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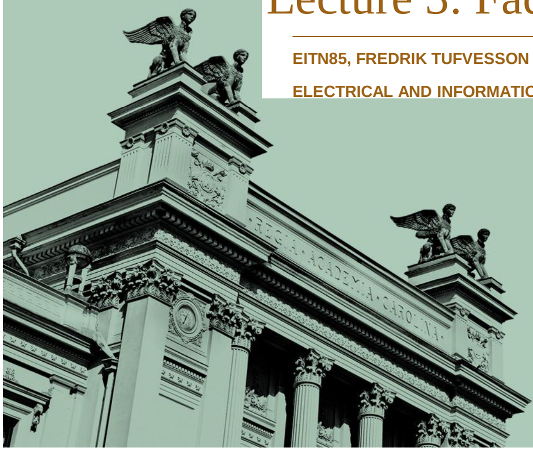
Wireless Communication Channels
Lecture 3: Fading

EITN85, FREDRIK TUFVESSON
ELECTRICAL AND INFORMATION TECHNOLOGY



Fading – Statistical description of the wireless channel

- Why statistical description
- Large scale fading
- Fading margin
- Small scale fading
  - without dominant component
  - with dominant component
- Statistical models
- Measurement example

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Why statistical description?

- Unknown environment
- Complicated environment
- Can not describe everything in detail
- Large fluctuations

- Need a statistical measure since we can not describe every point everywhere

"There is a x% probability that the amplitude/power will be above the level y"

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The WSSUS model Assumptions

A very common wide-band channel model is the WSSUS-model. Roughly speaking it means that the statistical properties remain the same over the considered time (or area)

Recalling that the channel is composed of a number of different contributions (incoming waves), the following is assumed:

The channel is Wide-Sense Stationary (WSS), meaning that the time correlation of the channel is invariant over time. (Contributions with different Doppler frequency are uncorrelated.)

The channel is built up by Uncorrelated Scatterers (US), meaning that the frequency correlation of the channels is invariant over frequency. (Contributions with different delays are uncorrelated.)

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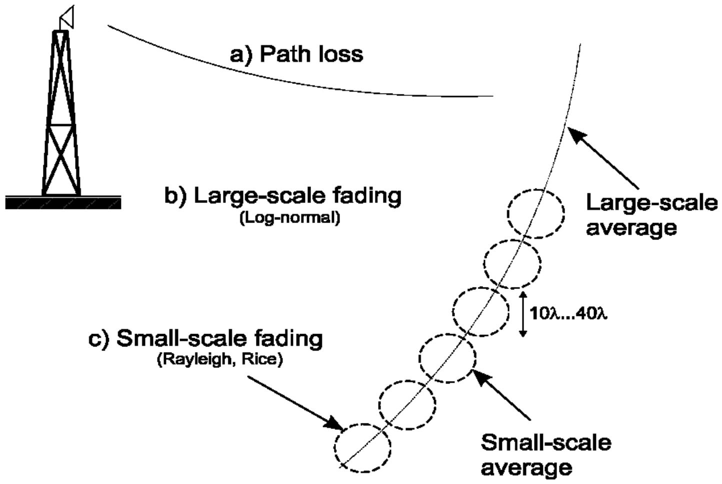
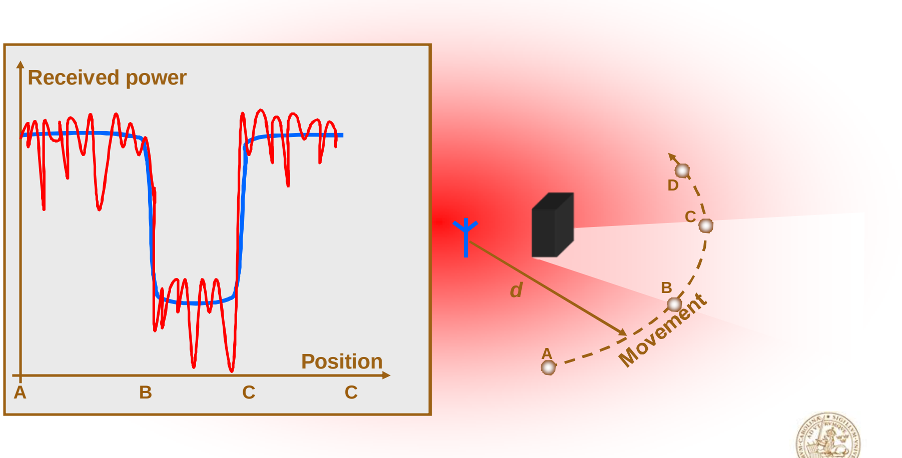
What is large scale and small scale?

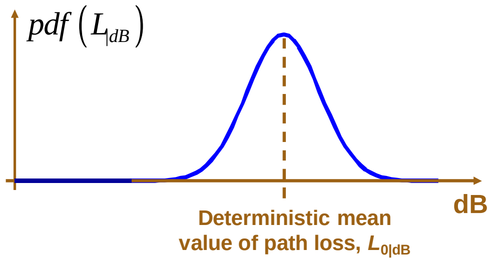


Large-scale fading
Basic principle



Large-scale fading
Log-normal distribution

A normal distribution in the dB domain.



$$pdf\left(L_{|dB}\right) = \frac{1}{\sqrt{2\pi}\sigma_{F|dB}} \exp\left(-\frac{\left(L_{|dB} - L_{0|dB}\right)^2}{2\sigma_{F|dB}^2}\right)$$

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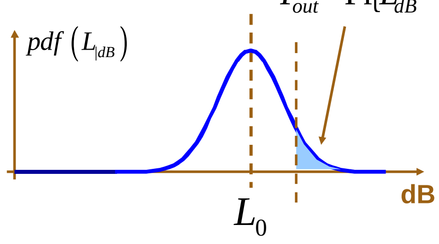
Large-scale fading - Example

What is the probability that the small scale averaged amplitude will be 10 dB below the mean if the large scale fading can be described as log-normal with a standard deviation of 5 dB?

$$P_{out} = \Pr\{L_{dB} \geq L_0 + 10\} = Q\left(\frac{10}{\sigma_{dB}}\right) = Q(2) \approx 0.02$$



What if the standard deviation is 10 dB instead?

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The Q(.)-function
Upper-tail probabilities

x	Q(x)
4.265	0.00001
4.107	0.00002
4.013	0.00003
3.944	0.00004
3.891	0.00005
3.846	0.00006
3.808	0.00007
3.775	0.00008
3.746	0.00009
3.719	0.00010
3.540	0.00020
3.432	0.00030
3.353	0.00040
3.291	0.00050
3.239	0.00060
3.195	0.00070
3.156	0.00080
3.121	0.00090

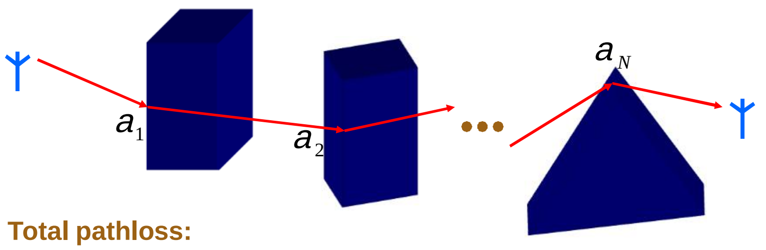
x	Q(x)
3.090	0.00100
2.878	0.00200
2.748	0.00300
2.652	0.00400
2.576	0.00500
2.512	0.00600
2.457	0.00700
2.409	0.00800
2.366	0.00900
2.326	0.01000
2.054	0.02000
1.881	0.03000
1.751	0.04000
1.645	0.05000
1.555	0.06000
1.476	0.07000
1.405	0.08000
1.341	0.09000

x	Q(x)
1.282	0.10000
0.842	0.20000
0.524	0.30000
0.253	0.40000
0.000	0.50000

Large-scale fading, why log-normal?

Many diffraction points adding extra attenuation to the pathloss.

This is, however, only one of several possible explanations.



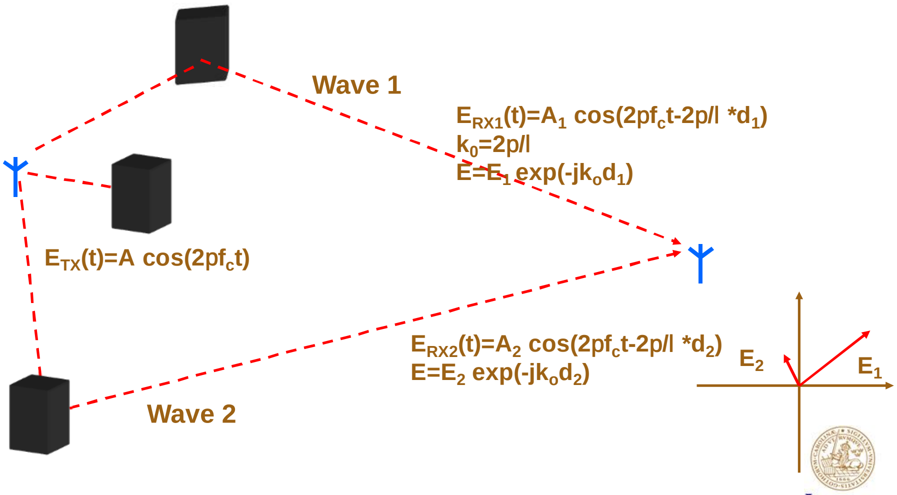
Total pathloss:

$$L_{tot} = L(d) \times a_1 \times a_2 \times \dots \times$$

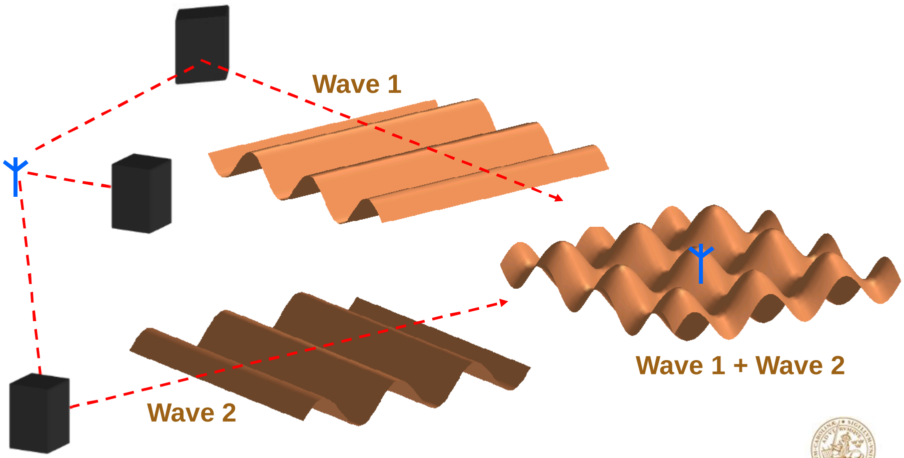
$$L_{tot|dB} = L(d)_{|dB} + a_{1|dB} + a_{2|dB} + \dots + {}_{N|dB}$$

If these are considered random and independent, we should get a normal distribution in the dB domain.

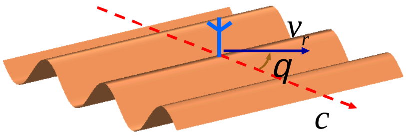
Small-scale fading
Two waves



Small-scale fading
Two waves



Small-scale fading
Doppler shifts



Receiving antenna moves with speed v_r at an angle θ relative to the propagation direction of the incoming wave, which has frequency f_0.

Frequency of received signal:

$$f = f_0 + n$$

where the doppler shift is

$$n = - f_0 \frac{v_r}{c} \cos(q)$$

The maximal Doppler shift is

$$n_{\max} = f_0 \frac{v}{c}$$

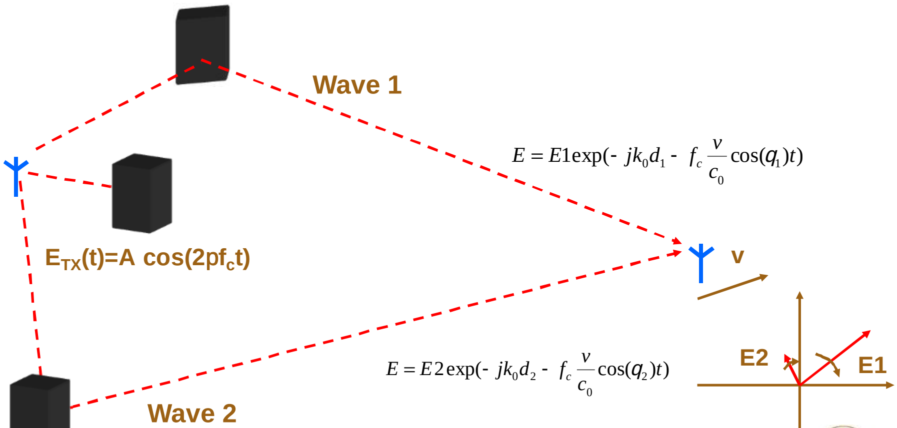
Small-scale fading
Doppler shifts

How large is the maximum Doppler frequency at pedestrian speeds for 5.2 GHz WLAN and at highway speeds using GSM 900?

$$n_{\max} = f_0 \frac{v}{c}$$

- f_0=5.2 10^9 Hz, v=5 km/h, (1.4 m/s) ➡ 24 Hz
- f_0=900 10^6 Hz, v=110 km/h, (30.6 m/s) ➡ 92 Hz

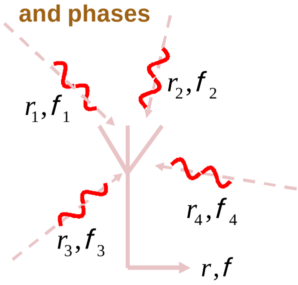
Small-scale fading
Two waves with Doppler



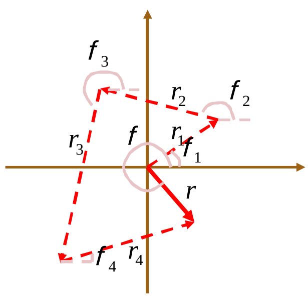
The two components have different Doppler shifts!
The Doppler shifts will cause a random frequency modulation

Small-scale fading
Many incoming waves

Many incoming waves with independent amplitudes and phases



Add them up as phasors



$$r\exp(jf) = r_1\exp(jf_1) + r_2\exp(jf_2) + r_3\exp(jf_3) + r_4\exp(jf_4)$$

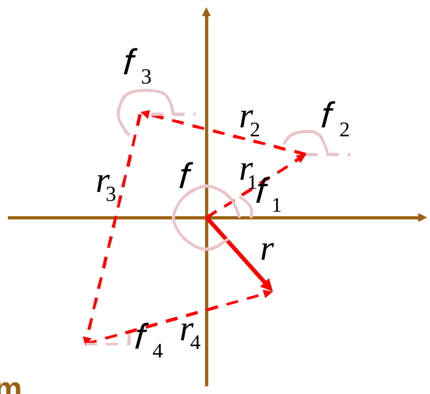
Small-scale fading
Many incoming waves

Re and Im components are sums of many independent equally distributed components

$$\mathrm{Re}(r) \in N(0, \sigma^2)$$

***Re(r)* and *Im(r)* are independent**

The phase of *r* has a uniform distribution



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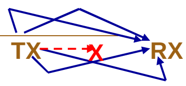
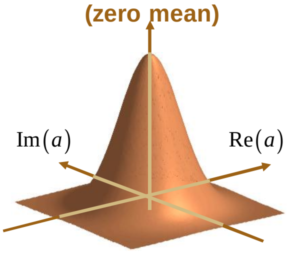
Small-scale fading
Rayleigh fading



No dominant component (non line-of-sight)

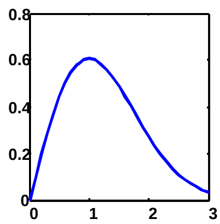
Tap distribution
2D Gaussian
(zero mean)



$$r = |a|$$

Amplitude distribution

Rayleigh



$$pdf(r) = \frac{r}{\sigma^2} \exp\left(-\frac{r^2}{2\sigma^2}\right)$$

No line-of-sight component

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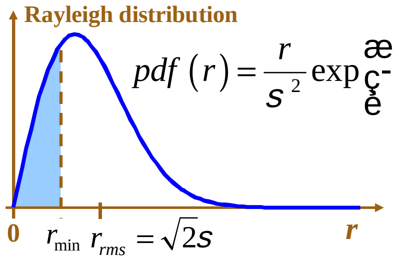
Small-scale fading
Rayleigh fading



$$pdf(r) = \frac{r}{\sigma^2}\exp\left(-\frac{r^2}{2\sigma^2}\right)$$

Probability that the amplitude is below some threshold $r_{\min}$:

$$\Pr(r < r_{\min}) = \int_0^{r_{\min}} pdf(r)\,dr = 1 - \exp\left(-\frac{r_{\min}^2}{r_{rms}^2}\right)$$

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Small-scale fading
Rayleigh fading – outage probability

- What is the probability that we will receive an amplitude 20 dB below the r_{rms}?

$$\Pr(r < r_{\min}) = 1 - \exp\left(-\frac{r_{\min}^2}{r_{rms}^2}\right) = 1 - \exp(-0.01) \approx 0.01$$

- What is the probability that we will receive an amplitude below r_{rms}?

$$\Pr(r < r_{\min}) = 1 - \exp\left(-\frac{r_{\min}^2}{r_{rms}^2}\right) = 1 - \exp(-1) \approx 0.63$$

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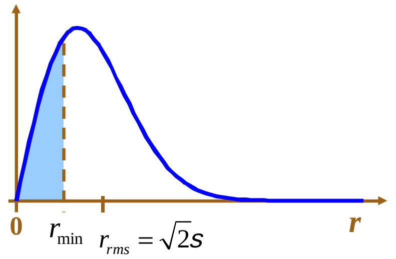
Small-scale fading
Rayleigh fading – fading margin

To Ensure that we in most cases receive enough power we transmit extra power – fading margin

$$M = \frac{r_{rms}^2}{r_{\min}^2}$$

$$M_{|dB} = 10\log_{10}\left(\frac{r_{rms}^2}{r_{\min}^2}\right)$$



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Small-scale fading
Rayleigh fading – fading margin

How many dB fading margin, against Rayleigh fading, do we need to obtain an outage probability of 1%?

$$\Pr\left(r < r_{\min}\right) = 1 - \exp\left(-\frac{r_{\min}^2}{r_{rms}^2}\right) = 1\% = 0.01$$

Some manipulation gives

$$1 - 0.01 = \exp\left(-\frac{r_{\min}^2}{r_{rms}^2}\right) \Rightarrow \ln\left(0.99\right) = -\frac{r_{\min}^2}{r_{rms}^2}$$

$$\Rightarrow \frac{r_{\min}^2}{r_{rms}^2} = -\ln\left(0.99\right) = 0.01 \Rightarrow M = \frac{r_{rms}^2}{r_{\min}^2} = 1/0.01 = 100$$

$$\Rightarrow \boxed{M_{|dB} = 20}$$

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Small-scale fading
Rayleigh fading – signal and interference

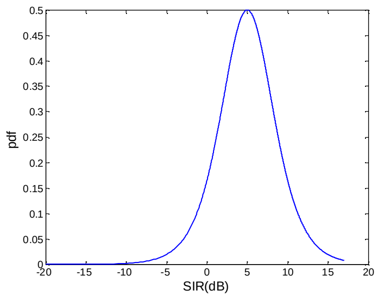
Both the desired signal and the interference undergo fading

For a single user inteferer and Rayleigh fading:

$$pdf_{SIR}(r) = \frac{2\bar{s}^2 r}{(\bar{s}^2 + r^2)^2}$$

$$cdf_{SIR}(r) = 1 - \frac{\bar{s}^2 r}{(\bar{s}^2 + r^2)}$$

where $\bar{s}^2 = \frac{s_2^2}{s_1^2}$ is the mean signal to interference ratio



pdf for 10 dB mean signal to interference ratio

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Small-scale fading
Rayleigh fading – signal and interference

What is the probability that the instantaneous SIR will be below 0 dB if the mean SIR is 10 dB when both the desired signal and the interferer experience Rayleigh fading?

$$\Pr\left(r < r_{\min}\right) = 1 - \frac{\bar{s}^2 r_{\min}}{(\bar{s}^2 + r_{\min}^2)} = 1 - \frac{10}{(10+1)} » 0.09$$

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Small-scale fading
one dominating component

In case of Line-of-Sight (LOS) one component dominates.

- Assume it is aligned with the real axis

$$\mathrm{Re}(r) \in N(A, \sigma^2) \quad \mathrm{Im}(r) \in N(0, \sigma^2)$$

- The recieved amplitude has now a Ricean distribution instead of a Rayleigh
  - The fluctuations are smaller
  - The phase is dominated by the LOS component
  - In real cases the mean propagation loss is often smaller due to the LOS
- The ratio between the power of the LOS component and the diffuse components is called Ricean K-factor

$$k = \frac{\text{Power in LOS component}}{\text{Power in random components}} = \frac{A^2}{2\sigma^2}$$

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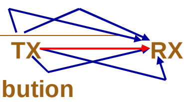
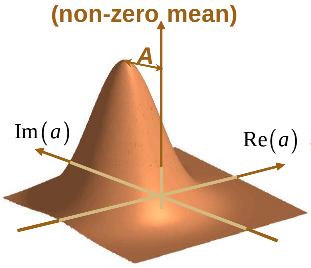
Small-scale fading
Rice fading



A dominant component (line of sight)

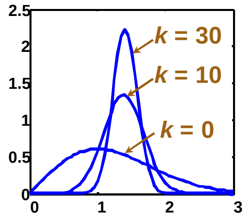
Tap distribution 2D Gaussian (non-zero mean)



$r = |a|$

Amplitude distribution

Rice



$$pdf(r) = \frac{r}{\sigma^2} \exp\left(-\frac{r^2 + A^2}{2\sigma^2}\right) I_0\left(\frac{rA}{\sigma^2}\right)$$

Line-of-sight (LOS) component with amplitude *A*.

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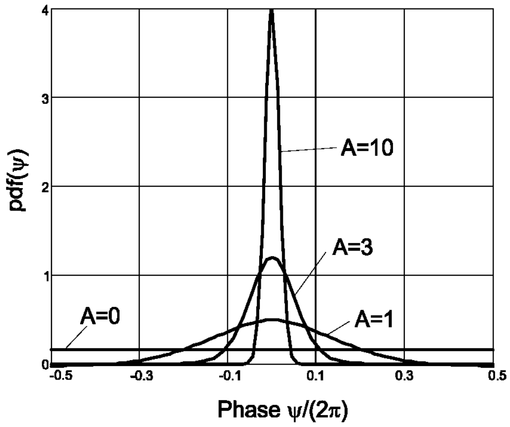
Small-scale fading
Rice fading, phase distribution

The distribution of the phase is dependent on the K-factor



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Small-scale fading
Nakagami distribution

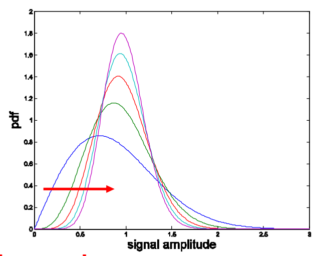
In many cases the received signal can not be described as a pure LOS + diffuse components

The Nakagami distribution is often used in such cases

$$pdf(r) = \frac{1}{\Gamma(m)}\left(\frac{m}{\sigma^2}\right)^m r^{2m-1} \exp\left(-\frac{m}{2\sigma^2}r^2\right)$$

where $\Gamma(m)$ is the Gamma function

$$m = \left(\frac{2\sigma^2}{r^2 - 2\sigma^2}\right)^2$$



increasing m
{1 2 3 4 5}

with *m* it is possible to adjust the dominating power

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Both small-scale and large-scale fading

Large-scale fading - lognormal fading gives a certain mean
Small scale fading – Rayleigh distributed given a certain mean
The two fading processes adds up in a dB-scale

Suzuki distribution:

$$pdf(r)=\int_0^\infty \frac{\pi r}{2\sigma^2} e^{-\frac{\pi r^2}{4\sigma^2}} \frac{20}{\ln(10)} \frac{1}{\sigma\sigma_F\sqrt{2\pi}} e^{-\frac{20\log(\sigma)-\mu}{2\sigma_F^2}}$$

log-normal mean

log-normal std

small-scale std for complex components

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Both small-scale and large-scale fading

An alternative is to add fading terms (fading margins) independently

- Pessimistic approach, but used quite often
- The communication link can be OK if there is a fading dip for the small scale fading but the log-normal fading have a rather small attenuation.

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Some special cases

Rayleigh fading

$$pdf(r) = \frac{r}{\sigma^2}\exp\left(-\frac{r^2}{2\sigma^2}\right)$$

Rice fading, *K*=0

$$pdf(r) = \frac{r}{\sigma^2}\exp\left(-\frac{r^2+A^2}{2\sigma^2}\right)I_0\left(-\frac{rA}{\sigma^2}\right) = \frac{r}{\sigma^2}\exp\left(-\frac{r^2}{2\sigma^2}\right)$$

Rice fading with K=0 becomes Rayleigh

Nakagami, *m*=1

$$pdf(r) = \frac{1}{\Gamma(m)}\left(\frac{m}{\sigma^2}\right)^m r^{2m-1}\exp\left(-\frac{m}{2\sigma^2}r^2\right) = \frac{r}{\sigma^2}\exp\left(-\frac{r^2}{2\sigma^2}\right)$$

Nakagami with *m*=1 becomes Rayleigh

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Example, shadowing from people

Two persons communicating with each other using PDAs, signal sometimes blocked by persons moving randomly

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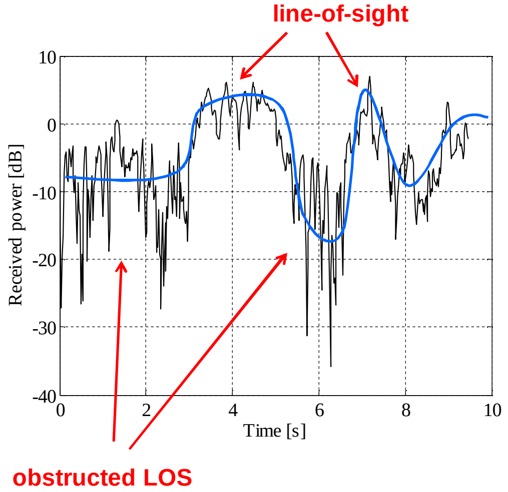
Example, shadowing from people



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