

$$R(u; x, r) = \sqrt{(u - x)^2 + r^2} \quad \Rightarrow \quad t(u; x, r) = \frac{2}{c} \sqrt{(u - x)^2 + r^2}$$

The translational invariance from the dependence on $u - x$ enables us to consider the generic response at $x = 0$.

Point spread response (PSR)

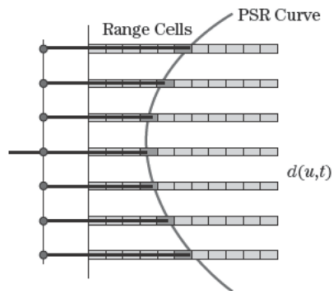


FIGURE 21-17 ■ The range variation to the scatterer over the dwell is described by the PSR. The PSR is manifested in the raw data as a hyperbolic migration of the return from a scatterer across compressed range bins.

The point spread response $t(u; x, r) = \frac{2}{c} \sqrt{(u - x)^2 + r^2}$ describes a hyperbola in the data domain $d(u, t)$, with a vertex depending on (x, r) .

Dependence of PSR on down-range

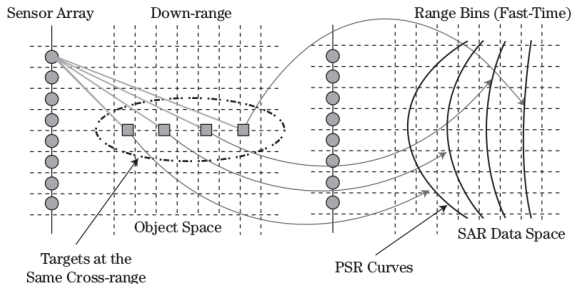


FIGURE 21-18 ■ Scatterers at different cross-track (down-range) locations have different PSR forms. Curvature decreases as range increases.

The PSR is not a simple translation when considered as a function of down-range r :

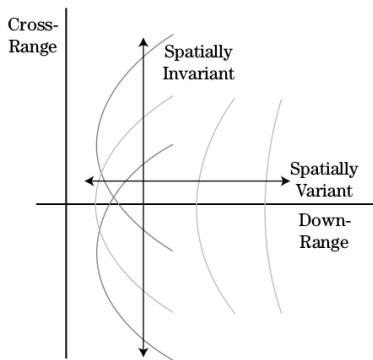
$$t(u; r + \Delta r) = \frac{2}{c} \sqrt{u^2 + (r + \Delta r)^2} = \frac{2}{c} \sqrt{u^2 + r^2 + 2r\Delta r + (\Delta r)^2}$$

$$\neq t(u; r) + \frac{2}{c} \Delta r$$

More bowing of the PSR for scatterers with close range.

Variations of PSR in cross-range and down-range

FIGURE 21-19 ■
Scatterers at different along-track (cross-range) location have the same PSR form, but with an along-track offset.



The shape invariance in cross-range (along-track) can be used for image formation.

Range-Doppler data

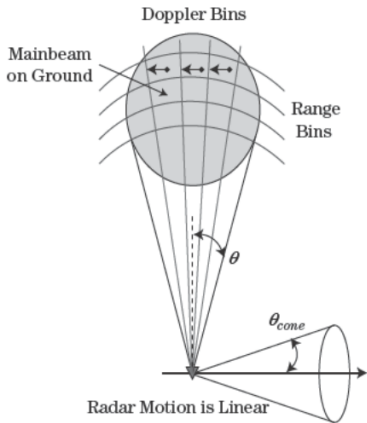


FIGURE 21-20 ■ Arranging ground returns as a function of slant range and Doppler yields a 2-D rendering of the scene of interest.

Range bins are formed as functions of fast time, and Doppler bins are formed as functions of slow time (phase shifts between pulses) due to Doppler shift

$$f_d = \frac{2v}{\lambda} \sin \theta$$

Short dwell time compared to range

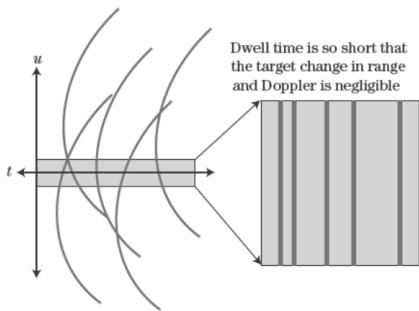


FIGURE 21-21 ■
For short collection times and coarse range bins each hyperbolic PSR is constrained to one range bin over the dwell.

At long range, the PSR splits into two terms:

$$\begin{aligned}t(u; x, r) &= \frac{2}{c} \sqrt{(u-x)^2 + r^2} = \frac{2r}{c} \sqrt{1 + \frac{(u-x)^2}{r^2}} \\ &\approx \frac{2r}{c} \left(1 + \frac{1}{2} \frac{(u-x)^2}{r^2} \right) = \frac{2}{c} \left(r + \frac{u^2}{2r} - \frac{ux}{r} + \frac{x^2}{2r} \right) \\ &= \frac{2}{c} \left(r + \frac{x^2}{2r} \right) + \frac{2}{c} \left(\frac{u^2}{2r} - \frac{ux}{r} \right) = t_0(x, r) + \Delta t(u; x, r)\end{aligned}$$

Cross-range position as spatial frequency

Assuming the u^2 term can be ignored (requires $D_{\text{SAR}} < \sqrt{r\lambda_c}$)

$$\Delta\psi = 2\pi f_c \Delta t(u; x, r) = \frac{4\pi}{\lambda_c} \left(\frac{u^2}{2r} - \frac{ux}{r} \right) \approx -u \frac{4\pi x}{\lambda_c r}$$

shows that the **phase change is linear as a function of platform position u** . The spatial frequency is proportional to x :

$$k_u = \frac{\partial}{\partial u} \Delta\psi = -\frac{4\pi x}{\lambda_c r}$$

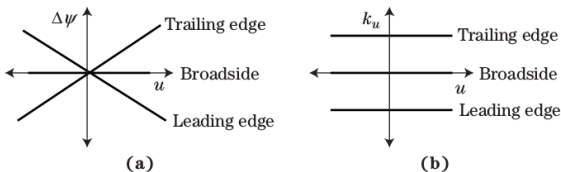


FIGURE 21-22 ■ In the DBS model, (a) phase progressions are approximately linear over the dwell, with slope mapping to cross-range location, and (b) spatial frequencies (Doppler) are approximately constant over the dwell, with frequency mapping to cross-range location.

Doppler Beam Sharpening (DBS) image formation

Fourier transform in platform position (slow time)

$$D(k_u, t) = \int_{-\infty}^{\infty} d(u, t) e^{-jk_u u} du$$

Coordinate mappings:

$$t = \frac{2R}{c} = \frac{2\sqrt{x^2 + r^2}}{c} \approx \frac{2}{c}r$$
$$k_u = -\frac{4\pi x}{\lambda_c r} \approx -\frac{4\pi x}{\lambda_c r_0}$$

Rescale coordinates to find final image:

$$f(x, r) = D \left[k_u \left(-\frac{\lambda_c r_0}{4\pi} \right) \rightarrow x, t \left(\frac{c}{2} \right) \rightarrow r \right]$$

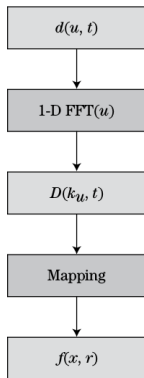


FIGURE 21-23 ■ DBS processing performs a spatial Fourier transform over the pulse history followed by mapping the data into scene coordinates.

DBS example 1

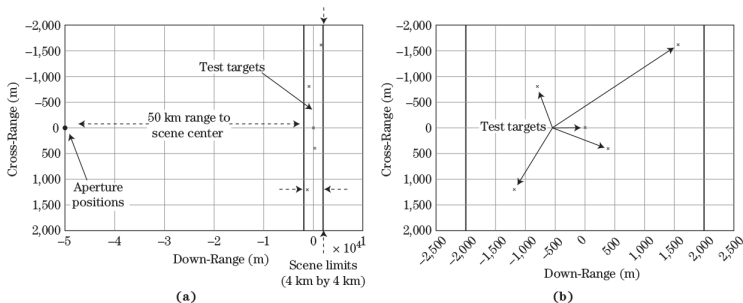


FIGURE 21-24 ■ Simulated DBS geometry emphasizes (a) the overall collection geometry, and (b) point scatterer locations in the scene of interest.

- ▶ Carrier frequency = 9.6 GHz.
- ▶ RF bandwidth = 3 MHz.
- ▶ Integration angle = 0.02° .
- ▶ Resolution = 50 m (both dimensions).
- ▶ Number of along-track sample points (pulses) = 80.
- ▶ Range to scene center = 50 km.

DBS example 1: raw data, absolute value

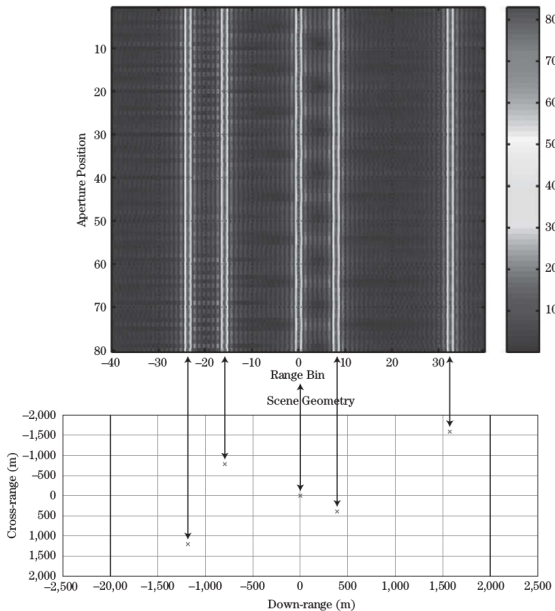
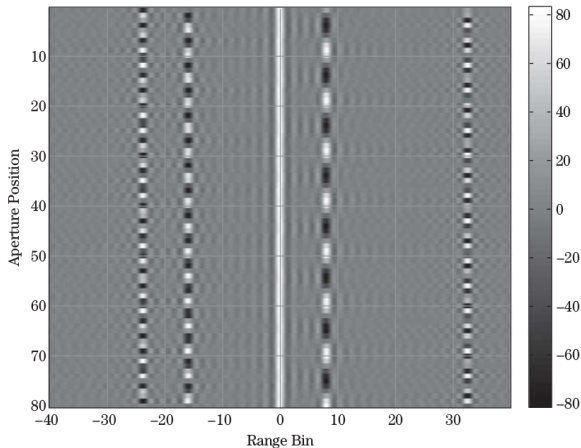


FIGURE 21-25 ■ Mapping of HRR profiles in the raw data to scatterer locations in the scene of interest.

DBS example 1: raw data, real part

FIGURE 21-26 ■
Real part of the HRR profiles, with positive voltages in white and negative voltages in black, indicates phase changes over the dwell and, therefore, Doppler frequency.



Phase shifts due to different cross-range positions of the targets.

DBS example 1: processed data

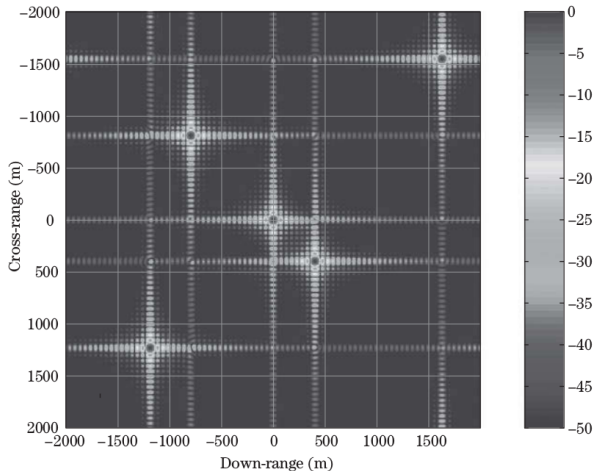


FIGURE 21-27 ■
DBS image of the
simulated point
targets.

The crosses correspond to two-dimensional sinc functions, typical from Fourier transformations of rectangular pulses. Can be suppressed by windowing, at the cost of reduced resolution.

DBS example 2: challenging the algorithm

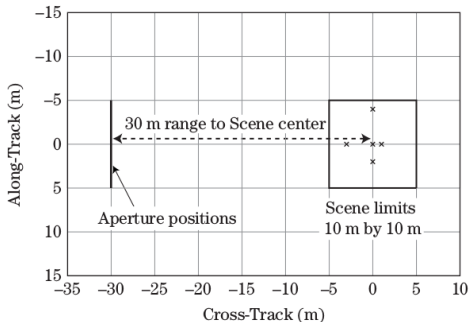
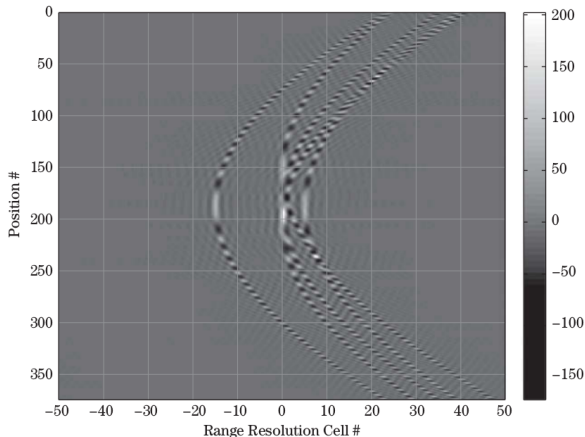


FIGURE 21-28 ■
A challenging,
near-range
collection geometry.

- ▶ Carrier frequency = 600 GHz.
- ▶ RF bandwidth = 750 MHz.
- ▶ Integration angle = 51° .
- ▶ Resolution = 0.2 m (both dimensions).
- ▶ Number of along-track sample points (pulses) = 275.
- ▶ Range to scene center = 30 m.

DBS example 2: raw data, real part

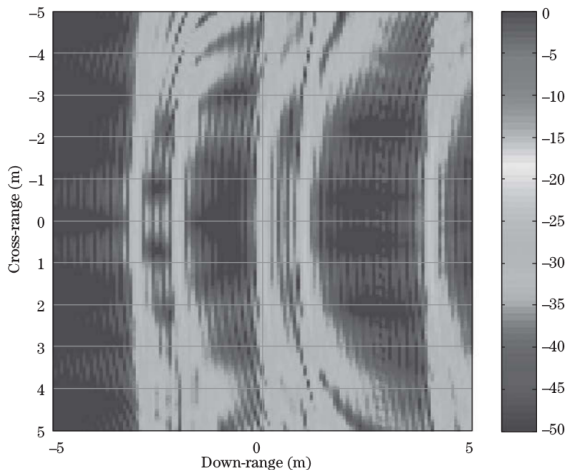
FIGURE 21-29 ■
Real part of the HRR
profiles for the
near-range
collection.
Tremendous range
migration and
higher-order phase
modulations are
apparent in the raw
data.



Significant range migration and higher order phase modulation.

DBS example 2: processed data

FIGURE 21-30 ■
The DBS image of the near-range point targets is distorted and unusable.



Standard DBS processing does not produce a useful image.

Matched filter imaging

More sophisticated imaging methods are necessary when the assumptions of the DBS are not valid. A reference signal d_{REF} is defined by the PSR

$$t(u; x, r) = \frac{2}{c} \sqrt{(u-x)^2 + r^2} \quad \Rightarrow \quad d_{\text{REF}}(u, t; x, r) = \delta_{\text{D}}(t - t(u; x, r))$$

The image can then be computed from the matched filter approach

$$f(x, r) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} d(u, t) d_{\text{REF}}^*(u, t; x, r) du dt$$

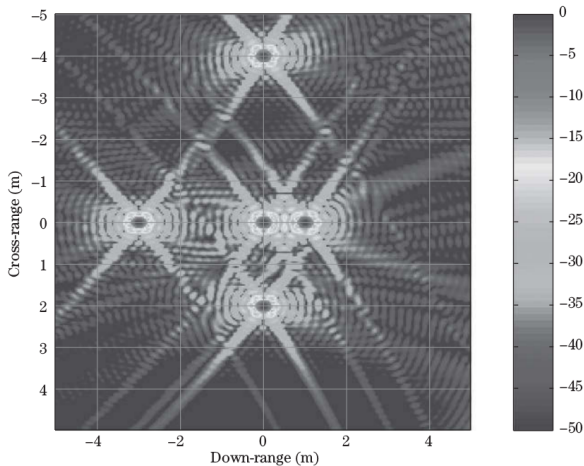
which reduces to the (time-domain) back-projection algorithm

$$f(x, r) = \int_{-\infty}^{\infty} d(u, t(u; x, r)) du$$

This usually entails resampling and interpolating the data to fit the discretized image coordinates (x, r) . Computationally intensive, but less geometry dependent than DBS.

Example 2: matched filter result

FIGURE 21-31 ■
Matched filter image
of the near-range
point targets is
accurate.



The constraints of the DBS assumptions are relaxed.

Image formation survey

TABLE 21-1 = Survey of SAR Image Formation Algorithms

Category	Features	Limitations	Algorithms	Notes
Doppler	Requires only a 1-D FFT after pulse compression	No accounting for migration of scatterers through range bins or phase orders above linear	Doppler beam sharpening	Fourier transform provides the "matched filter"
			DBS with azimuth dechirp	Accounts for QPE by applying a conjugate quadratic modulation to the data prior to DBS
Fourier Matched Filter	Match filtering with the PSR implemented efficiently by multiplication in the frequency domain	Data acquisition must be linear with uniform spacing between along-track samples	Range-Doppler algorithm	Assumes a single PSR is good for the entire scene, so focused swath depth is limited
			Range migration algorithm	Range Doppler algorithm combined with Stolt data interpolation provides error-free imaging over deep swaths
			Chirp scaling algorithm	Exploits direct sampling of linear FM waveforms to realize a computationally efficient approximation to the Stolt interpolation
			Range stacking algorithm	Like RMA, but less computationally efficient
Tomographic	Projection-slice view of data acquisition; efficient image formation follows	Scene size; data must be time delayed and phase corrected prior to image formation	Rectangular formatting algorithm	Analogous to DBS in simplicity; capable of fine resolution, but scene size severely restricted
			Polar formatting algorithm	2-D data interpolation provides quality imaging at fine resolutions over reasonable scene sizes; scene extents ultimately limited by geometric considerations
Inverse	A more holistic, mathematical view of data acquisition and image formation	Demanding processing requirements, and sometimes memory requirements as well	Matched filtering	Employs a product of the data and a reference response; implementable in either time-distance or frequency domains
			Constant aperture backprojection	Delay and sum imaging; same data records used for all pixels
			Constant integration angle backprojection	Like constant aperture backprojection, but different subset of the overall data applied to each pixel to realize a constant integration angle
			Filtered backprojection	Wideband/wide-angle data is preequalized to improve impulse response in the final image
			Fast backprojection	Family of algorithms that garner computational efficiencies by making minor approximations to basic backprojection
			Quad-tree	Fast backprojection using multiresolution topologies
Constrained inversion	SAR imaging treated as a mathematical inverse problem			

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No return areas (NRA): shadows

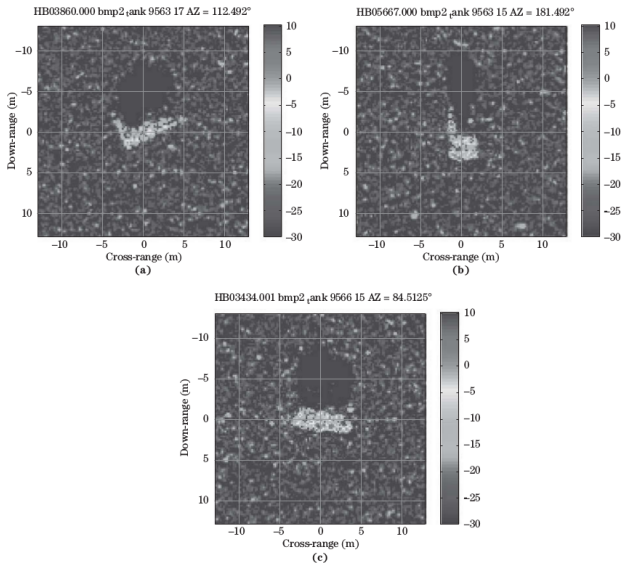


FIGURE 21-32 ■ Three SAR images of an armored vehicle with radar at the bottom of each image and the vehicle (a) at a non-cardinal pose angle, (b) broadside to the radar, and (c) end-on to the radar.

Paved areas, clutter, man-made objects

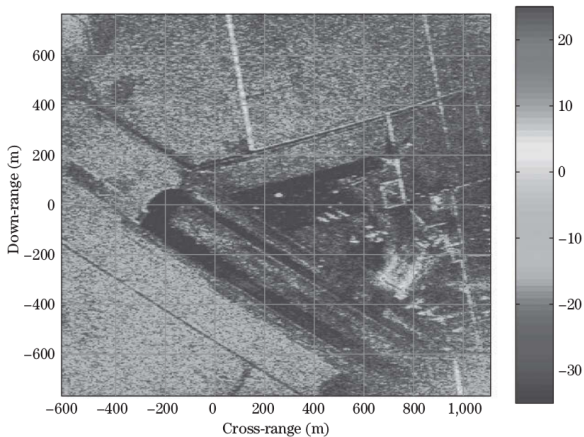


FIGURE 21-33 ■ SAR image of the Mojave Airport showing low-power areas corresponding to runways and tarmac, moderate-power areas off the runways due to natural clutter return, and strong man-made returns near the tarmac. (Data courtesy of Raytheon Corporation.)

Lakes

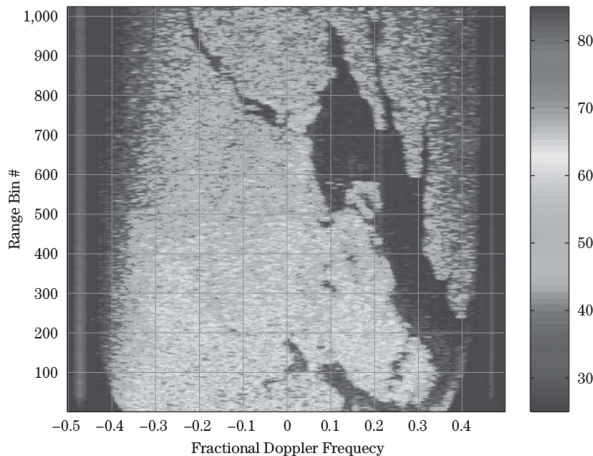
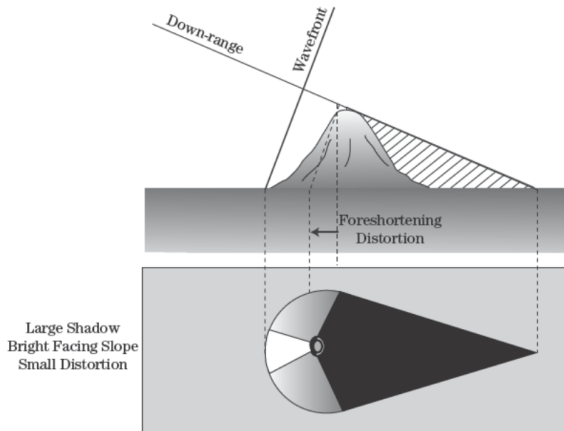


FIGURE 21-34 ■ Coarse-resolution DBS image emphasizing the contrast in power between land and water regions. (Data courtesy of Northrop Grumman Corporation.)

Still water has specular scattering, producing very little return.

Shadow, fore-shortening

FIGURE 21-35 ■
Foreshortening:
Targets having great height will appear collapsed in a SAR imaged collected with a significant grazing angle.

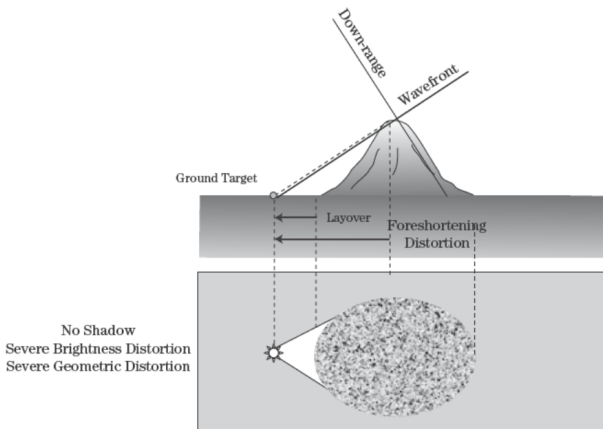


Shadow shapes provide information on target shapes. The increased slope of the terrain makes a stronger return than flat ground, causing a combination of bright and dark spots. Fore-shortening is the range compression of the illuminated hill.

Layover

FIGURE 21-36 ■

Layover: In collections having steep lookdown angles and significant height variations, scatterer order may appear reversed in the down-range direction.



Steep angles cause tall objects to appear earlier in the image than the expected ground return. Some effects can be studied at <http://www.nrcan.gc.ca/node/9325>.

Speckle

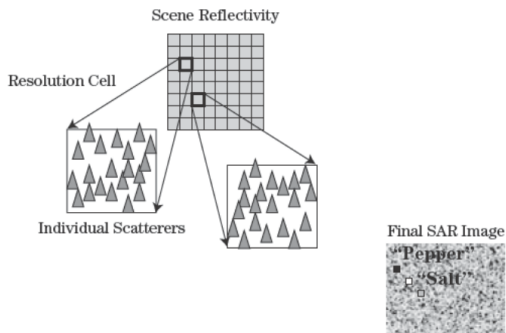


FIGURE 21-37 ■ Speckle is the natural consequence of coherent imaging on a scene having a large number of small, unresolved, randomly placed scatterers.

Constructive and destructive interference from many scatterers produce a speckled pattern, varying from pixel to pixel.

Speckle reduction by multilook and averaging

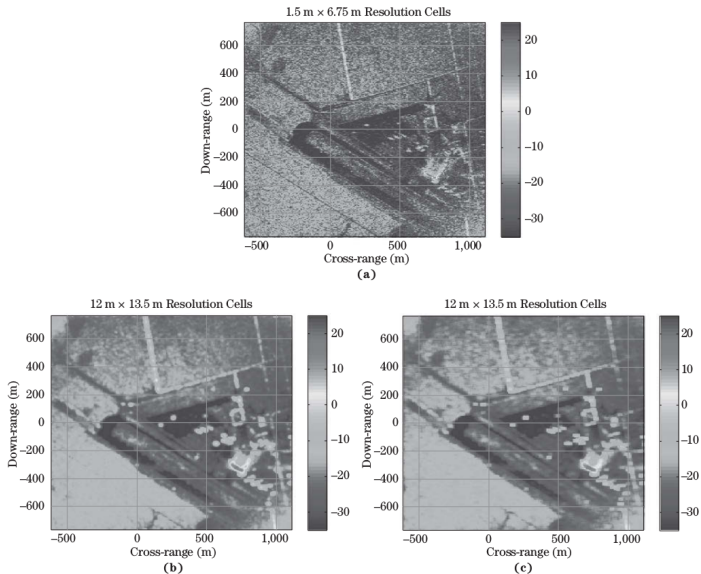


FIGURE 21-38 ■ Speckle reduction on (a) the original image using (b) multi-look processing and (c) image-domain filtering. (Data courtesy of Raytheon Corporation.)

Man-made returns

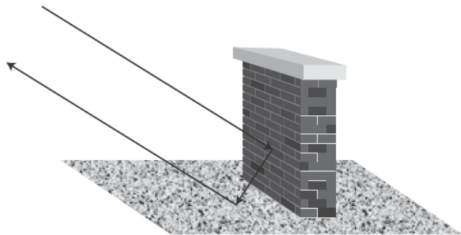


FIGURE 21-39 ■
Geometry for
two-plane
retro-reflection.

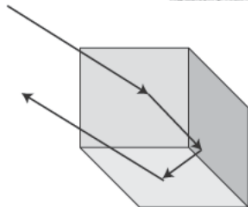


FIGURE 21-40 ■
Geometry for
three-plane
retro-reflection.

Man-made objects tend to have significant structure and finite extent, as compared to lawns, fields, forests etc. Typically metal structures, smooth surfaces, and right-angle corners, can cause strong reflections.

Man-made returns



FIGURE 21-41 ■
Armored vehicle
presents a
complicated and
faceted surface
(see images in
Figure 21-32).

Strong returns from the dihedral formed by ground and flat side. Several minor features in the vehicle may cause retroreflection.

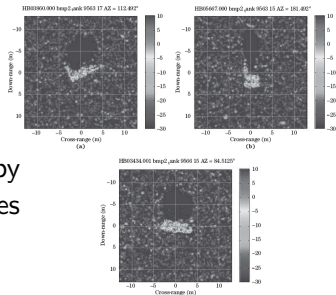


FIGURE 21-32 ■ Three SAR images of an armored vehicle with radar at the bottom of each image and the vehicle (a) at a non-cardinal pose angle, (b) broadside to the radar, and (c) end-on to the radar.

Signal-to-noise and clutter-to-noise ratios

The radar range equation can be rewritten in terms of parameters relevant to SAR imaging.

Signal-to-noise ratio

$$\text{SNR} = \frac{P_{\text{avg}} G_t G_r \lambda^3 \sigma}{2v \sin \theta_{\text{cone}} (4\pi)^3 R^3 N_0 L_s \Delta CR}$$

Clutter-to-noise ratio ($\sigma_c = \sigma^0 A_c = \sigma^0 \Delta CR \Delta R / \cos \delta$)

$$\text{CNR} = \frac{P_{\text{avg}} G_t G_r \lambda^3 \sigma^0 \Delta R}{2v \sin \theta_{\text{cone}} \cos \delta (4\pi)^3 R^3 N_0 L_s}$$

Noise-equivalent backscatter coefficient (σ^0 for which $\text{CNR} = 1$)

$$\sigma_n = \frac{2v \sin \theta_{\text{cone}} \cos \delta (4\pi)^3 R^3 N_0 L_s}{P_{\text{avg}} G_t G_r \lambda^3 \Delta R}$$

This limits the possible contrast between different terrain types.

Outline

- ① General imaging considerations
- ② Resolution relationships and sampling requirements
- ③ Data collection
- ④ Image formation
- ⑤ Image phenomenology
- ⑥ Conclusions**

Conclusions

- ▶ Radar images can be formed using synthetic aperture.
- ▶ Down-range resolution from pulse bandwidth, typically using pulse compression of chirped pulses.
- ▶ Cross-range resolution from along-track sampling.
- ▶ DBS algorithm suitable for long ranges compared to D_{SAR} , uses Fourier transform in cross-range.
- ▶ Matched filter or backprojection is more computationally demanding, but relaxes geometry constraints.
- ▶ Radar images tend to emphasize returns from man-made objects.